Dynamics in QCD like theories with infrared fixed point

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IR fixed point in two-loop β function of QCD like theories:

W. Caswell, PRL **33**, 244 (1974);
D. Jones, Nucl. Phys. **B75**, 531 (1974);
A. Belavin and A. Migdal, Pisma Zh. Eksp. Teor. Fiz. [JETP Letters] **19**, 317 (1974).

First discussion of consequences of existence of this IR fixed point:

T. Banks and A. Zaks, Nucl. Phys. **B196**, 189 (1982).

The two-loop solution:

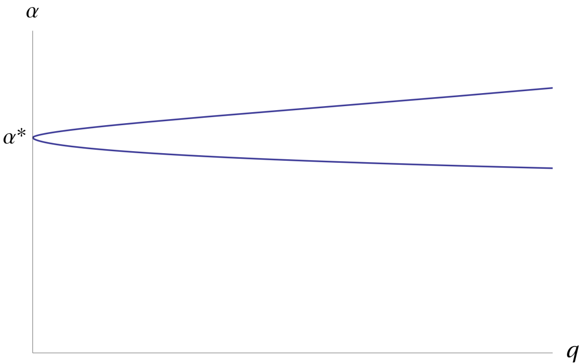
$$\frac{d\alpha}{d\ln\mu} = -b\alpha^2 - c\alpha^3.$$

If $b > 0$ ($N_f < N_f^{**} \equiv 11N_c/2$) and $c < 0$, the β function has a zero corresponding to a infrared-stable fixed point at

$$\alpha = \alpha^* = -\frac{b}{c}.$$

For $N_c = 3$, the condition $b > 0$ and $c < 0$ is valid for $N_f > N_f^* = 8$ and $N_f < N_f^{**} = 16.5$.

$\alpha^* \simeq 0.04,\ 0.14,\ 0.28,\ 0.47$ for $N_f = 16,\ 15,\ 14$, and 13, respectively ($N_c = 3$).



There exist two solutions!

Nonperturbative effects and the phase transition with respect to N_f

T. Appelquist, J. Terning, and L. C. R. Wijweardhana, PRL **77**, 1214 (1996);
V. M. and K. Yamawaki, PRD **55**, 5051 (1997)

The IR fixed point $\alpha^*(N_f)$ formally exists for all $N_f > N_f^*$ when $b(N_f) > 0$ and $c(N_f) < 0$. However, when

$$\alpha^*(N_f) > \alpha_{cr} \simeq \frac{2N_c}{N_c^2 - 1}\frac{\pi}{3},$$

quarks get a dynamical mass and decouple:
the IR fixed point is a fake in this case.

$N_f^* < N_f < N_f^{cr}$: $\alpha^*(N_f) > \alpha_{cr}$.

$N_f^{cr} < N_f < N_f^{**} = \dfrac{11N_c}{2}$: $\alpha^*(N_f) < \alpha_{cr}$.

For $N_c = 3$, $N_f^* = 8$ and $N_f^{cr} = 12$.

Phase transition in QCD at $N_f = N_f^{cr}$

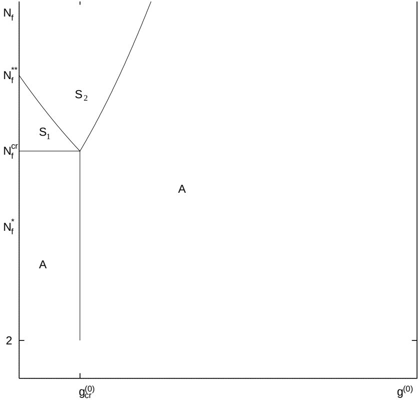
$$m_{dyn}^2 \sim \Lambda_{cr}^2 \exp\left(-\frac{C}{\sqrt{\frac{\alpha^*(N_f)}{\alpha_{cr}} - 1}} \right) :$$

$$\alpha_{cr} \simeq \frac{2N_c}{N_c^2 - 1}\frac{\pi}{3}, \qquad \Lambda_{cr}\colon\ \alpha(\mu)\big|_{\mu=\Lambda_{cr}} = \alpha_{cr},$$

$$N_f^{cr} \sim 4N_c > N_f^*.$$

The Phase Diagram in $SU(N_c)$ gauge model

V. M. and K. Yamawaki (1997)



The phase diagram in an SU(N_c) gauge model. The coupling constant $g^{(0)} = \sqrt{4\pi\alpha^{(0)}}$ and S and A denote symmetric and asymmetric phases, respectively.

A. Deuzeman, M. Lombardo and E. Pallante, PRD **82**, 074503 (2010)

Dynamics in the conformal window in QCD-like theories

The two loop β function leads to the following equation for the running coupling:

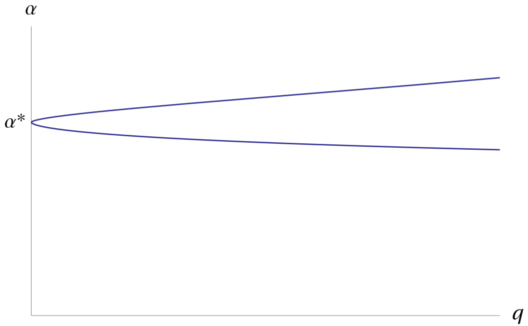
$$b \log\left(\frac{q}{\mu}\right) = \frac{1}{\alpha(q)} - \frac{1}{\alpha(\mu)} - \frac{1}{\alpha^*} \log\left(\frac{\alpha(q)(\alpha(\mu) - \alpha^*)}{\alpha(\mu)(\alpha(q) - \alpha^*)}\right).$$

The case $\alpha(\mu) < \alpha^*$ (T. Appelquist, J. Terning and R. Wijewardhana (1996)):

$$\alpha(q) \simeq \frac{1}{b \log(q/\Lambda)}, \qquad q \gg \Lambda \qquad (\alpha(\Lambda) \simeq 0.8\alpha^*)$$

$$\alpha(q) \simeq \frac{\alpha^*}{1 + e^{-1}(q/\Lambda)^{b\alpha^*}}, \qquad q \ll \Lambda$$

Asymptotically free theory with no chiral symmetry breaking.



The case with $\alpha(\mu) > \alpha^*$ (V. M. and K. Yamawaki (1997))

$$\frac{1}{\alpha(q)} = b \log \frac{q}{\tilde{\Lambda}} + \frac{1}{\alpha^*} \log \left(\frac{\alpha(q)}{\alpha(q) - \alpha^*} \right),$$

where $\tilde{\Lambda}$ is a Landau pole at which $\alpha(q)\big|_{q=\tilde{\Lambda}} = \infty$

Does a meaningful continuum limit exist in this case?
The answer is "yes".

Introduce UV cutoff M with a finite bare coupling
$\alpha^{(0)} \equiv \alpha(q)\big|_{q=M} > \alpha^*$.

Then, $M < \tilde{\Lambda}$ with $\alpha(\tilde{\Lambda}) = \infty$. The Landau pole is unreachable in the theory with cutoff M and $\alpha^{(0)} < \infty$.
As $M \to \infty$ with a fixed $\alpha^{(0)} = \alpha(q)\big|_{q=M}$,

$$\alpha(q) \simeq \frac{\alpha^*}{1 - e^{-1}(q/\tilde{\Lambda})^{b\alpha^*}} \to \alpha^*$$

<u>Nontrivial conformal field theory</u>

Deformation of theory: Introducing bare fermion mass

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Dynamics in the conformal window in QCD-like theories

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(Received 14 December 1998; published 26 March 1999)

The dynamics with an infrared stable fixed point in the conformal window in QCD-like theories (with a relatively large number of fermion flavors) is studied. The dependence of masses of colorless bound states on a bare fermion mass is described. In particular it is shown that in such dynamics glueballs are much lighter than bound states composed of fermions, if the value of the infrared fixed point is not too large. This yields a clear signature for the conformal window, which in particular can be useful for lattice computer simulations of these gauge theories. [S0556-2821(99)04408-2]

The absence of a mass for fermions and gluons is a key point for <u>not</u> creating bound states in S_1 and S_2 phases.

The situation changes dramatically if a bare fermion mass is introduced: even weak gauge, Coulomb-like, interactions easily produce bound states composed of massive constituents, as it happens, for example, in QED (positronium like bound states)

The main consequences of the presence of the bare mass:

a) Taking $(\alpha^{(0)} - \alpha^*) \ll \alpha^*$ (walking regime), the pole fermion mass m is expressed through $m^{(0)}$ as

$$m \simeq M\Big(\frac{m^{(0)}}{M}\Big)^{1/(1+\gamma_m)}$$

b) The mass of n-body bound state composed of fermions is

$M^{(n)} \sim nm \sim nM\big(\frac{m^{(0)}}{M}\big)^{1/(1+\gamma_m)}$.

c) At momenta $q < m$, fermions and their bound states decouple. There is a pure Yang-Mills theory with confinement. Its spectrum contains **light** glueballs, $M_{gl} \ll m$, if α^* is not too large:

$$M_{gl} \sim \Lambda_{YM} \sim m \exp\Big(-\frac{1}{\bar{b}\alpha^*}\Big), \qquad \bar{b} = \frac{11}{6\pi}N_c.$$

For $N_c = 3$, $\exp(-1/\bar{b}\alpha^*)$ is 6×10^{-7}, 2×10^{-2}, 10^{-1}, and 3×10^{-1} for $N_f = 16$, 15, 14, and 13, respectively

The signature of the conformal window for lattice computer simulations

1. The universal scaling: $M^{(n)} \sim \left(m^{(0)}\right)^{1/(1+\gamma_m)}$
2. Light glueballs: $M_{gl} \sim \left(m^{(0)}\right)^{1/(1+\gamma_m)}$ and $M_{gl} \ll M^{(n)}$

Z. Fodor, K. Holland, J. Kuti, D. Nógrádi, C. Schroeder, Phys. Lett. B **681**, 353 (2009)

T. DeGrand and A. Hasenfratz , PRD **80**, 034506 (2009)

L. Del Debbio, B. Lucini, A. Patella, C. Pica and A. Rago, PRD **80**, 074507 (2009)

A. Deuzeman, M. Lombardo and E. Pallante, PRD **82**, 074503 (2010)

T. Appelquist, G. Fleming, M. Lin, E. Neil and D. Schaich, PRD **84**, 054501 (2011)

Kenji Ogawa et al., arXiv:1111.1575 [hep-lat]

Y. Aoki, T. Aoyama, M. Kurachi, T. Maskawa, K. Nagai, H. Ohki, A. Shibata, K. Yamawaki, T. Yamazaki, arXiv:1201.4157 [hep-lat]; arXiv:1202.4916 [hep-lat].

The structure of the β-function in QCD in the $\overline{\mathrm{MS}}$ scheme

F. Chishtie, V. Elias, V. Miransky, and T. Steele,
Prog. Theor. Phys. **104**, 603 (2000)

The questions:

1. What is the N_f^{cr} in the $\overline{\mathrm{MS}}$ scheme?
2. What is the form of the β-function for $N_f > N_f^{cr}$ in the $\overline{\mathrm{MS}}$ scheme?
3. What is the form of the β-function for $N_f < N_f^{cr}$ in the $\overline{\mathrm{MS}}$ scheme?

Padé-improvement for β-function

J. Ellis, I. Jack, D. Jones, M. Karliner, M. Samuel,
Phys. Rev. D **57**, 2665 (1998)

A k-loop truncation:

$$\beta^{(k)}(x) = -\beta_0 x^2 (1 + R_1 x + \cdots + R_{k-1} x^{k-1}),$$

$$x = \frac{\alpha}{\pi}, \quad \beta_0 = \frac{11 - \frac{2}{3} N_f}{4}.$$

Then, the $[N/M]$ Padé-approximant $(N + M = k - 1)$ is

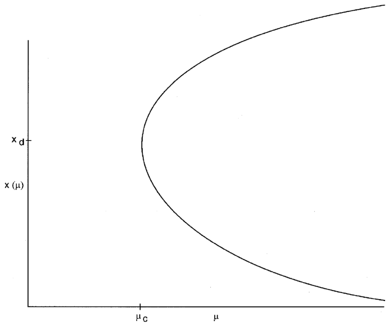
$$\beta^{[N/M]} = -\beta_0 x^2 \left(\frac{1 + a_1 x + \cdots + a_N x^N}{1 + b_1 x + \cdots + b_M x^M} \right)$$

The $N + M$ coefficients $a_1, \ldots, a_N, b_1, \ldots, b_M$ are determined by the requirement that the first k terms in the Taylor expansion of $\beta^{[N/M]}$ replicate $\beta^{(k)}(x)$.

$SU(N_c)$ SUSY gluodynamics

I. Kogan and M. Shifman, PRL **75**, 2085 (1995)

$$\beta(x) = -\frac{3N_c x^2}{4}\left[\frac{1}{1 - N_c x/2}\right]; \quad x \equiv \frac{\alpha_s}{\pi}$$



There is an UV stable fixed point at $x = \infty$ in strong coupling phase. The denominator zero $x_d = \frac{2}{N_c}$ corresponds to an infrared attractor.

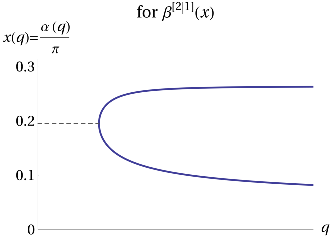
Gluodynamics

Four-loop $\overline{\mathrm{MS}}$ β-function:

$$\beta^{(4)}(x) = -\frac{11}{4}x^2\left[1 + 2.31818x + 8.11648x^2 + 41.5383x^3\right],$$

$$\beta^{[2|1]}(x) = -\frac{11}{4}x^2\left[\frac{1 - 2.7996x - 3.7475x^2}{1 - 5.1178x}\right],$$

$$\beta^{[1|2]}(x) = -\frac{11}{4}x^2\left[\frac{1 - 5.9672x}{1 - 8.2854x + 11.091x^2}\right],$$



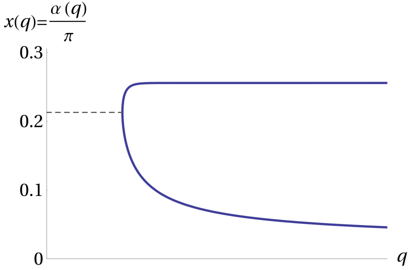
The denominator zero x_d (infrared attractor) precedes the numerator zero (x_n):
$x_d = 0.195 < x_n = 0.264$ and $x_d = 0.151 < x_n = 0.168$ in $[2|1]$ and $[1|2]$ approximant, respectively. If taken seriously, x_n zero corresponds to UV fixed point.

F. Sannino and J. Schechter, Phys. Rev. D **82**, 096008 (2010)
Rytov-Sannino β function for Gluodynamics

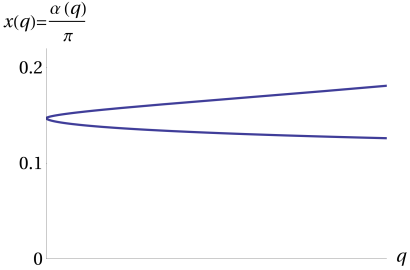
QCD with fermions

- Below an approximant-dependent flavor threshold ($6 \leqslant N_f^{thr} \leqslant 8$), the approximants $[2|1]$ and $[1|2]$ always exhibit a positive pole prior to the occurence of their first positive zero, precluding any identification of this zero as a IR fixed point.
- Above the threshold N_f^{thr}, those approximants exhibit a IR fixed point. Its value becomes less than $x_{cr} = \frac{1}{4}$ at $N_f = 11$. This value decreases with increasing N_f and is close to the 2-loop value for $N_f > 12$.

$\beta^{[2|1]}$ for $N_f = 3$



$\beta^{[2|1]}$ for $N_f = 12$



UV fixed point in strong coupling gauge theories
(recent discussions):

D. Kaplan, J. Lee, D. Son and M. Stephanov,
PRD **80**, 125005 (2009)

Conclusion

Dynamics in the conformal window is a new territory
and one can expect interesting surprises there.