The low-energy spectra of 12 and 16 flavored QCD on the lattice

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March, 17, 2012

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Introduction

Oringin of the electroweak symmetry breaking

- **Technicolor (dynamical symmetry breaking model, alternative to Higgs mechanism in SM)**

Electroweak symmetry breaking

-> techni-fermion Q condensation (scale up of QCD)

(c.f. Chiral symmetry breaking in QCD)

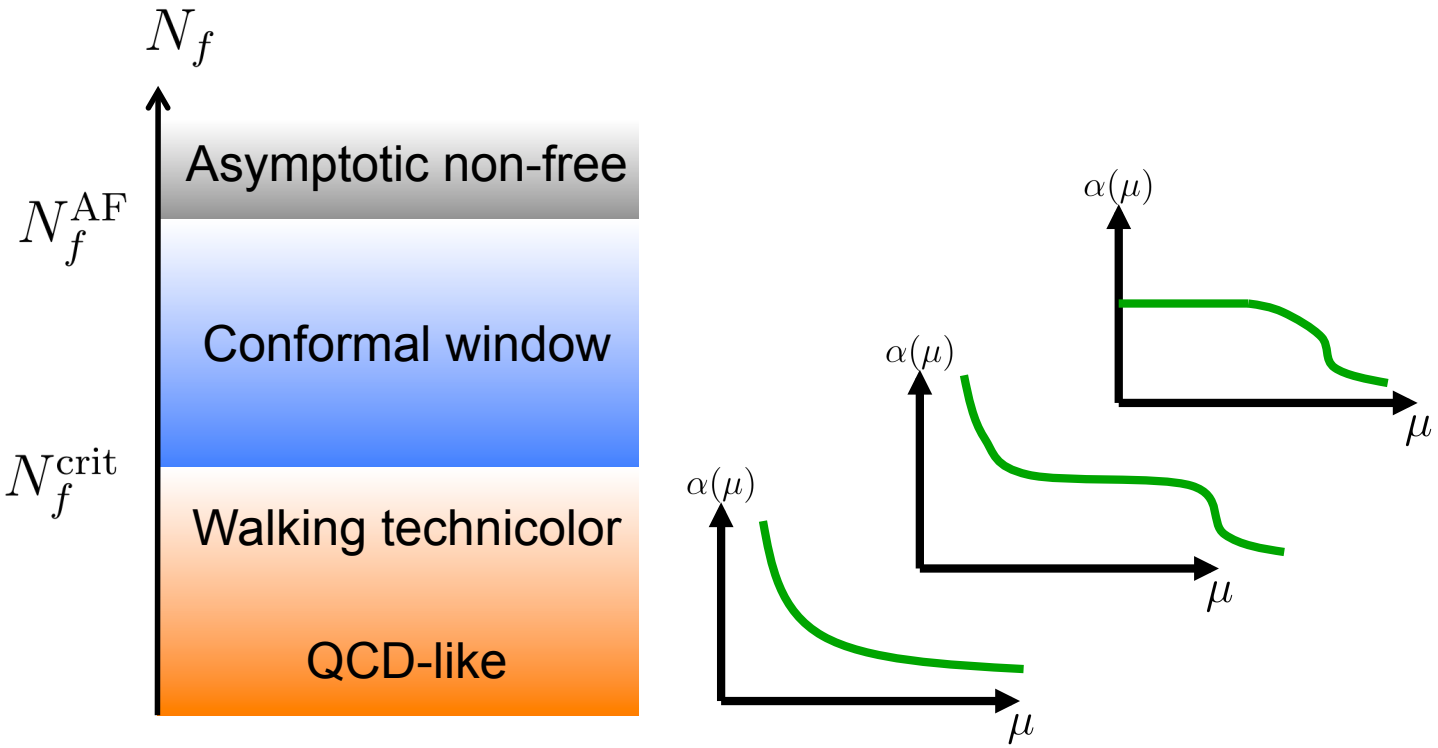
a new strong interaction

Ex. $SU(N)_{TC} \times SU(3)_{color} \times SU(2)_L \times U(1)_Y$

$$\langle \bar{Q}Q \rangle = \Lambda_{TC}^3 \qquad \Lambda_{TC} \sim 250 \text{ GeV}$$

Walking (conformal) behavior -> non-perturbative dynamics

Large flavor QCD: benchmark test of walking dynamics



- Walking technicolor could be realized just below conformal window.
- What the value of the anomalous dimensions?
- Rich hadron structures may be observed.

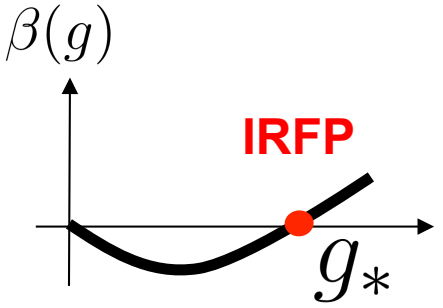
■Possible signals of the conformal behavior (on lattice)

•Measurement of (hadron) mass spectrums & decay constant with finite small bare mass
-> Hyper scaling [Miransky '96]

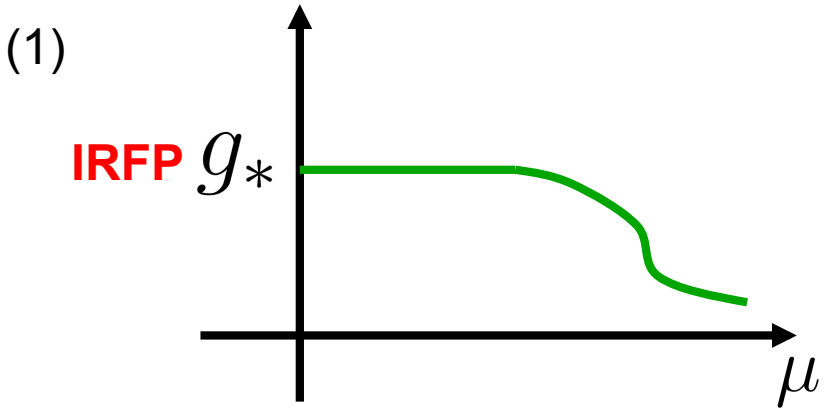
Introducing the bare mass (explicit breaking of conformality)

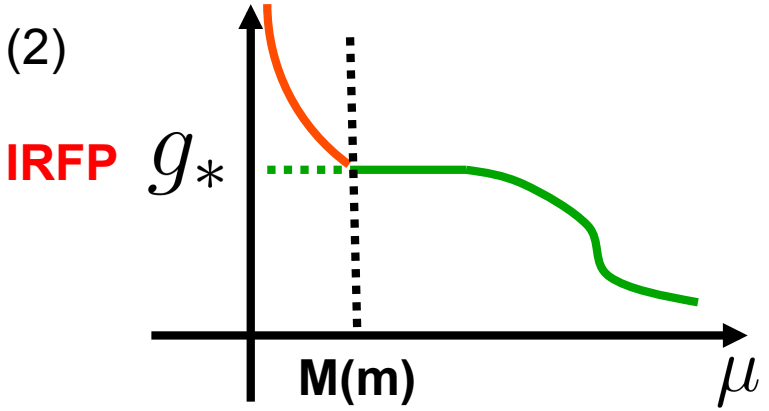
$$\delta\mathcal{L} = -m\bar{q}q$$

If the theory is in the conformal window,



1. m->0, L->∞, CFT is realized. (no mass gap)
2. m≠0, this theory has mass gap at low energy (M) and give rise to asymptotic state (bound state).





Rich hadronic structures

■QCD

- Pion (pNG-boson)
- rho meson (Λ_{QCD})
- Scalar

■technicolor

- Techni-pion
- Techni-rho

} a few hundred GeV - TeV (almost degenerate)

- Techni-dilation, (techni-) glueball

could be lighter than other mesons due to approximate scale inv.

(These could be observed in LHC)

■Conformal

- Universal behavior for above all bound state masses (hyper scaling)

e.g.

$$m_\pi \propto m_f^{\frac{1}{1+\gamma_*}} \quad f_\pi \propto m_f^{\frac{1}{1+\gamma_*}} \qquad \gamma^* : \text{mass anomalous dimension}$$

- Finite volume formula for hyper scaling (Del Debbio et.al.)

$$\text{scaling variable} : x = Lm^{1/(1+\gamma_*)}$$

$$Lm_\pi = G(x) \qquad Lf_\pi = F(x)$$

SU(3) with fundamental fermions

Systematic study of flavor dependence using single setup of the lattice simulation

Our goals:

- Understand the flavor dependence of the theory
- Find the conformal window
- Find the walking regime and investigate the anomalous dimension

Status (lattice):

- Nf=16: likely conformal
- Nf=12: controversial

} This talk

- Nf=8: studies suggests no conformal behavior or walking behavior?
- Nf=4: chiral broken and enhancement of chiral condensate

} Next talk

Lattice gauge theory + numerical simulation : powerful tool

Our work

- use of improved staggered action

  Highly improved staggered quark action [HISQ]

  - to get nearly continuum results from non-zero lattice spacing
  - to reduce flavor violation for good SU(N) chiral symmetry
  - bound to N_f=4 n
- We use MILC version of HISQ action

  use tree level Symanzik gauge action

  no $(ma)^2$ improvement (no interest to heavy quarks)= HISQ/tree

- Measurement of meson spectrum

  in particular Goldstone pion mass and decay constant varying volume

- The results shown are preliminary

KMI computer

- non GPU nodes
  - 148 nodes
  - 2x Xenon 3.3 GHz
  - 24 TFlops (peak)
- GPU nodes
  - 23 nodes
  - 3x Tesla M2050
  - 39 TFlops (peak)

simulation setup

- **Parameters search**

- **SU(3), Nf=12 flavor**

Volume (= L^3 x T)

- **L =18, T=24**
- **L =24, T=32**
- **L =30, T=40**

} L:T =3:4

Bare coupling constant ($\beta = \frac{6}{g^2}$)

- **beta=3.7, 4**

bare quark mass

- **Mf=0.04, 0.05, 0.08, 0.1, 0.12, 0.16, 0.2**

ChPT v.s. conformal hypothesis

CHPT hypothesis

- In usual QCD chiral symmetry is spontaneously broken.

Low energy effective theory (ChPT) is described by the pion (NG-boson).

leading order $m_\pi^2 \propto m_q$ (PCAC relation)

Including higher order $m_\pi^2 = c_1 m_q + c_2 m_q^2 + \cdots$

Pion decay constant $f_\pi = c_0 + c_1 m_q + \cdots$

(c0=0 -> chiral restoration)

Conformal hypothesis

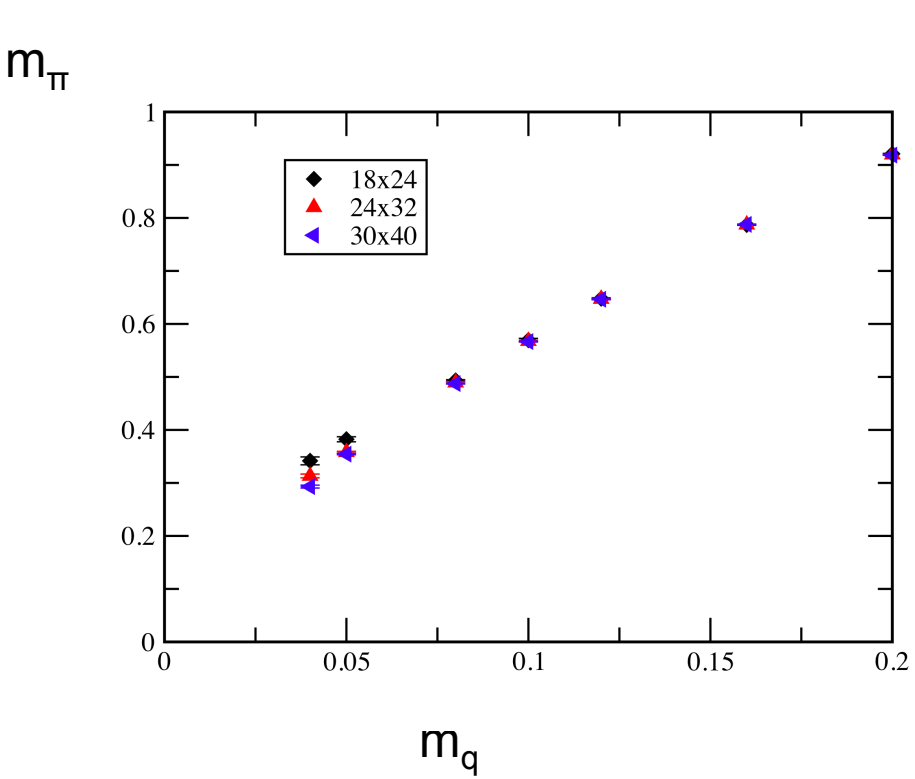
- Universal behavior for all bound state masses.

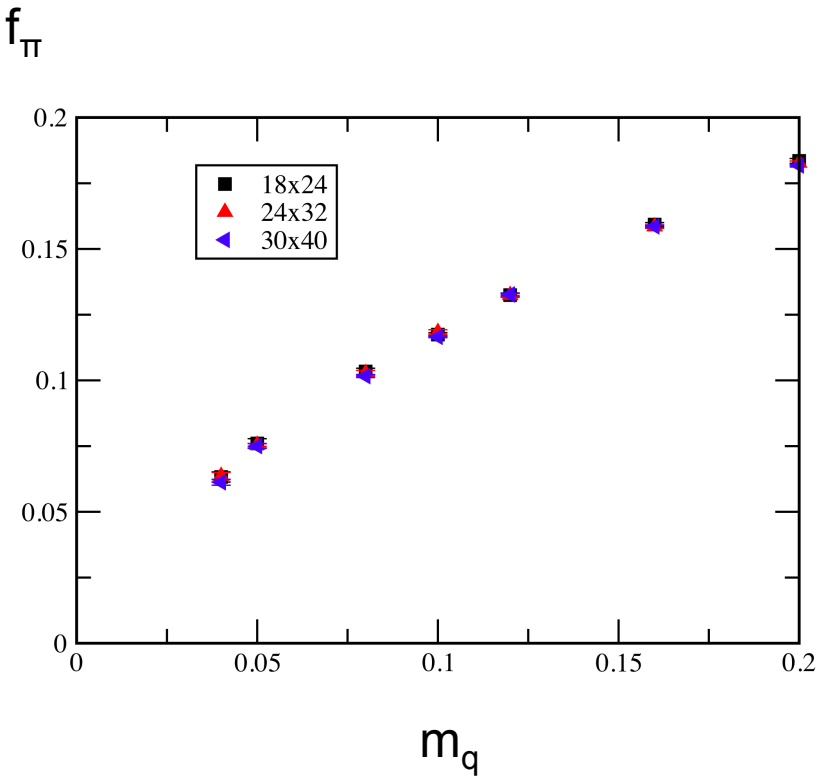
$$m_H \sim (am)^{1/y_m}$$

$$f_\pi \sim (am)^{1/y_m} \qquad (y_m = 1 + \gamma_*)$$

Lattice results Nf=12

■Raw data of Spectra (beta=3.7)





- **analysis**

- **Taking infinite volume limit**
- **CHPT fit (Polynomial fit)**
- **Study in the chiral limit**

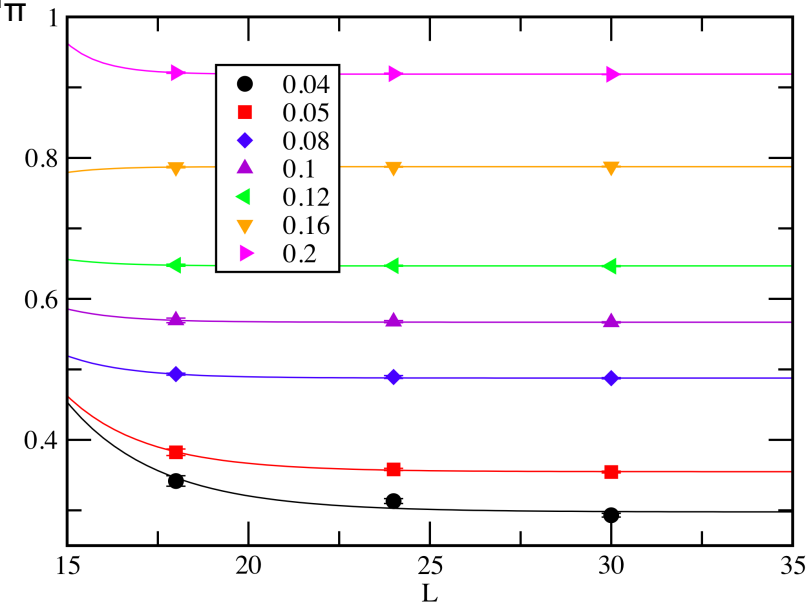
■Infinite volume limit (beta=3.7)

ChPT-like finite volume scaling

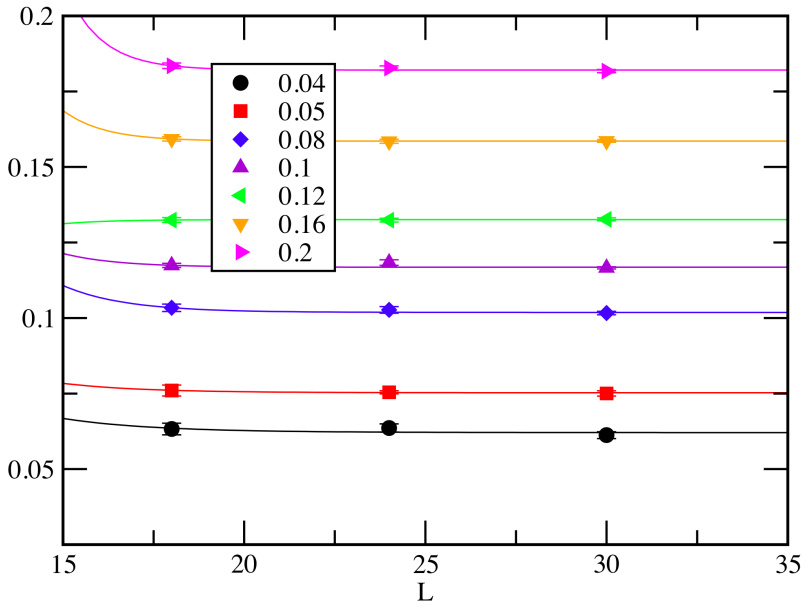
$$M_\pi(L) - M_\pi = c_{M_\pi} \frac{e^{-LM_\pi}}{(LM_\pi)^{3/2}} + \mathcal{O}(e^{-\sqrt{2}LM_\pi})$$

$$F_\pi(L) - F_\pi = c_{F_\pi} \frac{e^{-LM_\pi}}{(LM_\pi)^{3/2}}, + \mathcal{O}(e^{-\sqrt{2}LM_\pi})$$

m_π



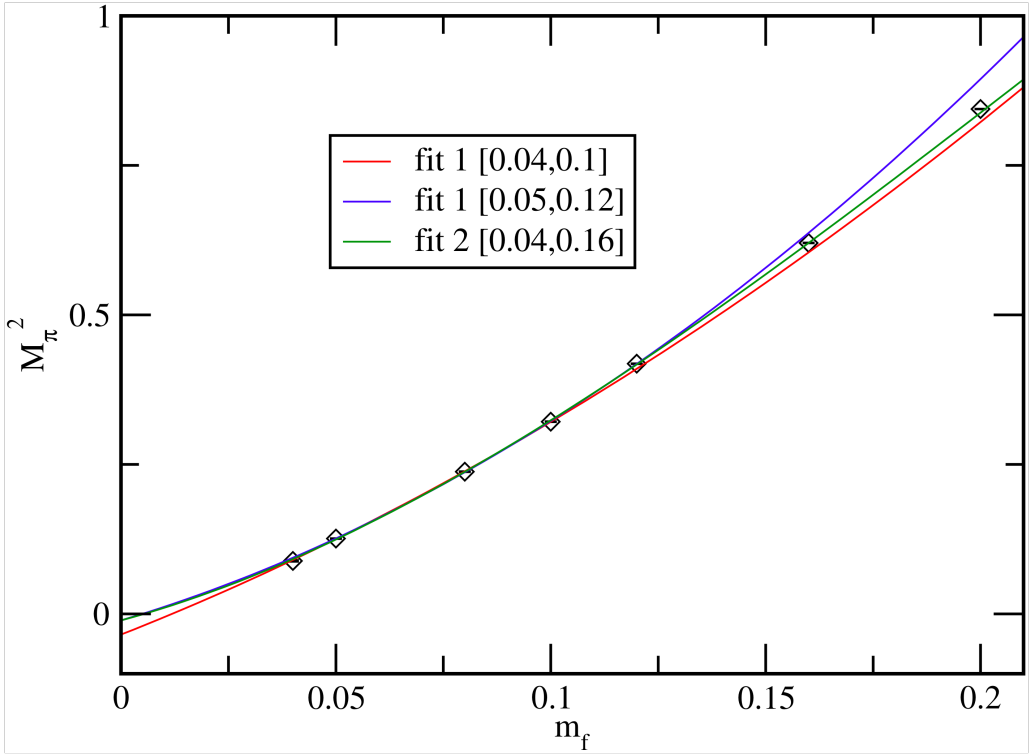
f_π



- **results**

- **reasonably fitted**
- **The value at infinite volume is consistent with one at the largest L(=30)**

■Fit result on pion mass (beta=3.7)



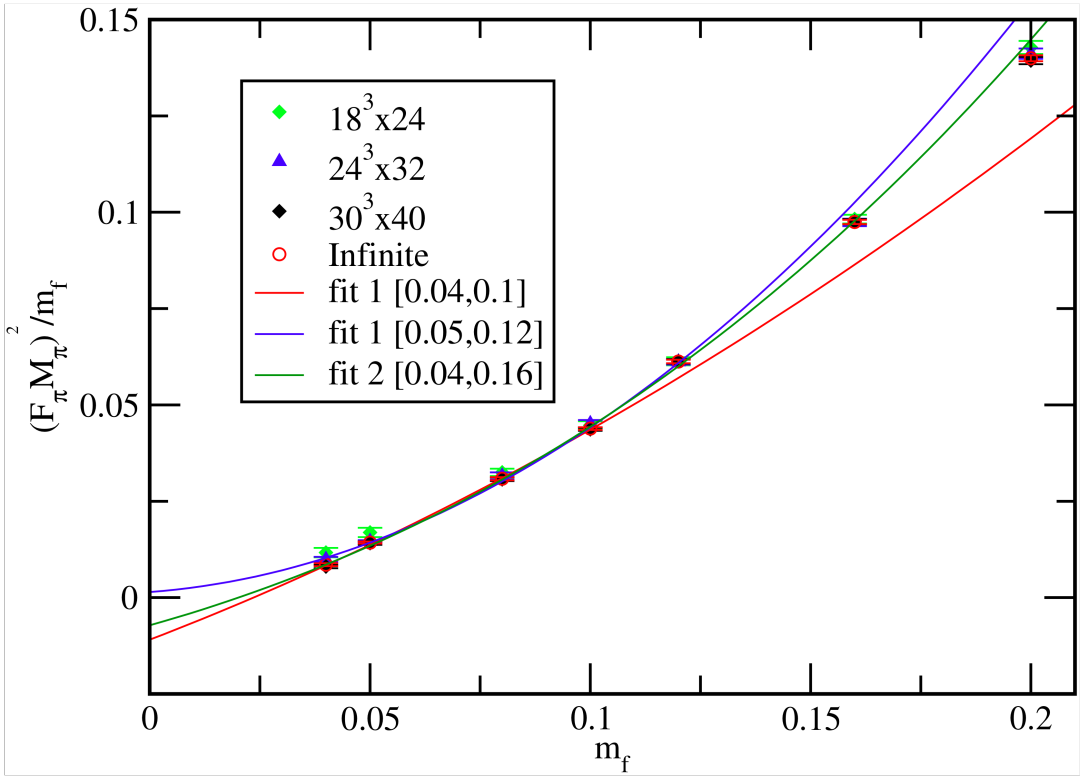
$$M_\pi^2 = \begin{cases} c_0 + c_1 m_f + c_2 m_f^2 & \cdots \text{fit1} \\ c_0 + c_1 m_f + c_2 m_f^2 + c_3 m_f^3 & \cdots \text{fit2} \end{cases}$$

fit range	c_0	c_1	c_2	c_3	χ^2/dof
fit 1 : $[0.04, 0.1]$	-0.0345(33)	2.83(19)	7.2(1.4)	-	3.22
fit 1 : $[0.05, 0.12]$	-0.0109(47)	2.14(12)	11.91(69)	-	4.09
fit 2 : $[0.04, 0.16]$	-0.0109(75)	1.95(27)	16.5(2.9)	-25.0(9.1)	10.0

Fit results

- **Negative constant C0= -0.0345(33)**
- **If we fix c0- > zero Chi^2/dof=3.22-> 17**

■Chiral condensation (beta=3.7)



$$\frac{(F_\pi M_\pi)^2}{m_f} = \begin{cases} c_0 + c_1 m_f + c_2 m_f^2 & \cdots \text{fit1} \\ c_0 + c_1 m_f + c_2 m_f^2 + c_3 m_f^3 & \cdots \text{fit2} \end{cases}$$

fit range	c_0	c_1	c_2	c_3	χ^2/dof
fit 1 : $[0.04 - 0.1]$	-0.0109(29)	0.440(90)	1.05(64)	-	1.98
fit 1 : $[0.05 - 0.12]$	0.0015(28)	0.086(73)	3.42(45)	-	3.58
fit 2 : $[0.04 - 0.16]$	-0.0072(40)	0.32(16)	1.7(1.7)	2.6(5.6)	5.78

GMOR relation

$$-\langle\bar{\psi}\psi\rangle = \frac{(F_\pi M_\pi)^2}{m_f}$$

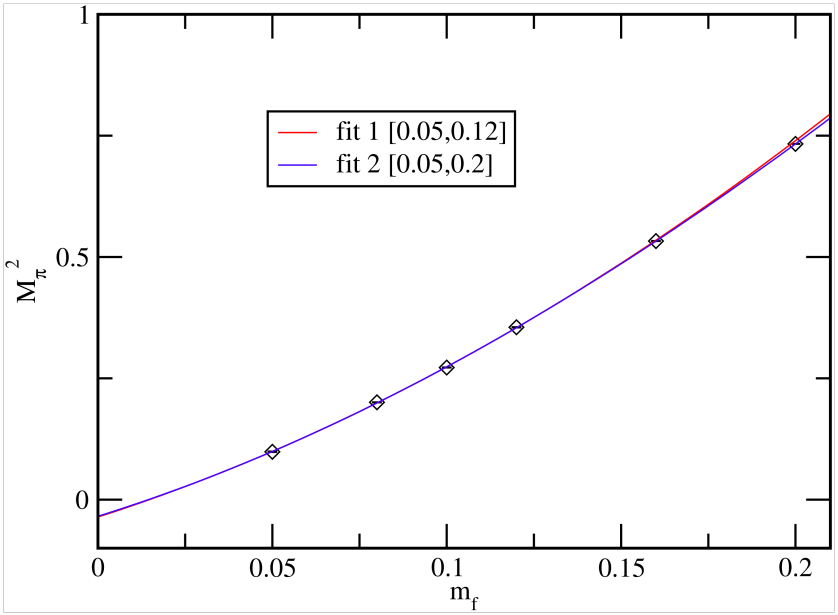
Fit results

- **Negative constant**

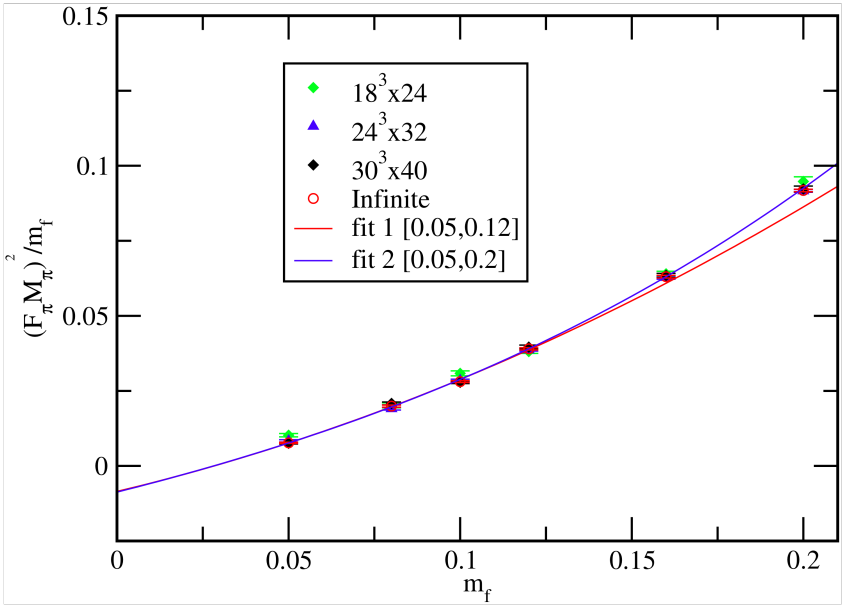
C0= -0.0109 (29)
or consistent with zero

■Results at beta=4

M_π^2



$(F_\pi M_\pi)^2/m_f$



short summary of CHPT fit

- ChPT fit of Mpi^2 and (fpi mpi)^2/mf does not work well.
  - Non-zero constant (negative) for pion mass is obtained
  - Chiral condensation in massless limit is negative or zero consistent
- Our results do not favor ChPT behavior
- **Two analyses for beta=3.7 & 4 give similar results.**

- **We need to investigate smaller masses and larger volumes.**

-> Next we also try to fit with hyper scaling hypothesis

Test of Finite size hyper scaling

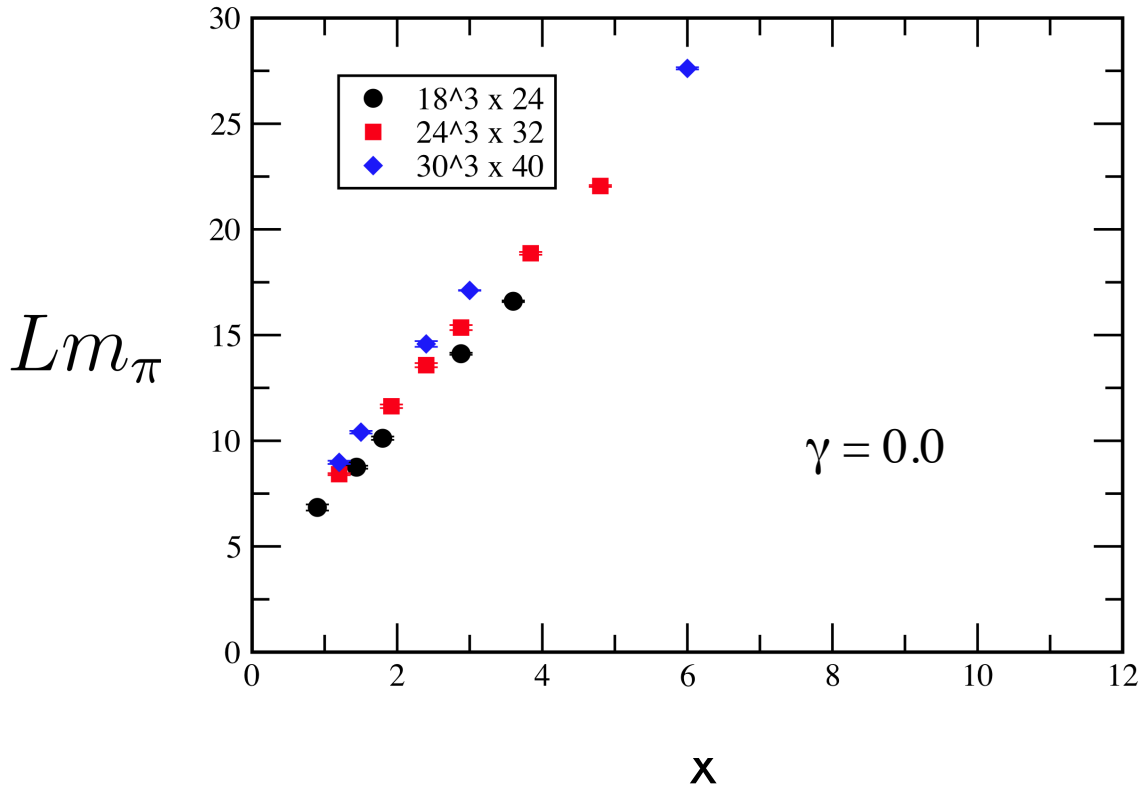
Conformal hypothesis

- Universal behavior for all hadron masses.

- Scaling variable x

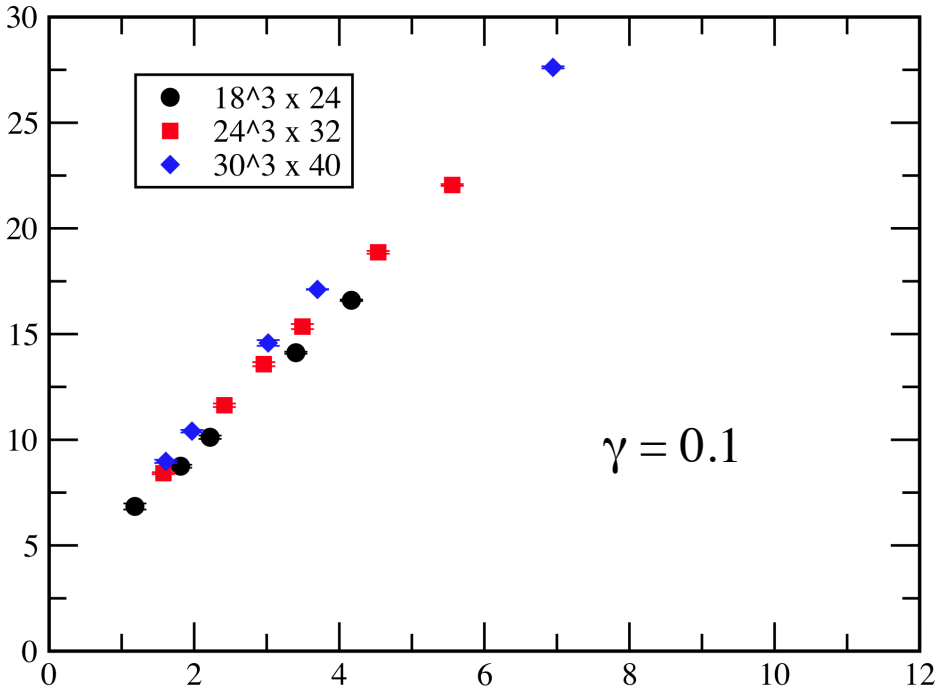
$$LM_\pi = f(x) \qquad x = Lm^{1/(1+\gamma_*)}$$

We test the finite hyper-scaling for our data at L=18, 24, 30.
If the theory is inside the conformal window,
the data should be described by one scaling parameter x.


Lm_π
18^3 x 24
24^3 x 32
30^3 x 40
$\gamma = 0.0$
x
0
5
10
15
20
25
30
0
2
4
6
8
10
12

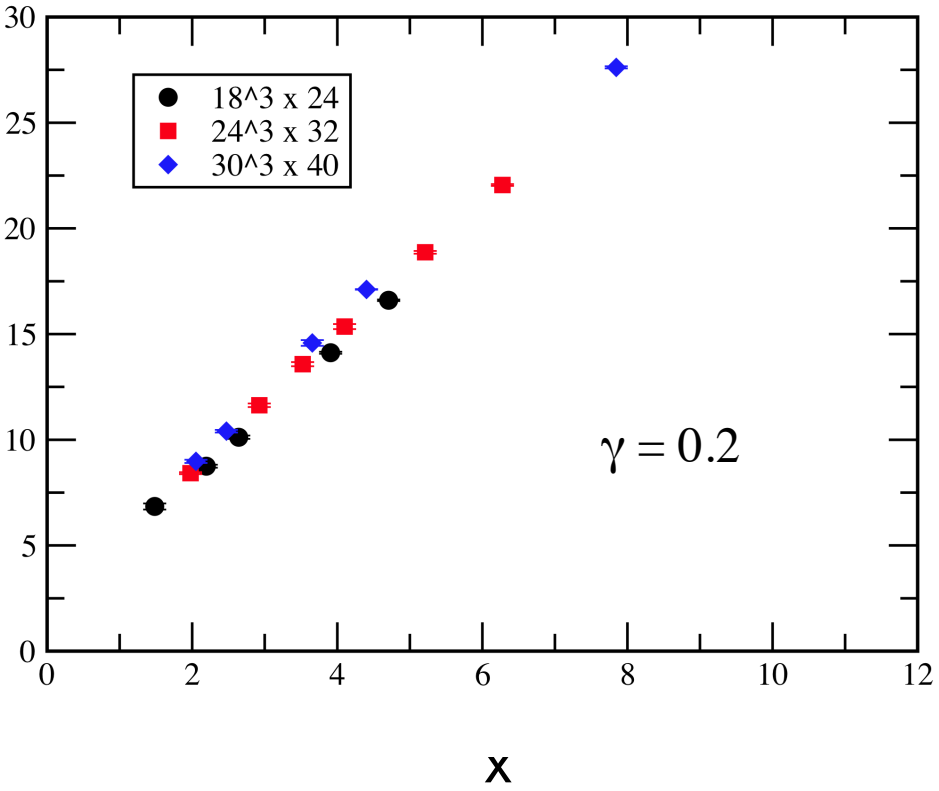
$$x = Lm^{1/(1+\gamma_*)}$$

Lm_π

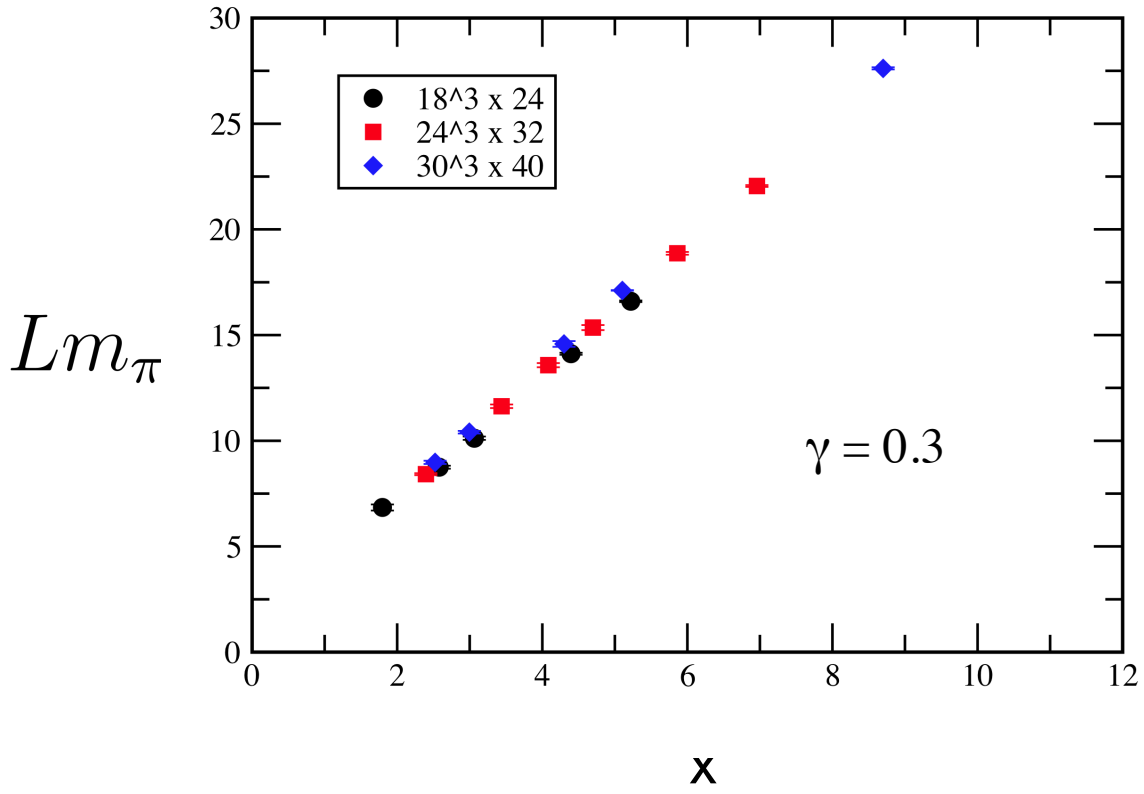


$$x = Lm^{1/(1+\gamma_*)}$$

Lm_π

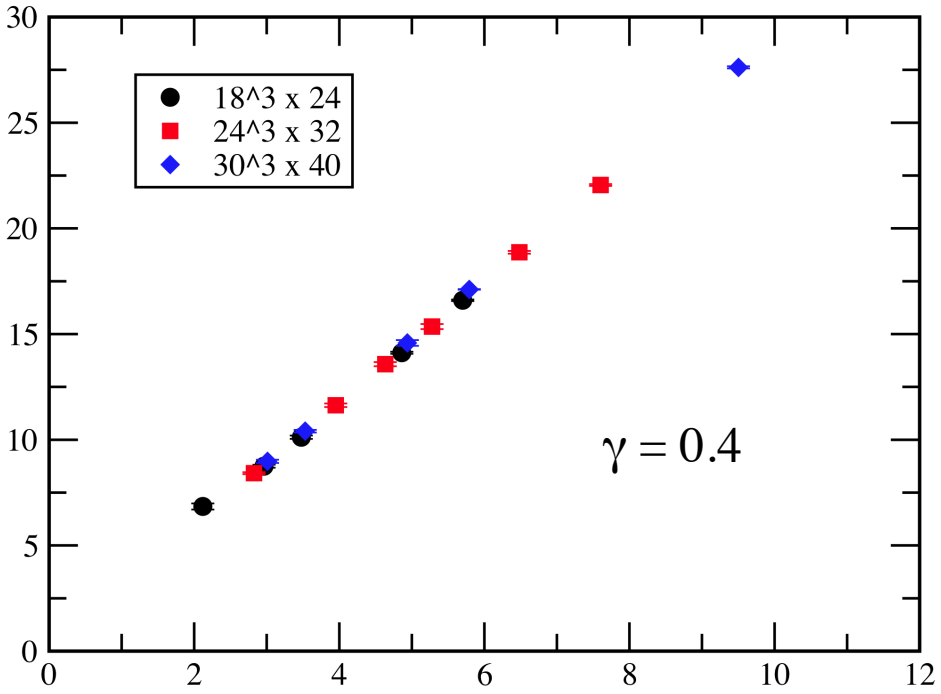


$$x = Lm^{1/(1+\gamma_*)}$$


Lm_π
18^3 x 24
24^3 x 32
30^3 x 40
$\gamma = 0.3$
x
0
2
4
6
8
10
12
0
5
10
15
20
25
30

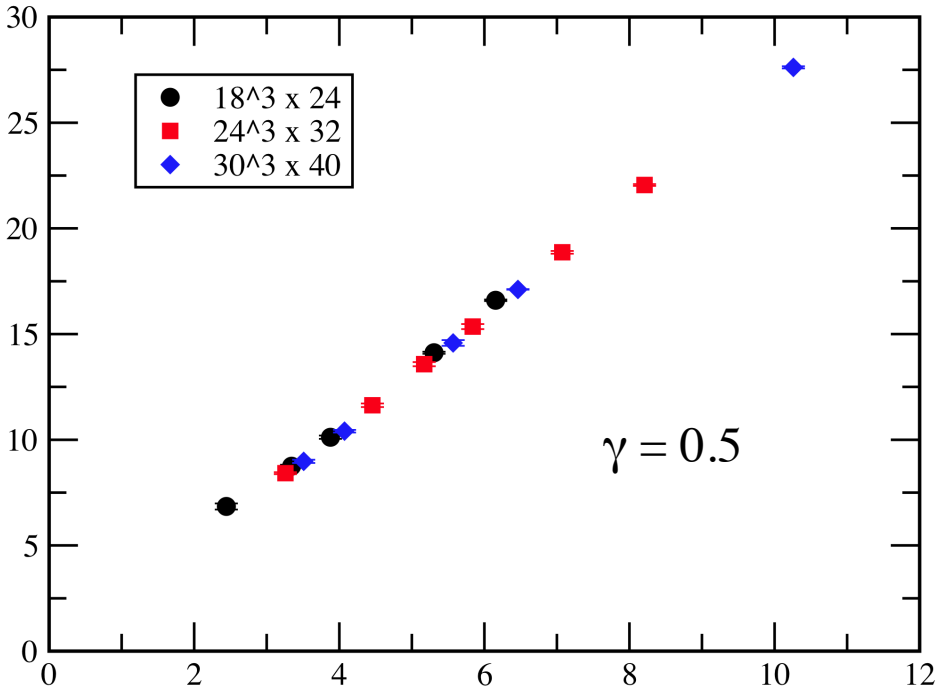
$$x = Lm^{1/(1+\gamma_*)}$$

Lm_π



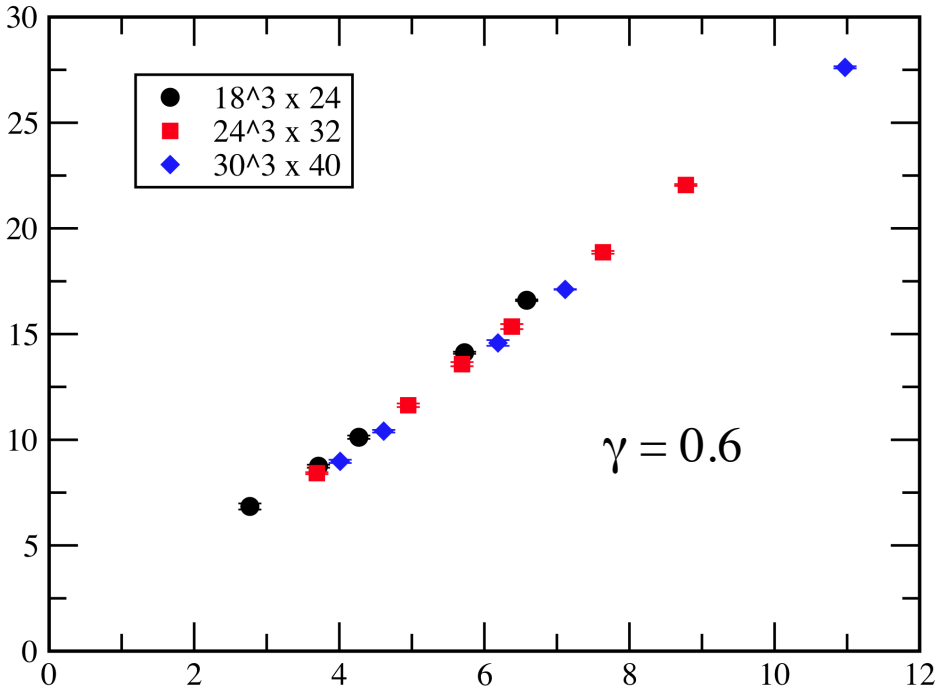
$$x = Lm^{1/(1+\gamma_*)}$$

Lm_π



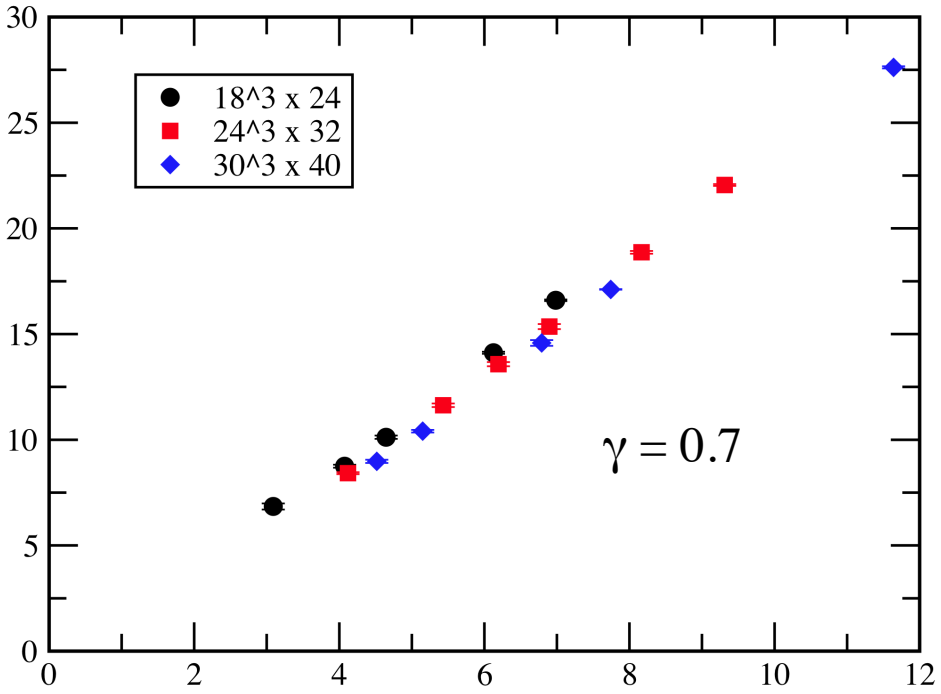
$$x = Lm^{1/(1+\gamma_*)}$$

Lm_π



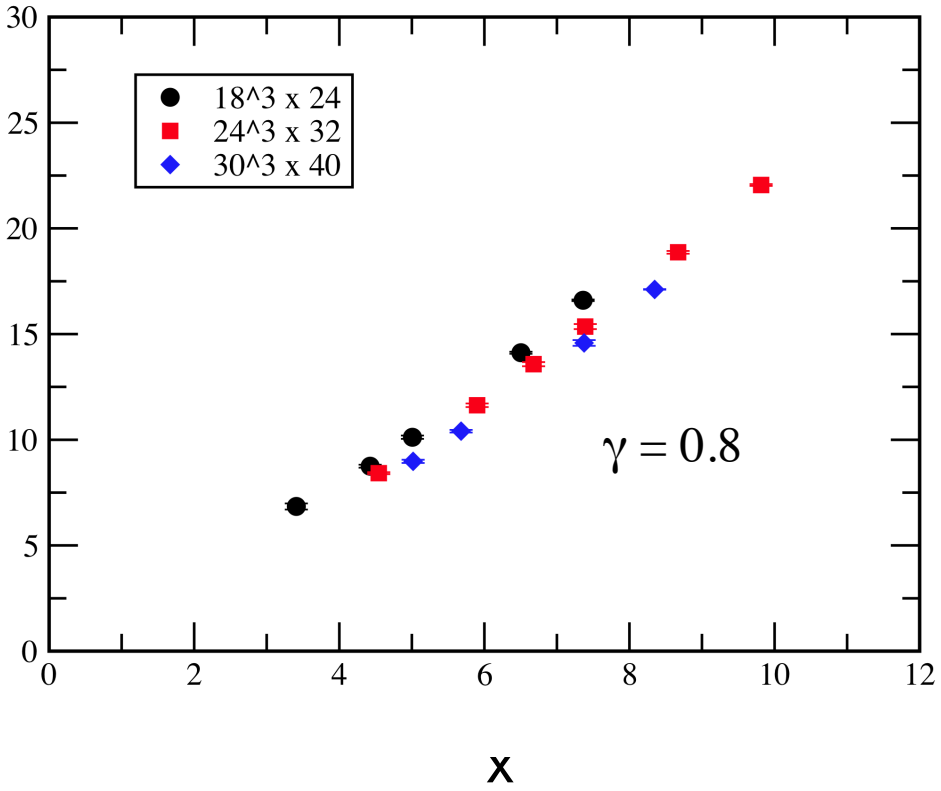
$$x = Lm^{1/(1+\gamma_*)}$$

Lm_π



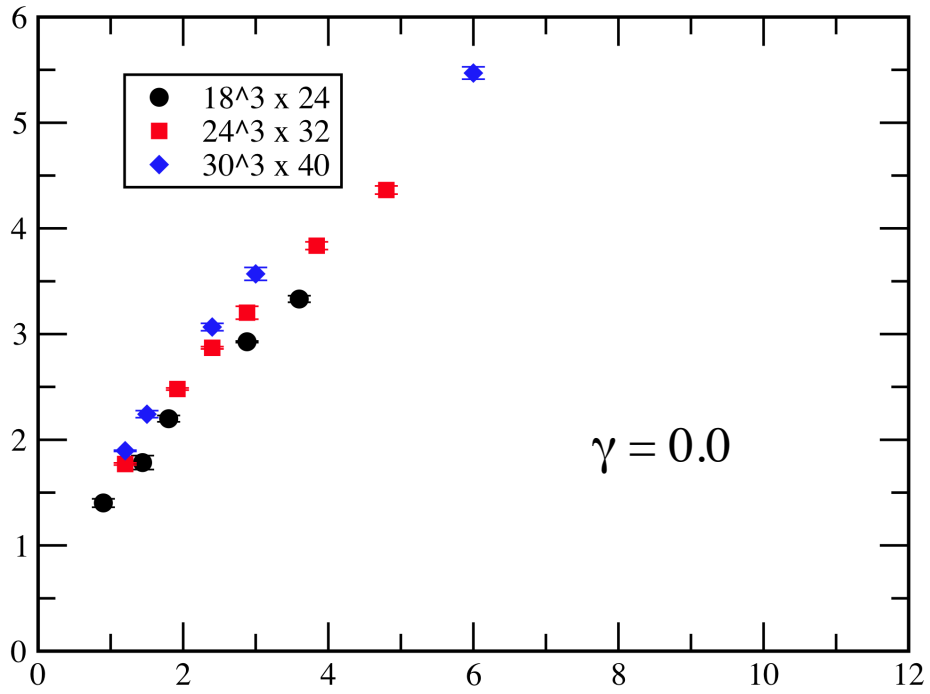
$$x = Lm^{1/(1+\gamma_*)}$$

Lm_π



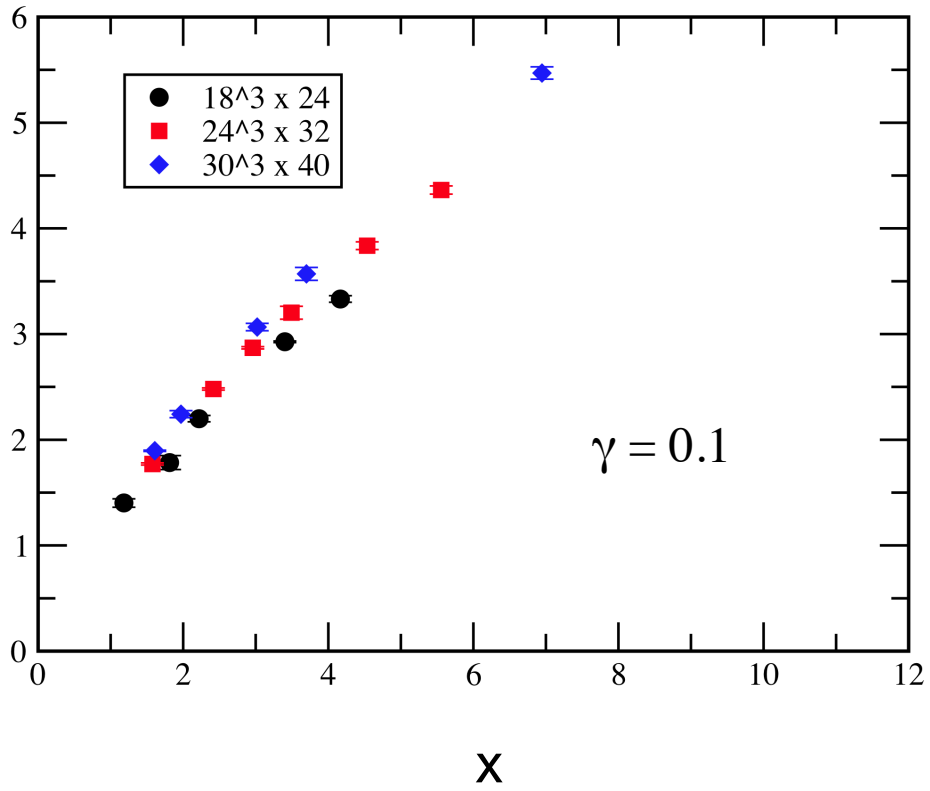
$$x = Lm^{1/(1+\gamma_*)}$$

Lf_π



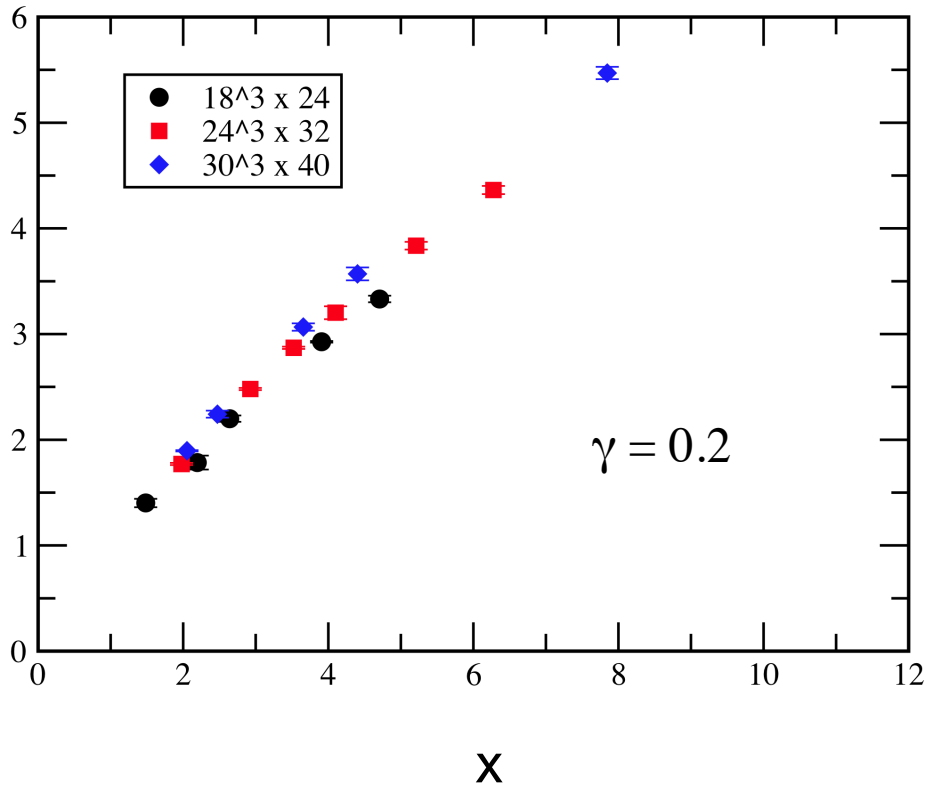
$$x = Lm^{1/(1+\gamma_*)}$$

Lf_π



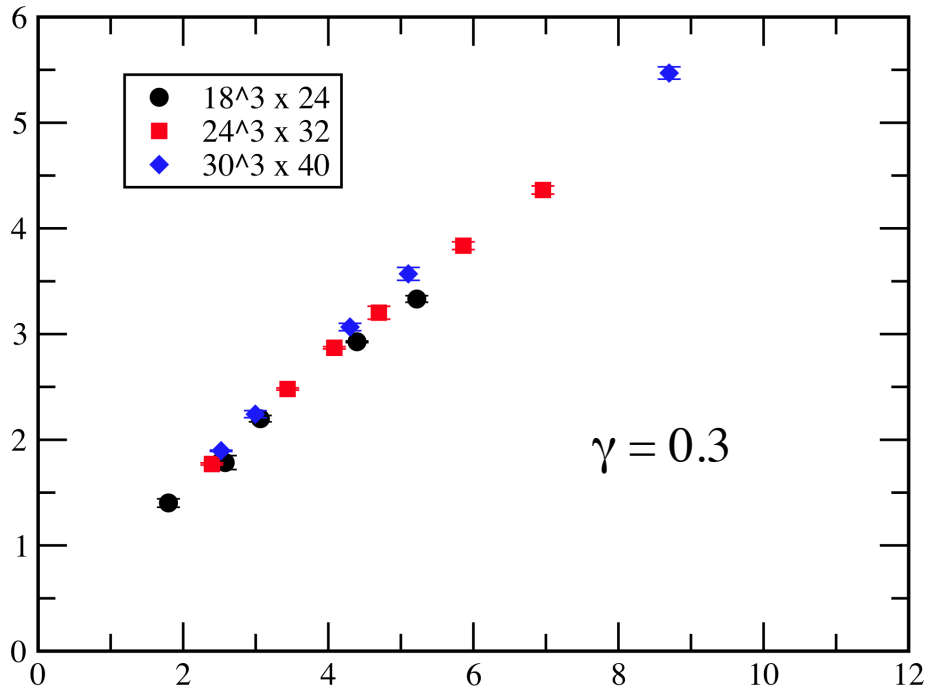
$$x = Lm^{1/(1+\gamma_*)}$$

Lf_π



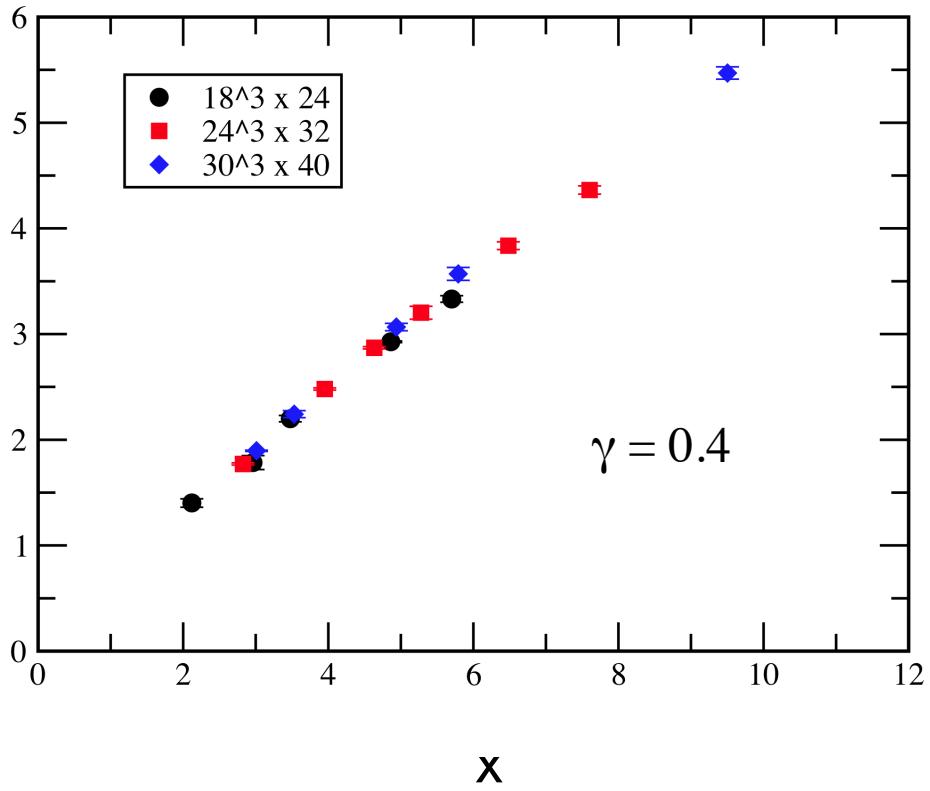
$$x = Lm^{1/(1+\gamma_*)}$$

Lf_π



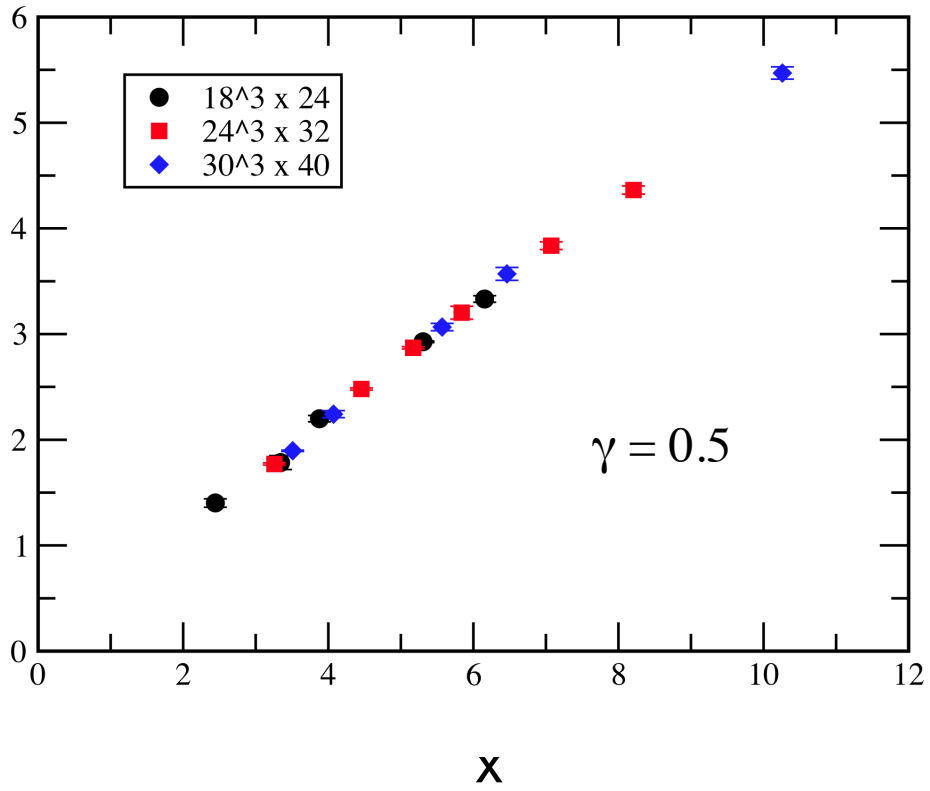
$$x = Lm^{1/(1+\gamma_*)}$$

Lf_π



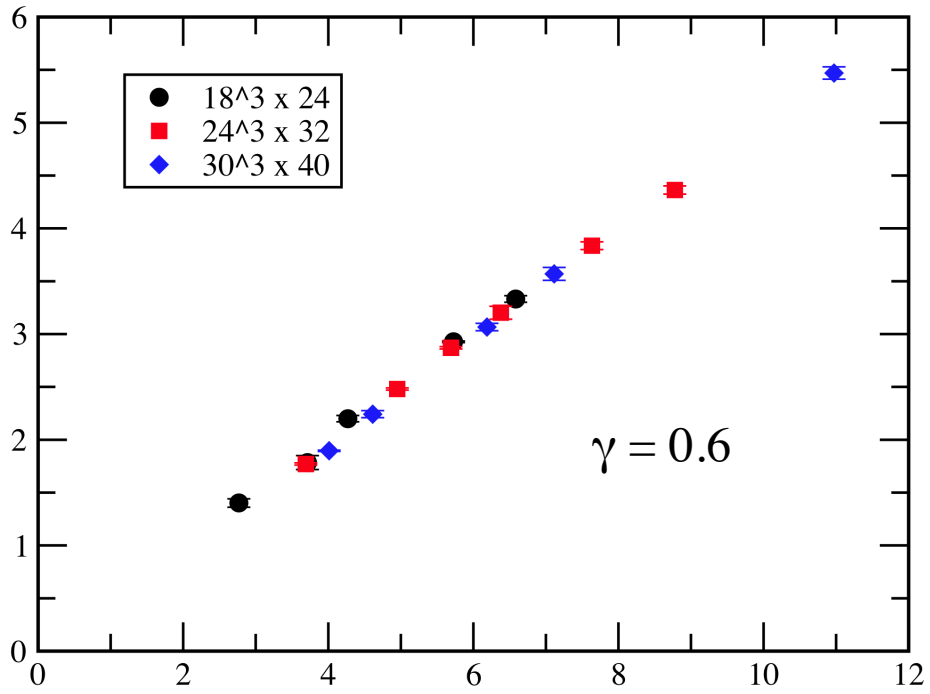
$$x = Lm^{1/(1+\gamma_*)}$$

Lf_π



$$x = Lm^{1/(1+\gamma_*)}$$

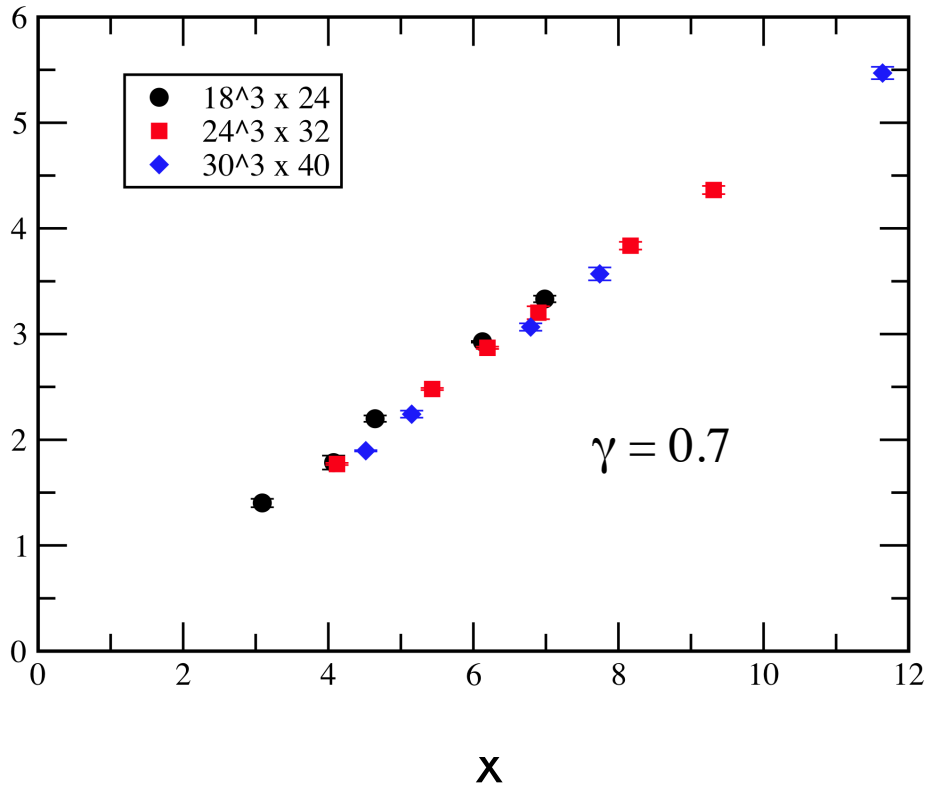
Lf_π



x

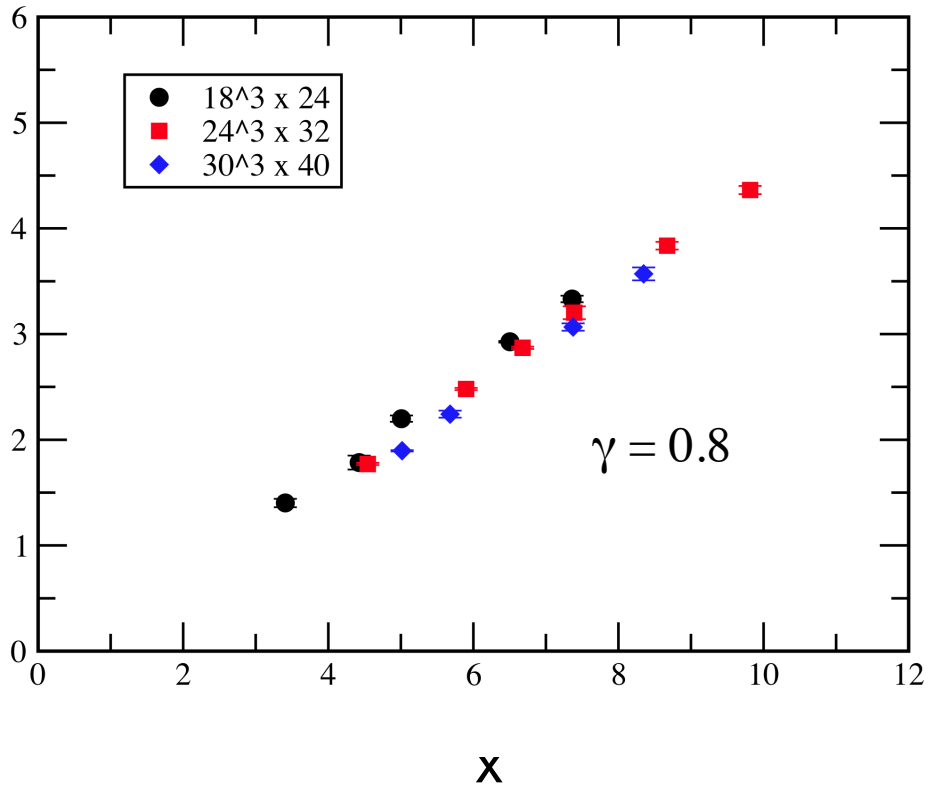
$$x = Lm^{1/(1+\gamma_*)}$$

Lf_π



$$x = Lm^{1/(1+\gamma_*)}$$

Lf_π



$$x = Lm^{1/(1+\gamma_*)}$$

■Fit analysis

Near the optimal value of γ, the data become to align,
where linearity is observed.

$$\xi = c_0 + c_1 L m_f^{1/(1+\gamma)}$$

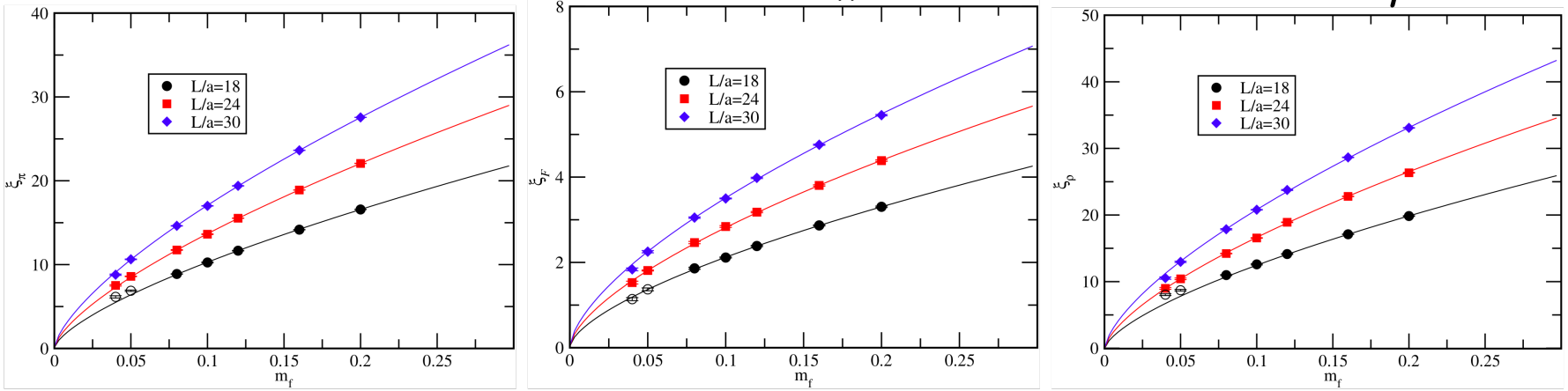
$$x = L m_f^{1/(1+\gamma)}$$

$$\xi = LM_\pi,\ LF_\pi,\ LM_\rho$$

- L-> infinity, the (infinite volume) hyper-scaling relation is obtained.
- 3 fit parameters : c0, c1, γ

■Results (beta=3.7)

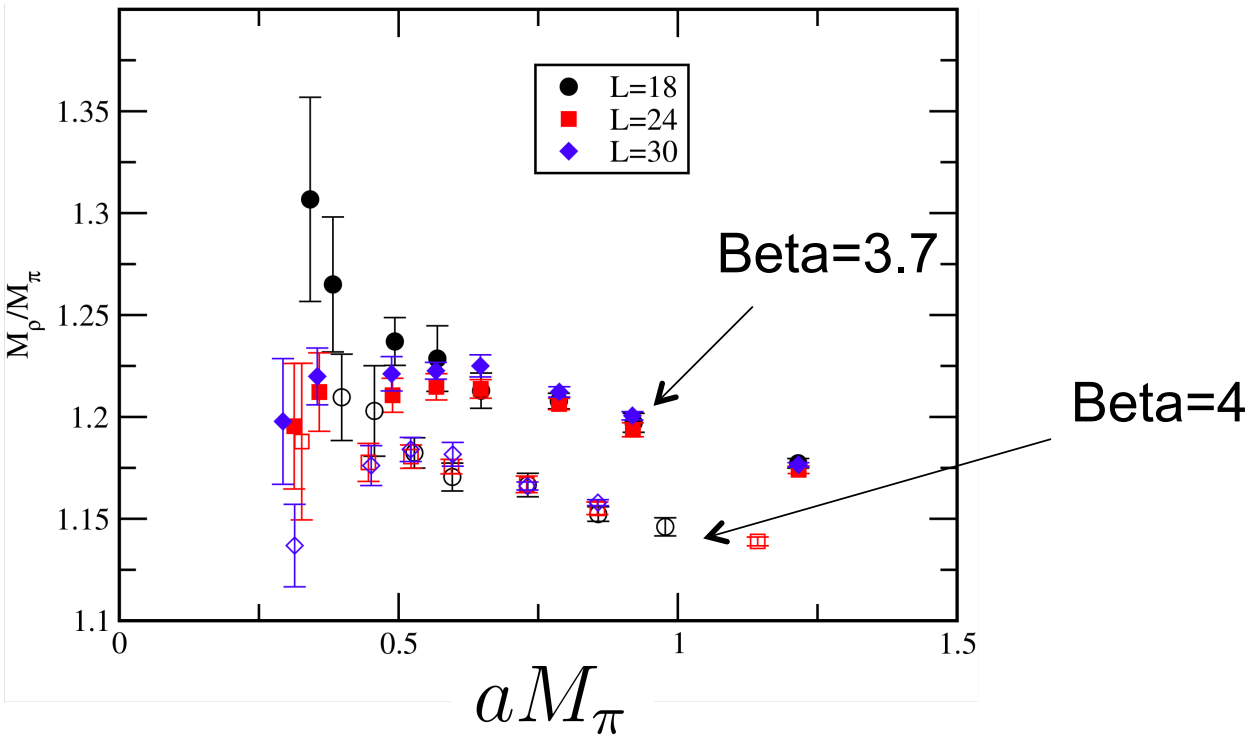
LM_π LF_π $LM\rho$



- The data with empty symbols are not used in the fit

	γ	c_0	c_1	χ^2/dof
M_π	0.4412(31)	0.1039(28)	2.7950(53)	3.69
F_π	0.544(12)	0.0348(18)	0.5150(34)	1.80
M_ρ	0.4915(64)	-0.055(66)	3.255(13)	4.83

■ρ/π ratio



- The data near the chiral limit -> almost constant.
  (c.f. it becomes blow-up in QCD.)
- This ratio is useful as the definition of constant physics
- a(β=3.7) > a(β=4).

=> Our results suggests asymptotically free region for beta=3.7-4.

Lattice results Nf=16

Nf=16

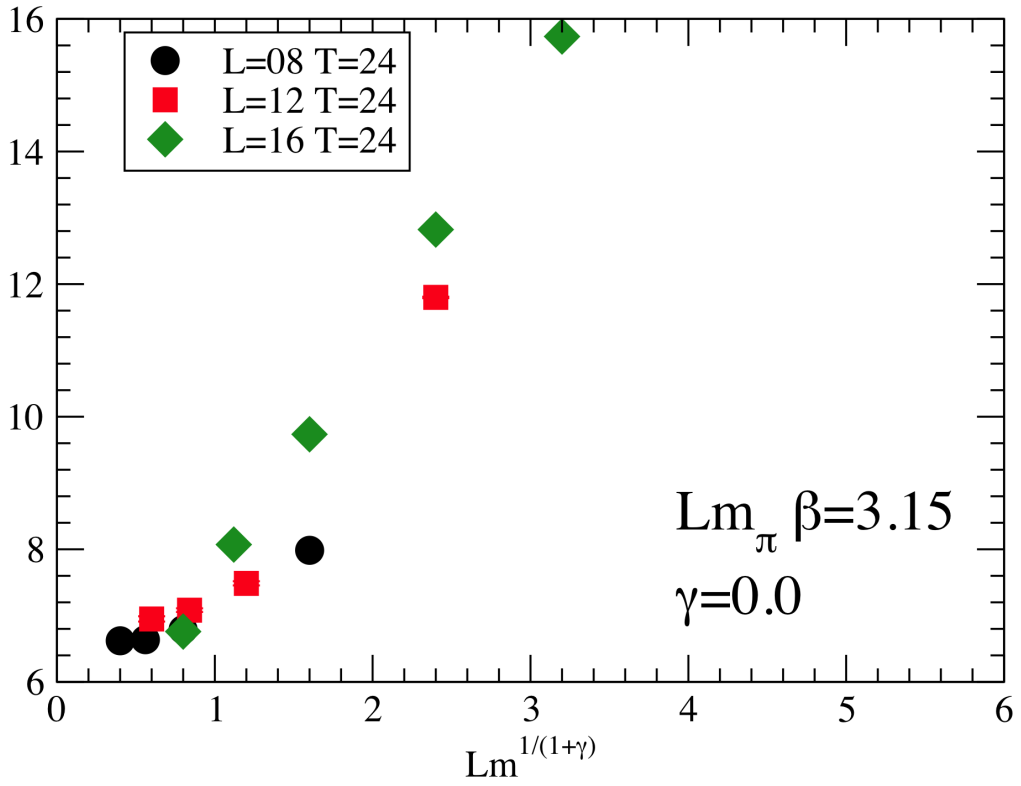
- **This theory should be in the deep conformal window**
- **Two loop perturbation suggests the existence of the IRFP in very weakly couling region.**

Bare coupling constant
beta=3.15, 3.5, 5

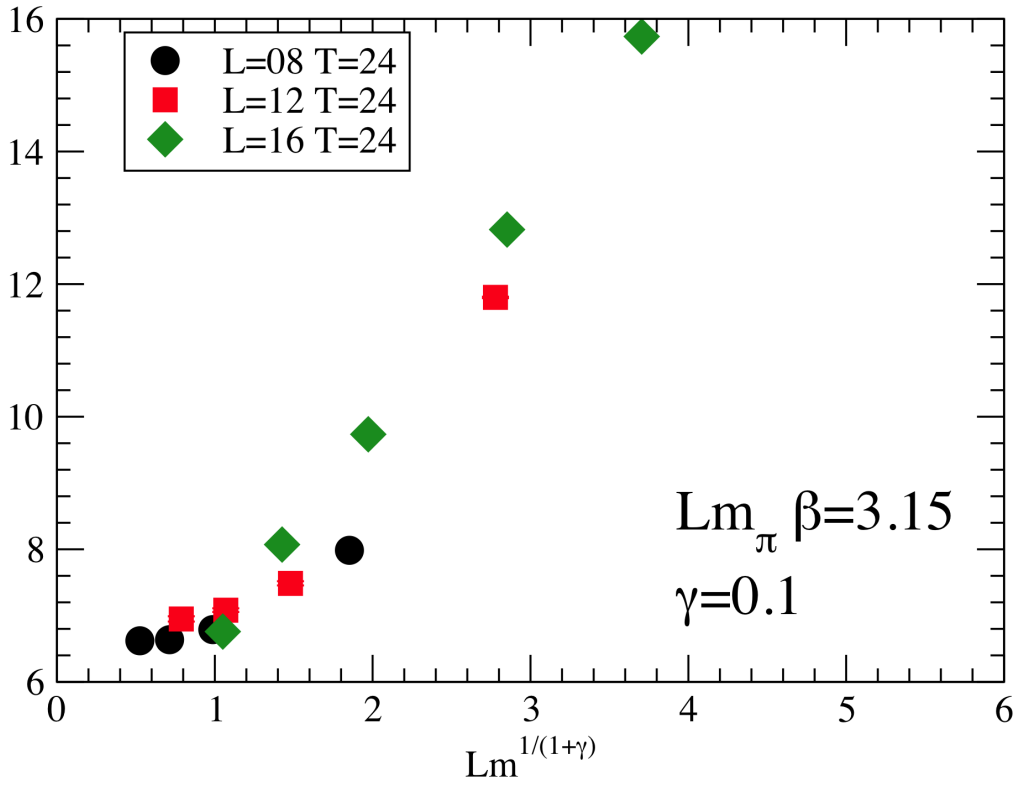
Volume (= L^3 x T)
- **L =8, T=24**
- **L =12, T=24**
- **L =16, T=24**
- **L=24, T=32**

- **Test of the finite size hyperscaling for pion mass**

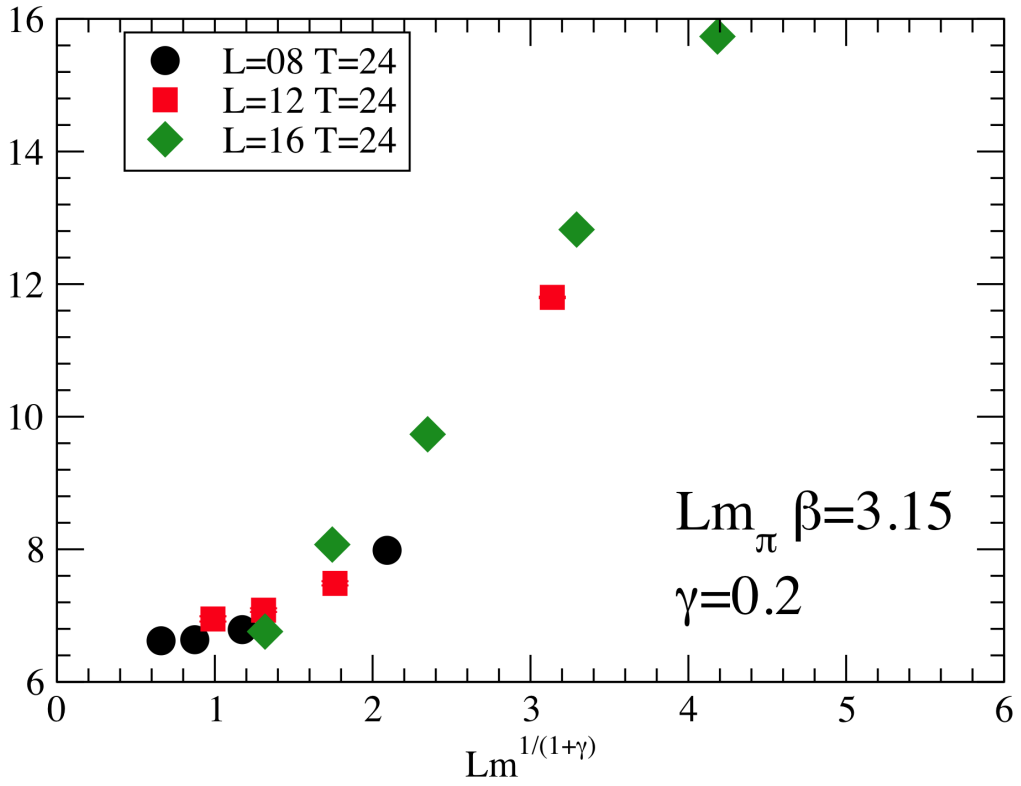
■Pion mass (beta=3.15)



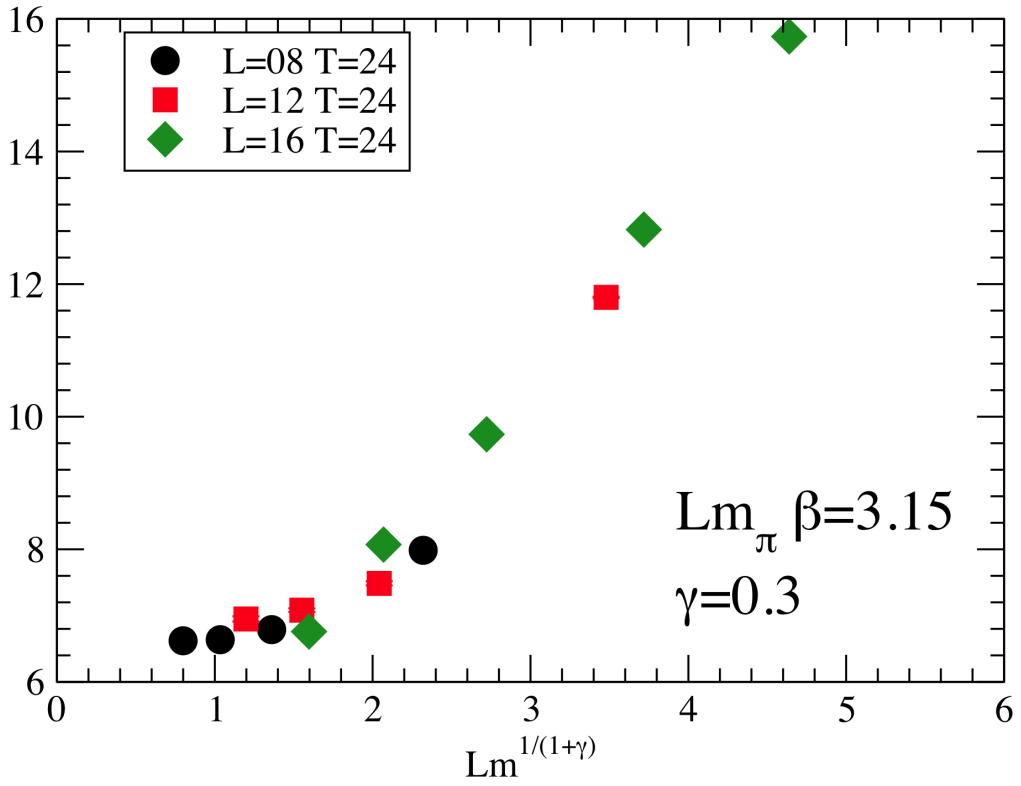
■Pion mass (beta=3.15)



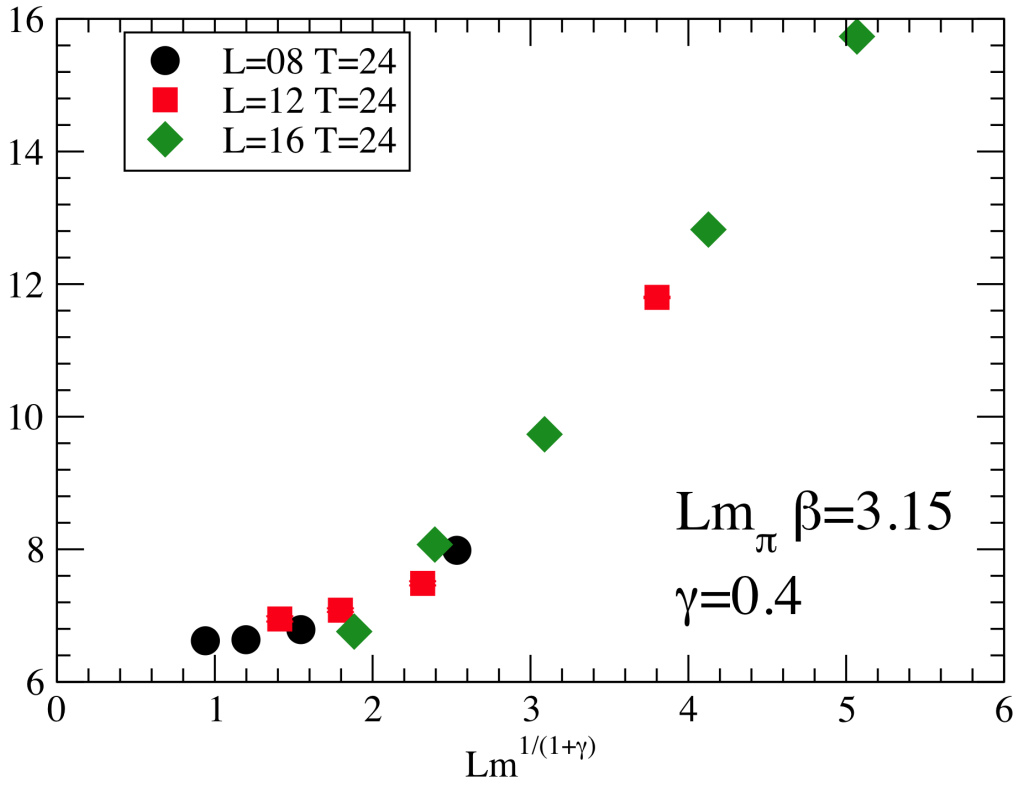
■Pion mass (beta=3.15)



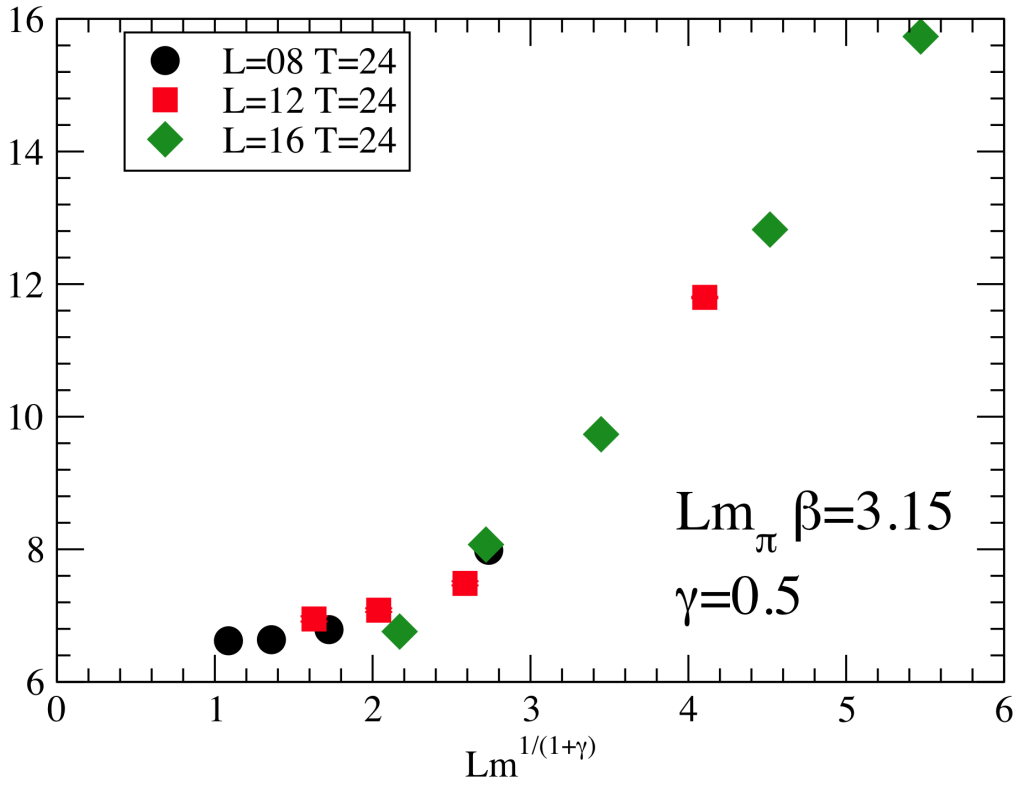
■Pion mass (beta=3.15)



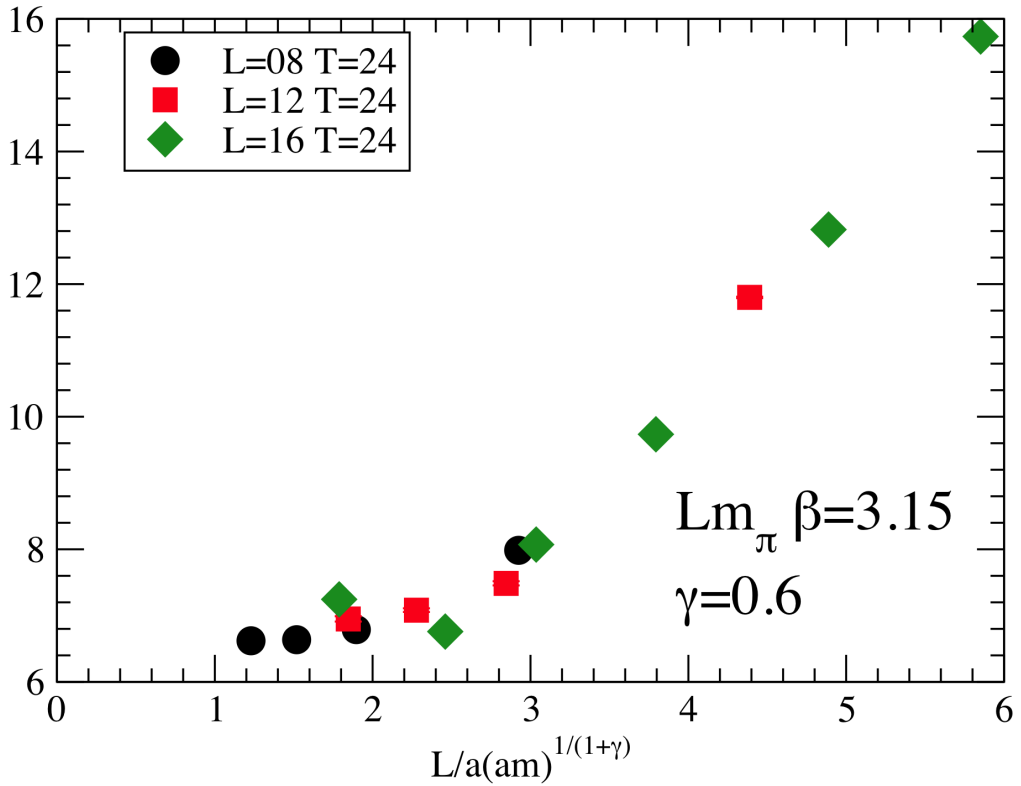
■Pion mass (beta=3.15)



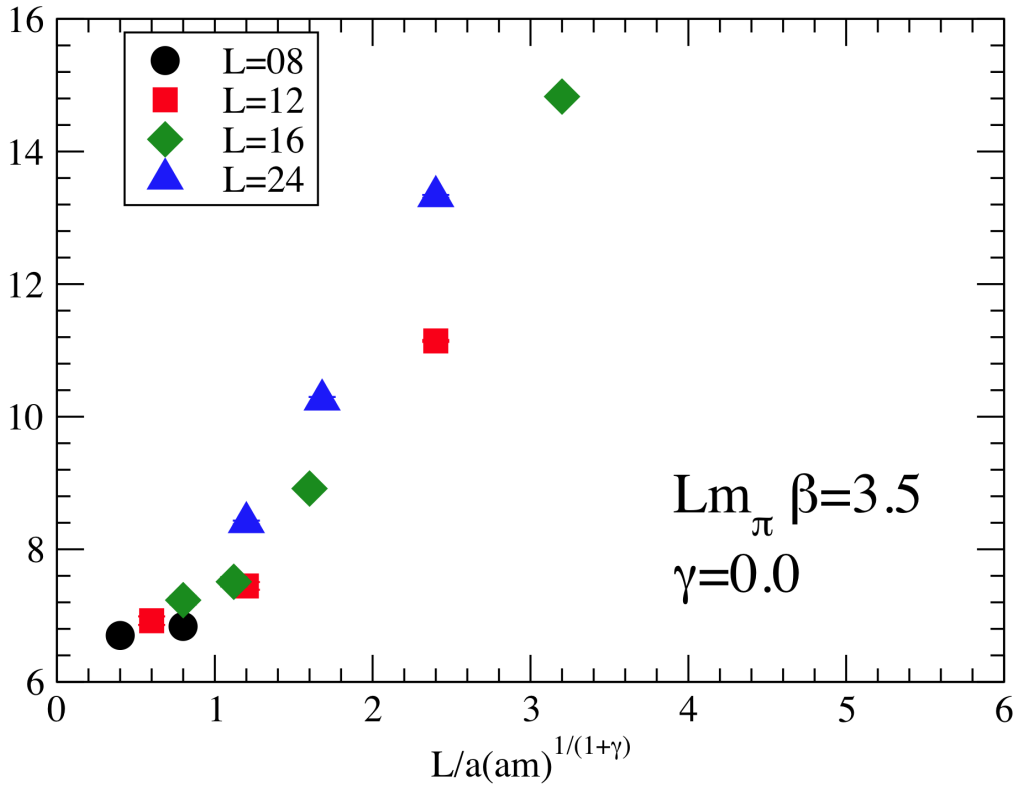
■Pion mass (beta=3.15)



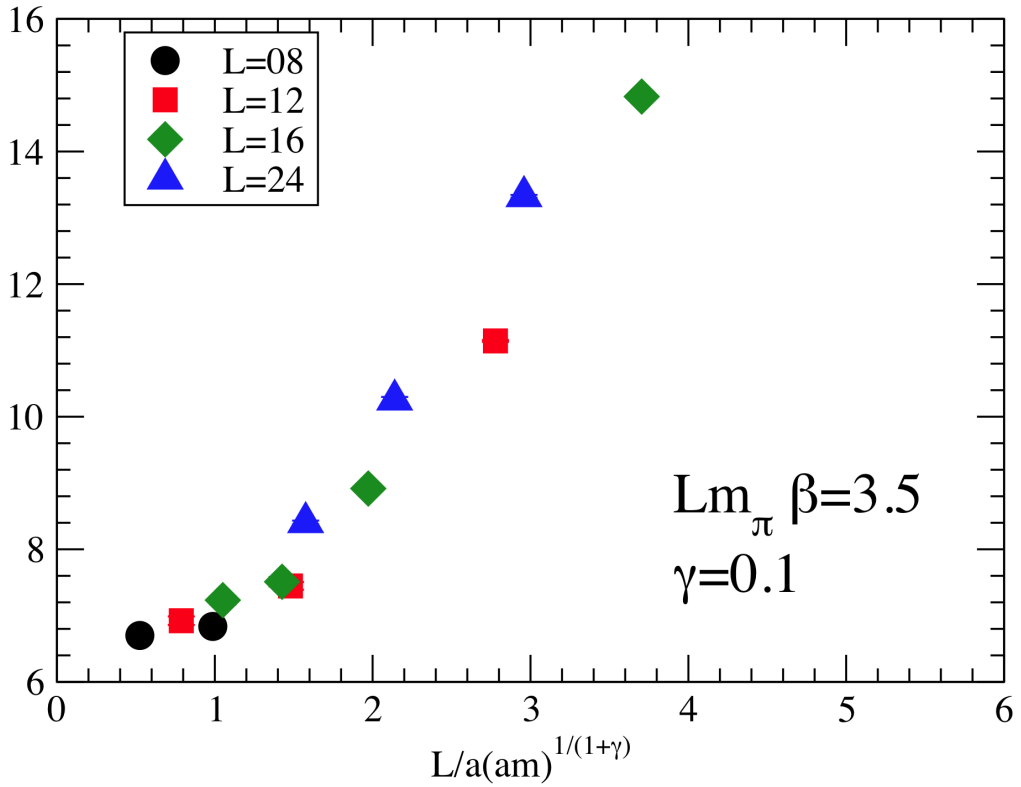
■Pion mass (beta=3.15)



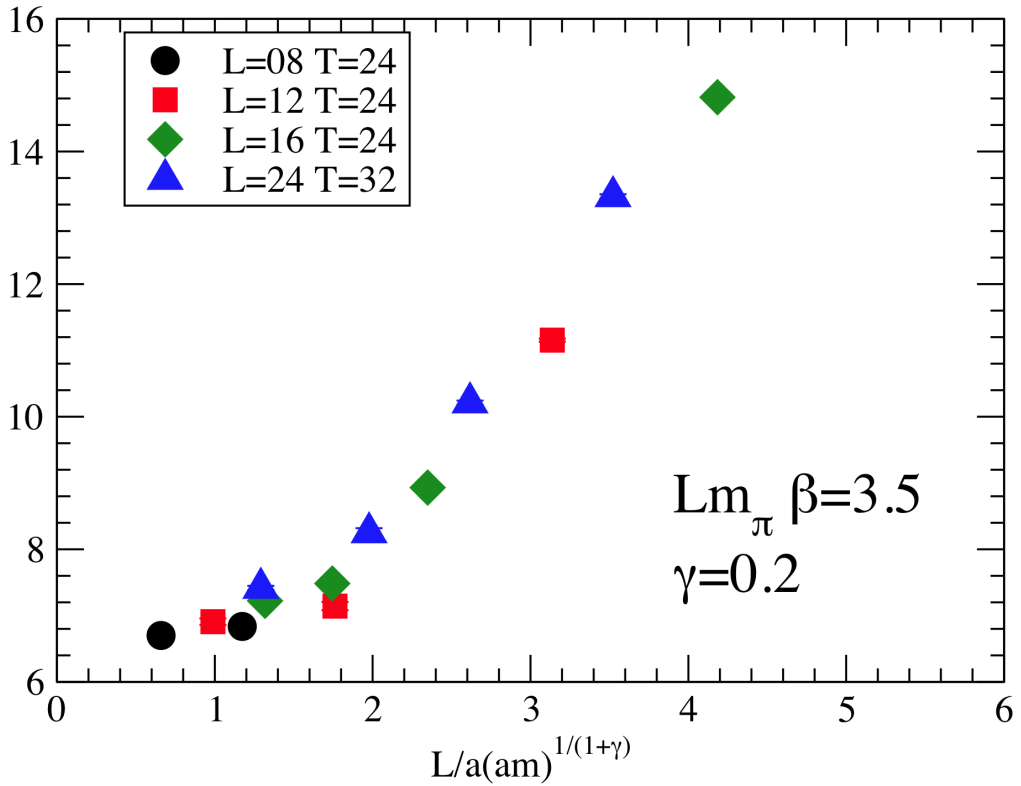
■Pion mass (beta=3.5)



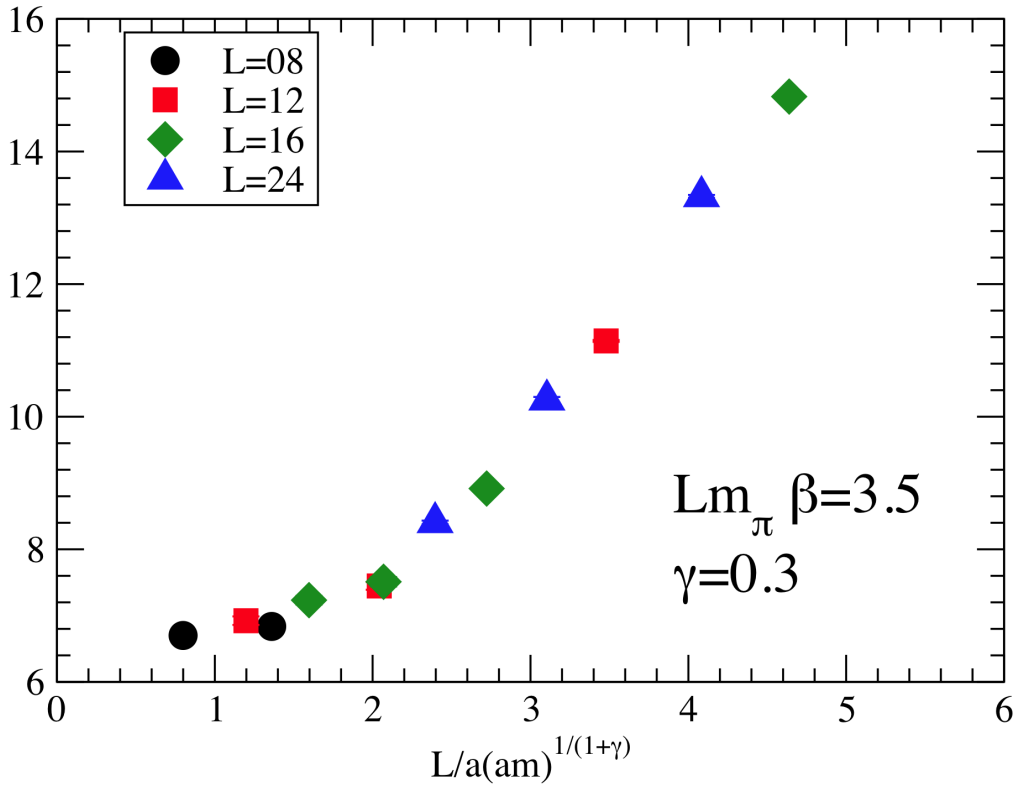
■Pion mass (beta=3.5)



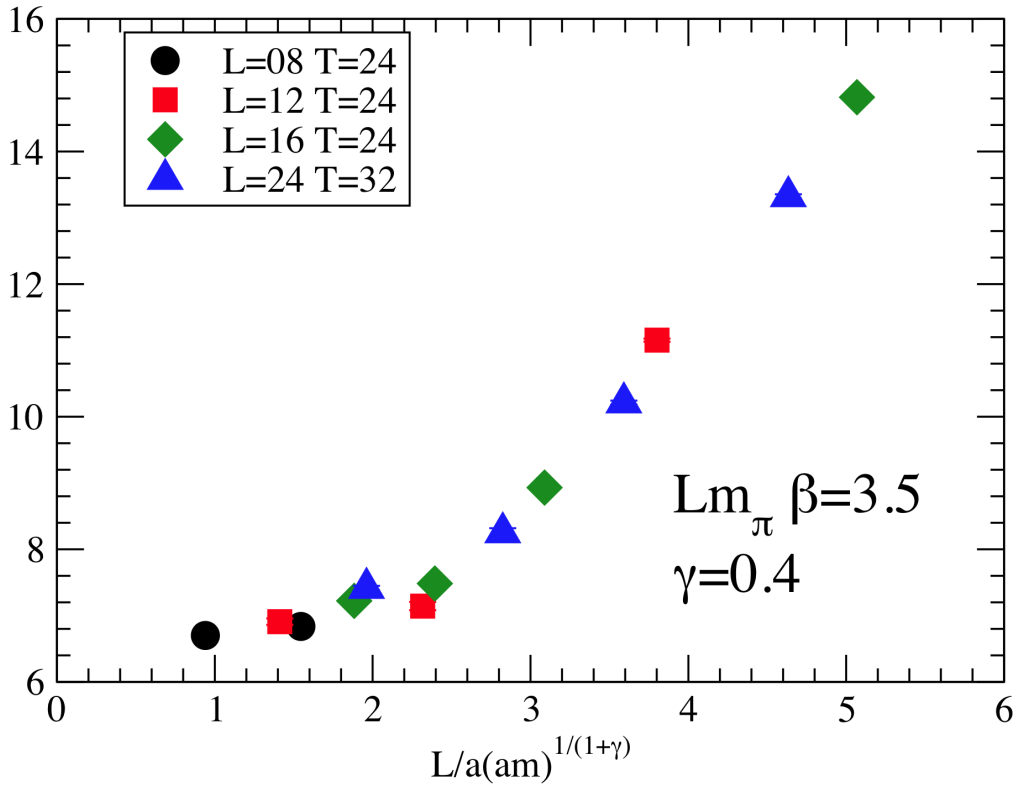
■Pion mass (beta=3.5)



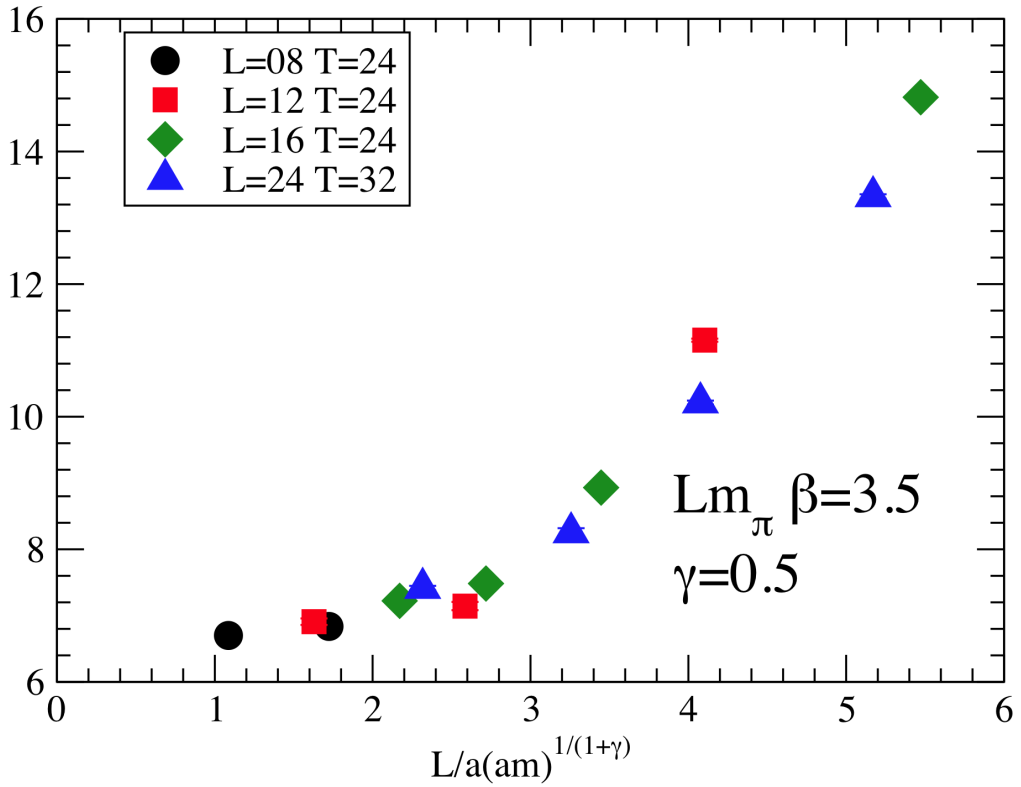
■Pion mass (beta=3.5)



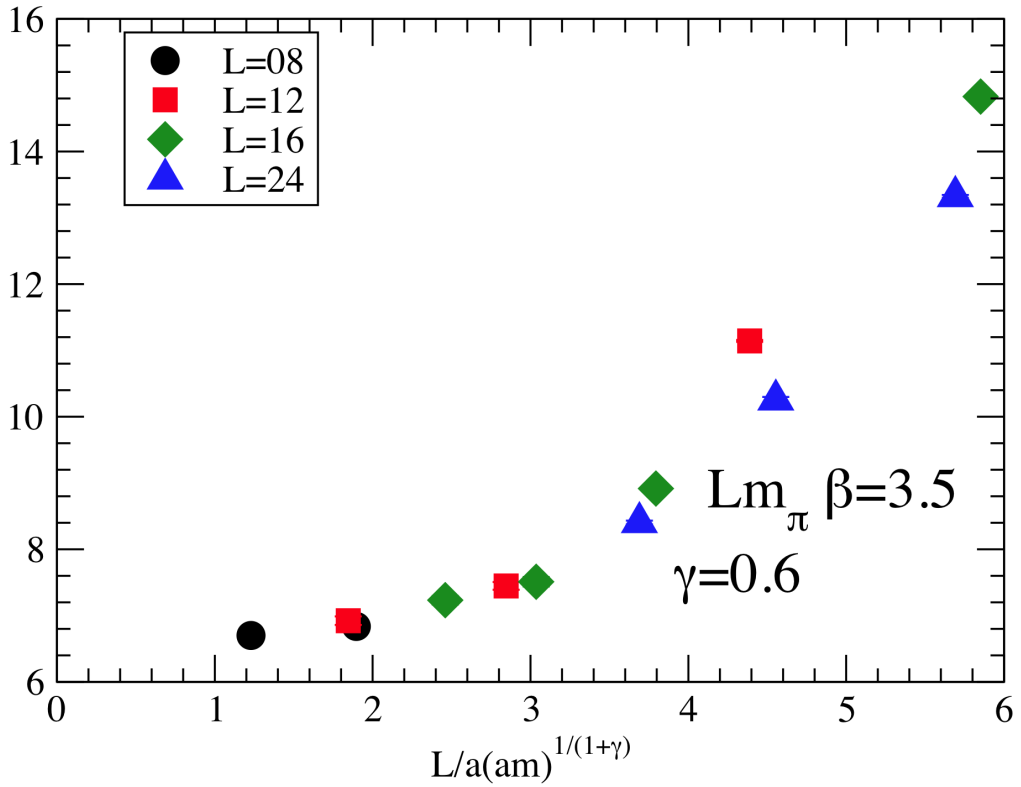
■Pion mass (beta=3.5)



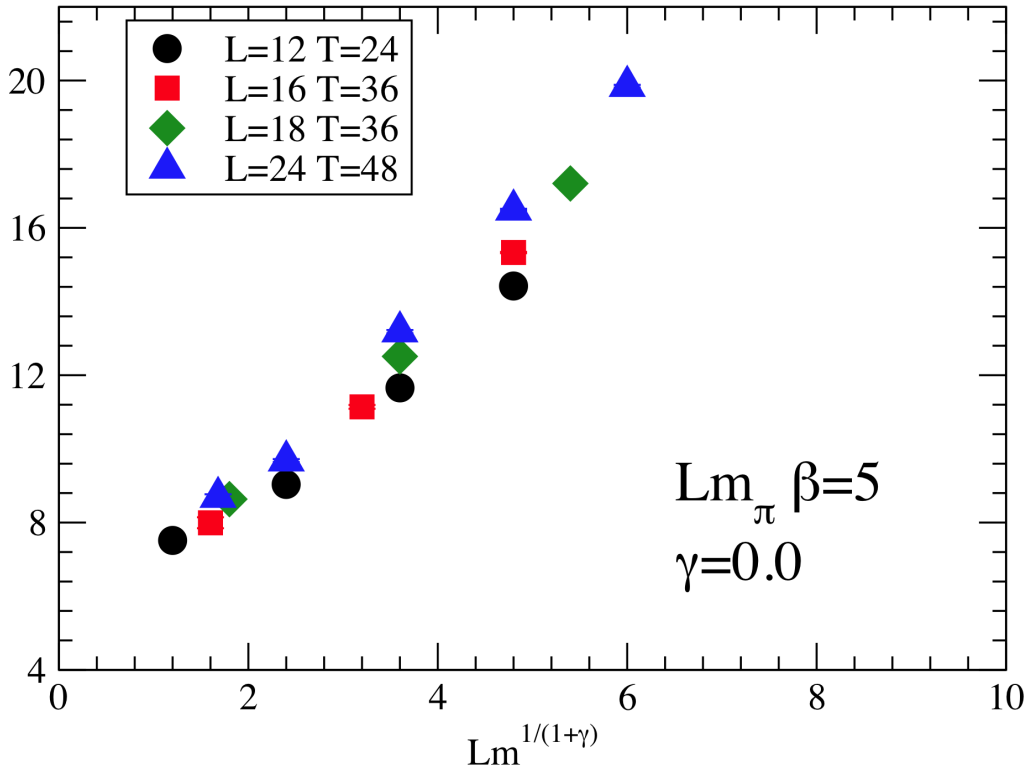
■Pion mass (beta=3.5)



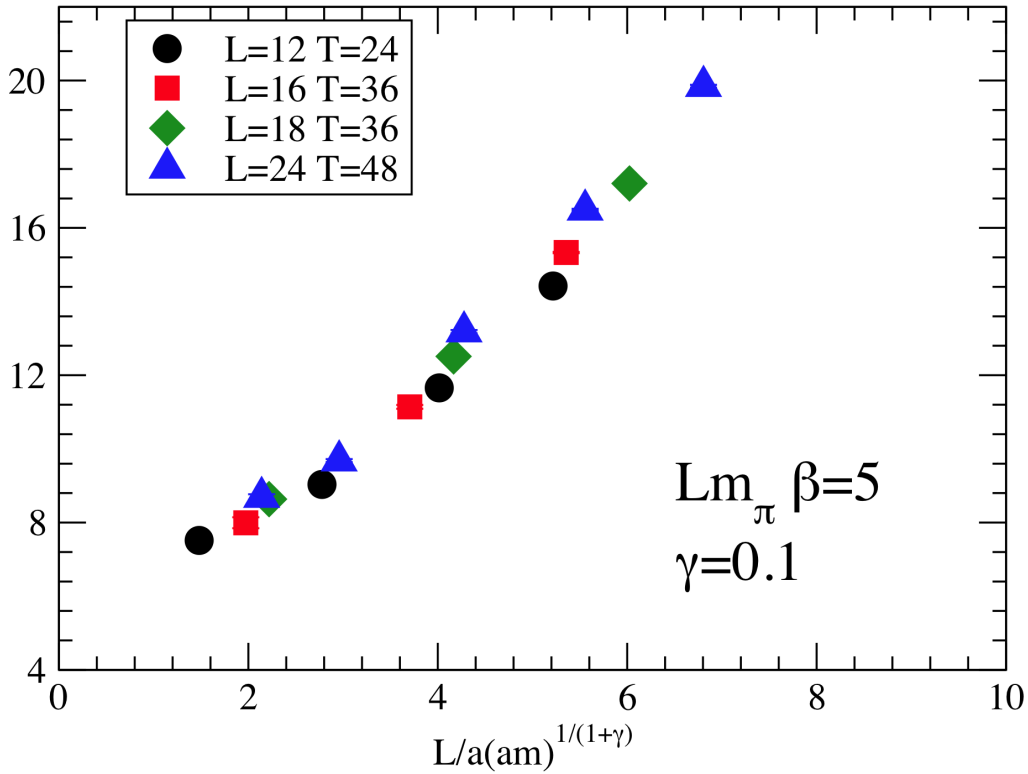
■Pion mass (beta=3.5)



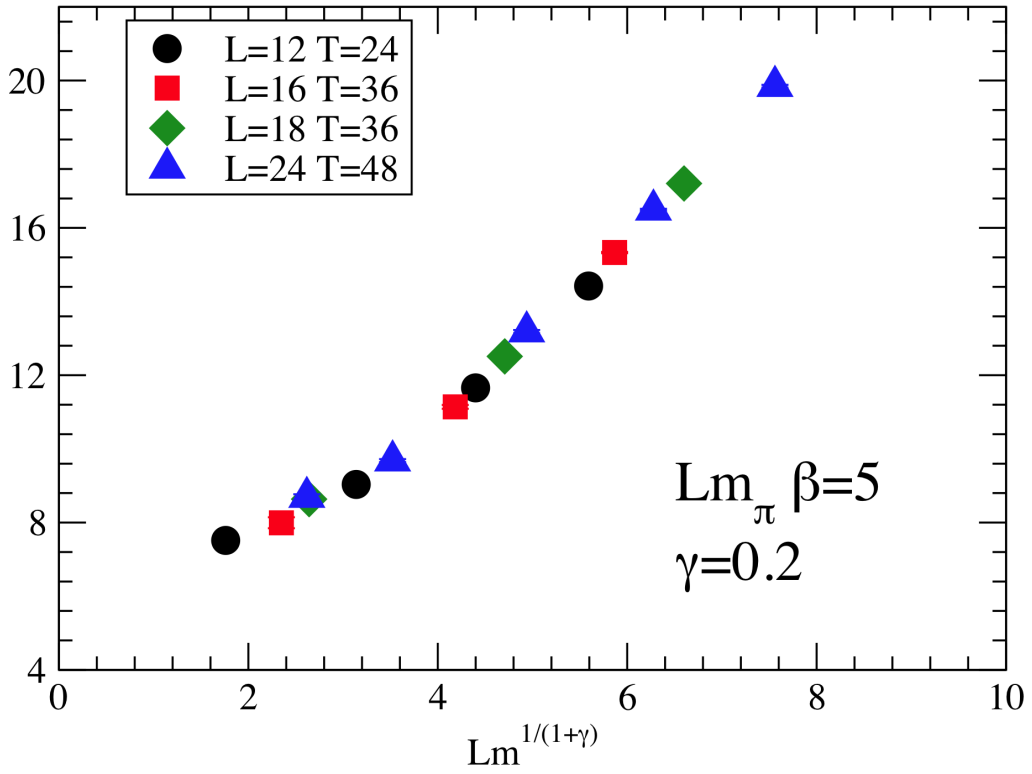
■Pion mass (beta=5)



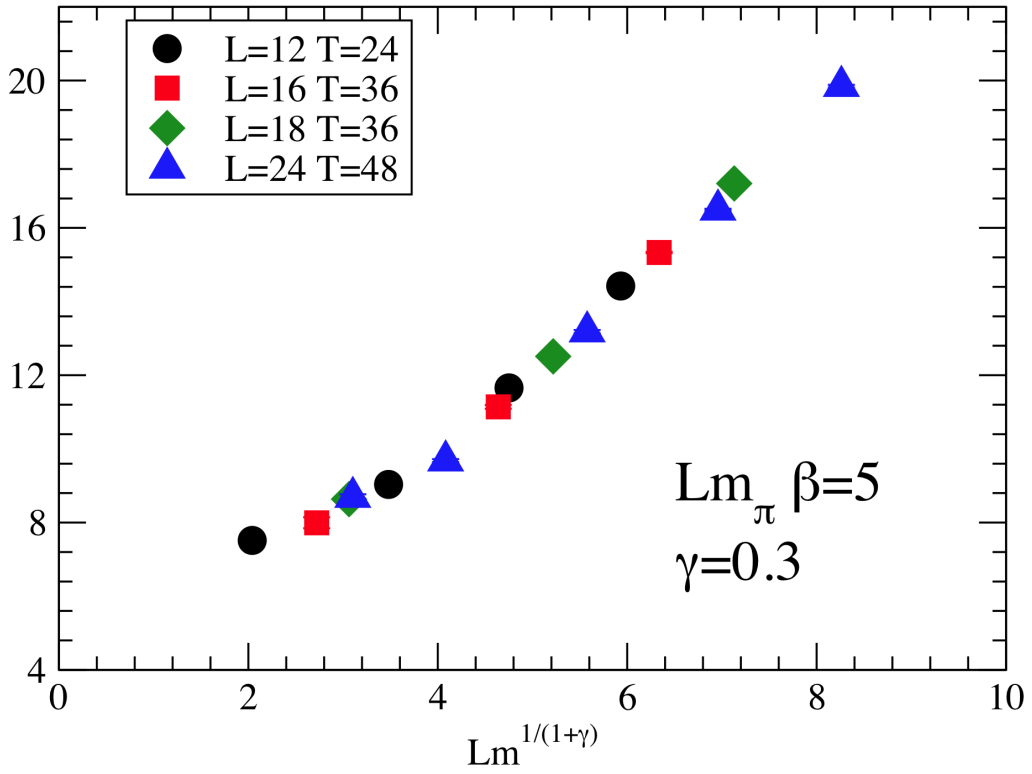
■Pion mass (beta=5)



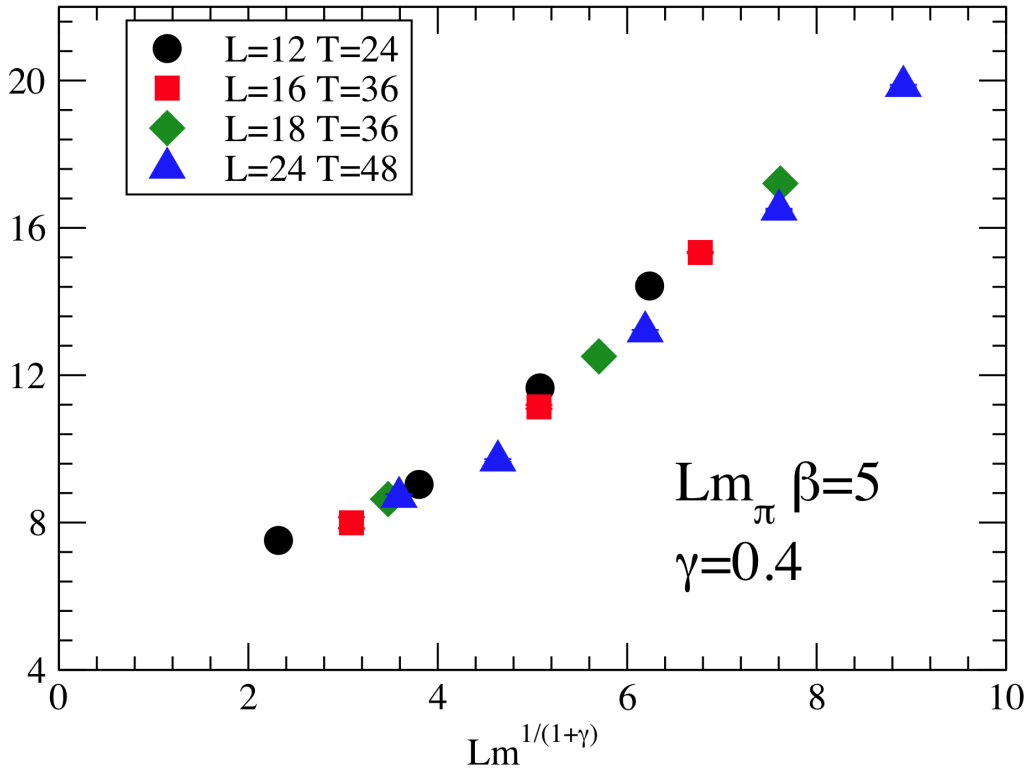
■Pion mass (beta=5)



■Pion mass (beta=5)



■Pion mass (beta=5)



- **The preliminary result of gamma**

beta	**3.15**	**3.5**	**5**
γ	0.42	0.35	0.25

Summary

- **Large flavor SU(3) gauge theory is being investigated.**
- **We focus on the Nf=12,16 case.**
- **We also study other models with fermions #=4,8 (talk by Nagai)**

- **We measure the pion (NG-boson) mass and decay constant.**
- **Finite size hyper scaling is observed.**
- **Nf=12 &16 favor conformal gauge theory**

Future prospect

- Questions

In Nf=12

difference of γ between several observables

<- due to mass and volume corrections? or walking?

In Nf=16

Large γ compared with perturbation theory

Beta dependence of γ

->need to take continuum limit or

very small IRFP (large beta simulations)

- Study of other spectra (ongoing)

scalar

glueball mass

END

Thank you