Holographic Walking Technicolor

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Introduction

Strongly coupled fermions play an important role in many physical systems:

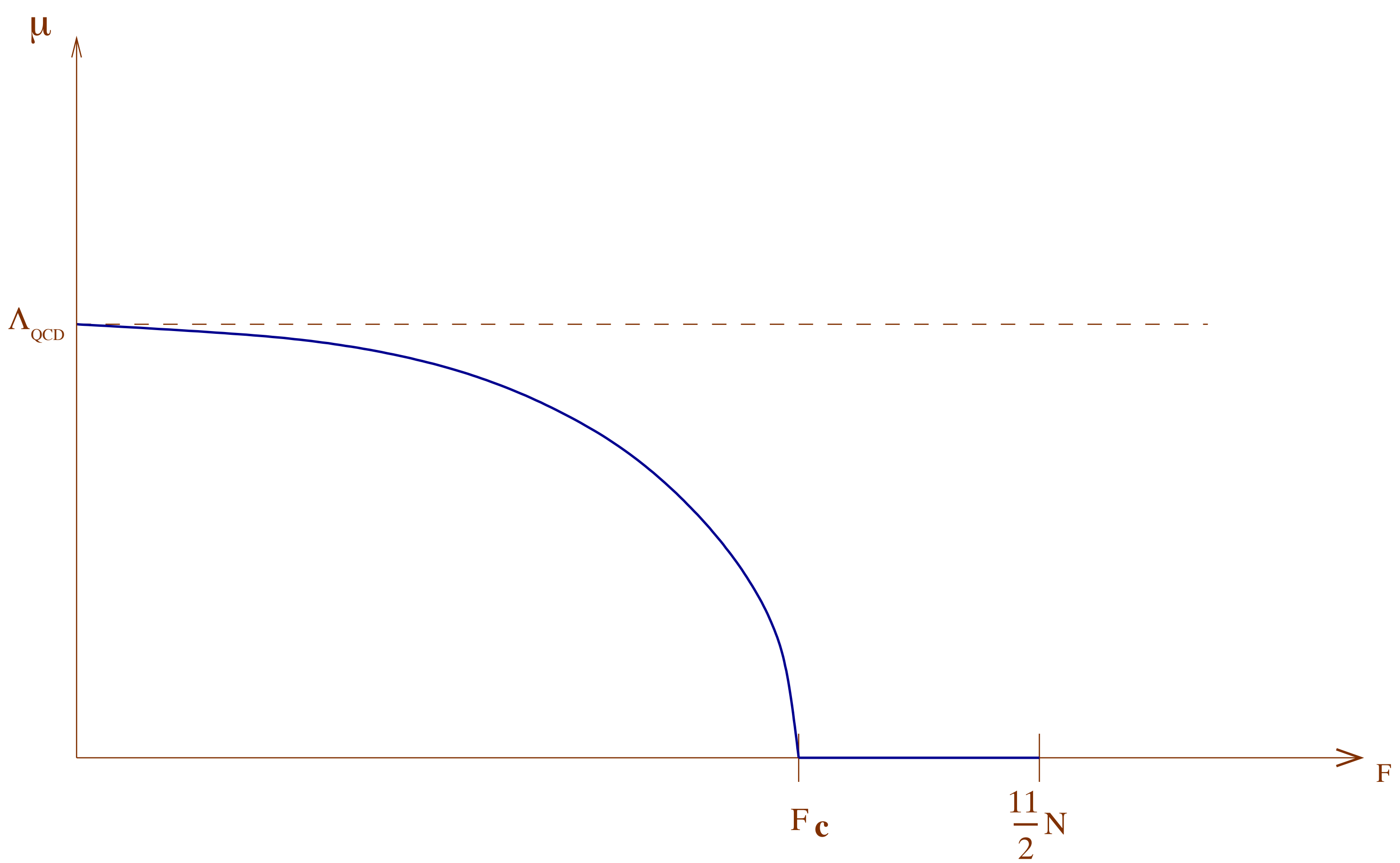
- QCD
- Technicolor
- Condensed matter

Typically, at weak coupling the dynamics is simple, while at strong coupling one finds many interesting phenomena, such as dynamical symmetry breaking, mass generation and confinement.

The focus of this lecture will be the transition between the two regimes. Examples of systems in which such transitions occur are:

QCD

SU(N) gauge theory coupled to F Dirac fermions in the fundamental representation. Expected phase structure:



- For F slightly below 11N/2 the theory is weakly coupled at all scales (Banks-Zaks).

- For small F , the theory looks like real world QCD: one scale, Λ_{QCD} ; well above that scale the theory is free; hadron masses are at that scale as well.

- Near the transition, $F \simeq F_c$, scale separation, $\mu \ll \Lambda_{QCD}$.

The physics near the transition is not well understood. It is believed that it is driven by the fermion bilinear $\bar{\psi}\psi$.

The UV scaling dimension of this operator is three, but its IR scaling dimension is lower due to gauge interactions. It is believed that (at large N) as $F \to F_c$, the IR dimension approaches two and for $F < F_c$ it becomes complex.

This behavior signals an instability of the Coulomb phase due to a non-vanishing beta function for the ``double trace" operator $(\mathrm{Tr}\bar{\psi}\psi)^2$, which leads to the appearance of the small mass gap μ .

Near the transition one has:

$$\mu \sim \Lambda_{QCD} \exp(-a/\sqrt{F - F_c})$$

Miransky scaling

Conformal Phase Transition.

Defect fermions

Another class of systems that exhibits conformal phase transitions is systems of fermions localized on a d-1 dimensional defect interacting with gauge fields and scalars propagating in the bulk. In that case the parameter controlling the transition is the gauge coupling, which can be treated as a free parameter by taking the bulk theory to have a line of fixed points (e.g. N=4 SYM).

Such systems can be embedded in string theory as low energy theories on D-branes. The gauge fields live on D-branes, while the fermions are localized at intersections. Their dynamics can be studied using standard string theory techniques. In some regimes in parameter space this analysis can be done and one finds Miransky scaling at a critical value of the `t Hooft coupling λ.

More generally, conformal phase transitions occur when two fixed points of the RG approach each other, merge, and move off into the complex plane (Kaplan, Lee, Son, Stephanov).
In the examples mentioned above, the two fixed points that merge differ by the coefficient of a double trace operator $\mathcal{O}^2$, with $\mathcal{O}$ a fermion bilinear.

In general such transitions are difficult to study since the fixed points in question are strongly coupled. It is natural to ask whether holography can be used to study them. The purpose of this work is to develop such a description.

The fact that the transition is continuous makes it natural to look for a description in terms of the order parameter $\mathcal{O}$. In the bulk this operator correspondence to a scalar field T whose mass approaches the Breitenlohner-Freedman (BF) bound. Thus, to describe the transition in d dimensional QFT, we need to study the dynamics of T in AdS_{d+1} .

What Lagrangian should we take for T ? Intuition from Landau-Ginzburg theory of phase transitions might lead one to expect that we only need to consider the behavior of the Lagrangian for small and slowly varying T, but this is not true, essentially because CPT's are infinite order transitions, as is clear from the Miransky scaling formula.

We will pick a class of Lagrangians and study their dynamics. This should be understood as a bottom-up approach that is useful for exploring possible universality classes. Which universality class corresponds to a particular transition is a separate question, that we will not address. One advantage of this class is that any CPT that occurs in a system which has a (probe) D-brane description is described by a Lagrangian of this sort.

TDBI description

In the examples mentioned above, the bulk field T is dual to a bilinear in fields trasforming in the fundamental representation of the gauge group.
Thus, it is an open string tachyon.
The dynamics of such tachyons has been extensively studied, and is known to be well described by the Tachyon Dirac-Born-Infeld action.

For a real tachyon, it takes the form:

$$\mathcal{S} = -\int d^{d+1}x V(T)\sqrt{-G} = -\int d^{d+1}x\sqrt{-g}V(T)\sqrt{1+g^{MN}\partial_M T\partial_N T},$$

$$G_{MN} = g_{MN} + \partial_M T \partial_N T$$

where g_{MN} is the bulk (AdS) metric, and V(T) is the tachyon potential.

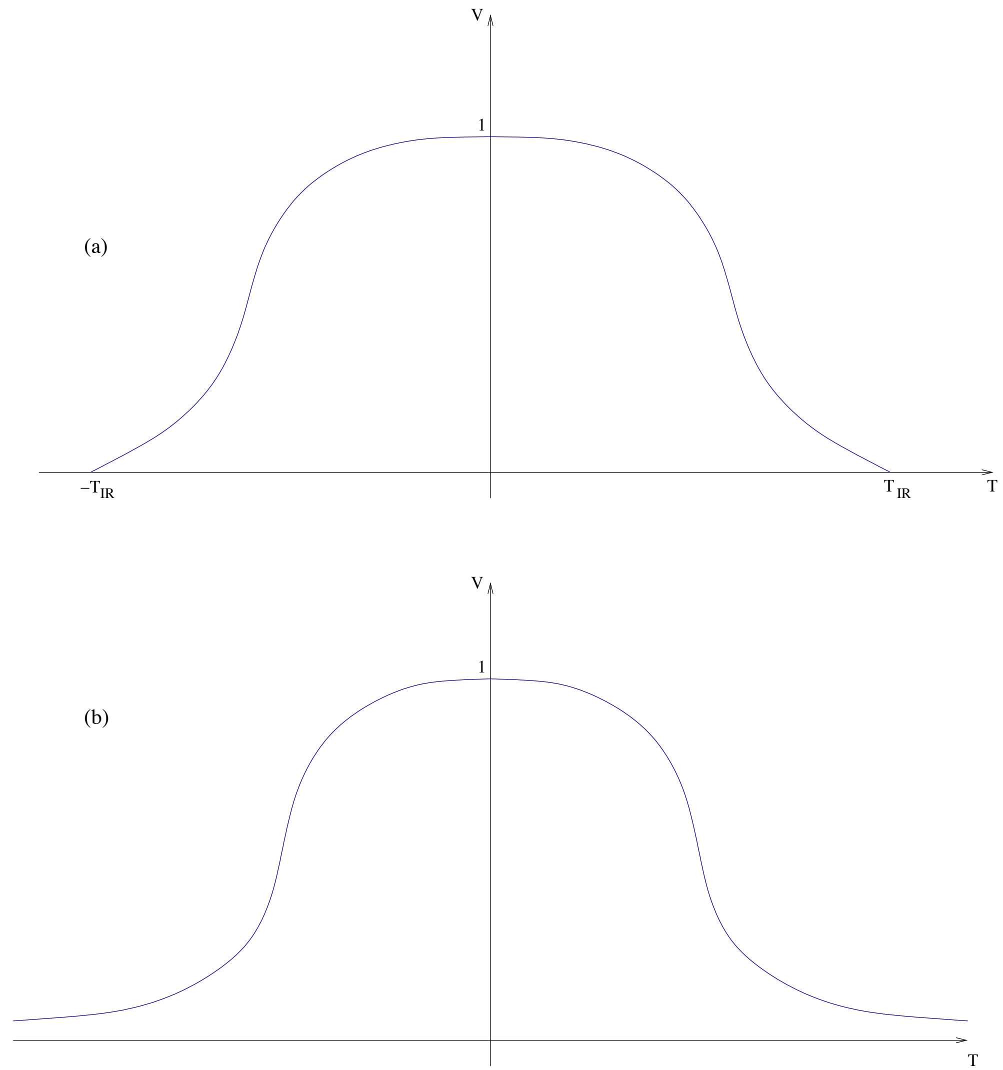
For small T it behaves like

$$V(T) = 1 + \frac{1}{2}m^2T^2 + \cdots$$

where the mass m is taken to be close to the BF bound

$$m_{BF}^2 \ = \ -d^2/4$$

For larger T it has the form



We will see that there is a qualitative difference between the two cases.

In the remainder of the talk I will describe the physics of the phase transition described by this class of Lagrangians (see papers for details).
I will discuss:

(1) Vacuum structure.

(2) Small excitations (mesons).

(3) Finite temperature phase transition.

Vacuum structure

To find the vacuum we need to minimize the energy function,

$$\mathcal{E} = \int drr^{d-1}V(T)\sqrt{1+r^2T'(r)^2}.$$

where r is the radial direction in AdS.

The boundary conditions are:

UV: $T(r=\Lambda)=0$

IR: $\frac{\partial r}{\partial T}(T=T_{IR})=0$

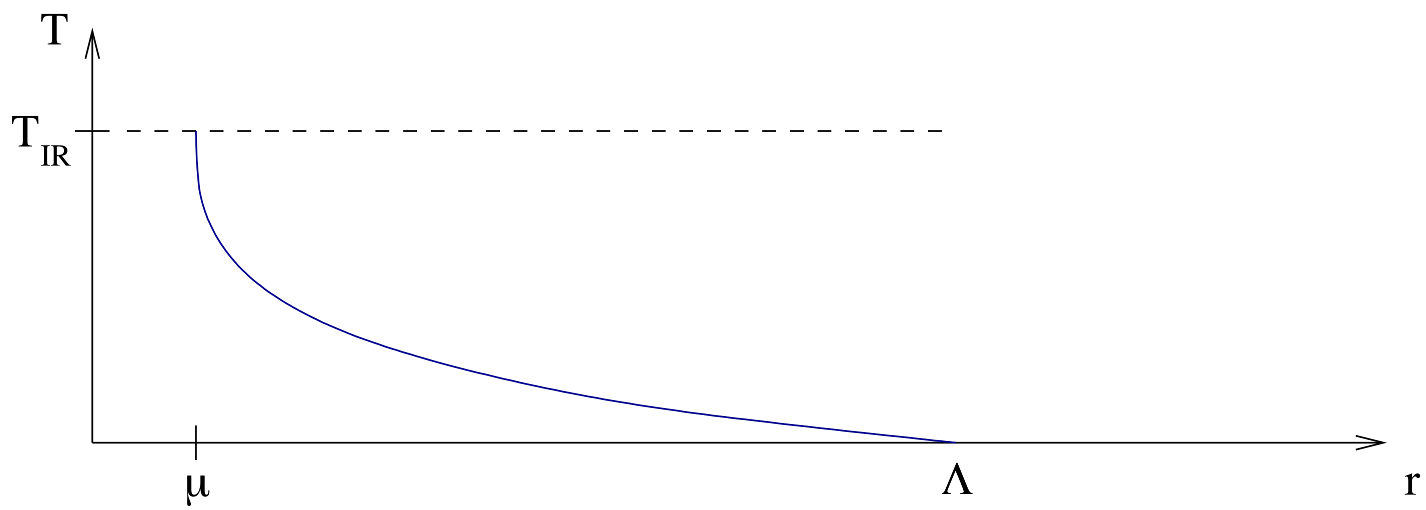
The form of the vacuum depends on the sign of

$$\kappa = m_{BF}^2 - m^2$$

For $\kappa < 0$ the vacuum is trivial, $T(r) = 0$, and conformal symmetry is unbroken.

For $\kappa > 0$ the trivial vacuum is unstable, the lowest energy configuration has $T(r) \neq 0$ and the conformal symmetry is broken.

The form of the vacuum solution is different for the two classes of potentials mentioned above. For potentials with finite T_{IR}, the vacuum is



In particular, a region of size μ is excised from the space. The resulting models are similar to hard wall models of AdS/QCD.

The dynamically generated scale μ is given by

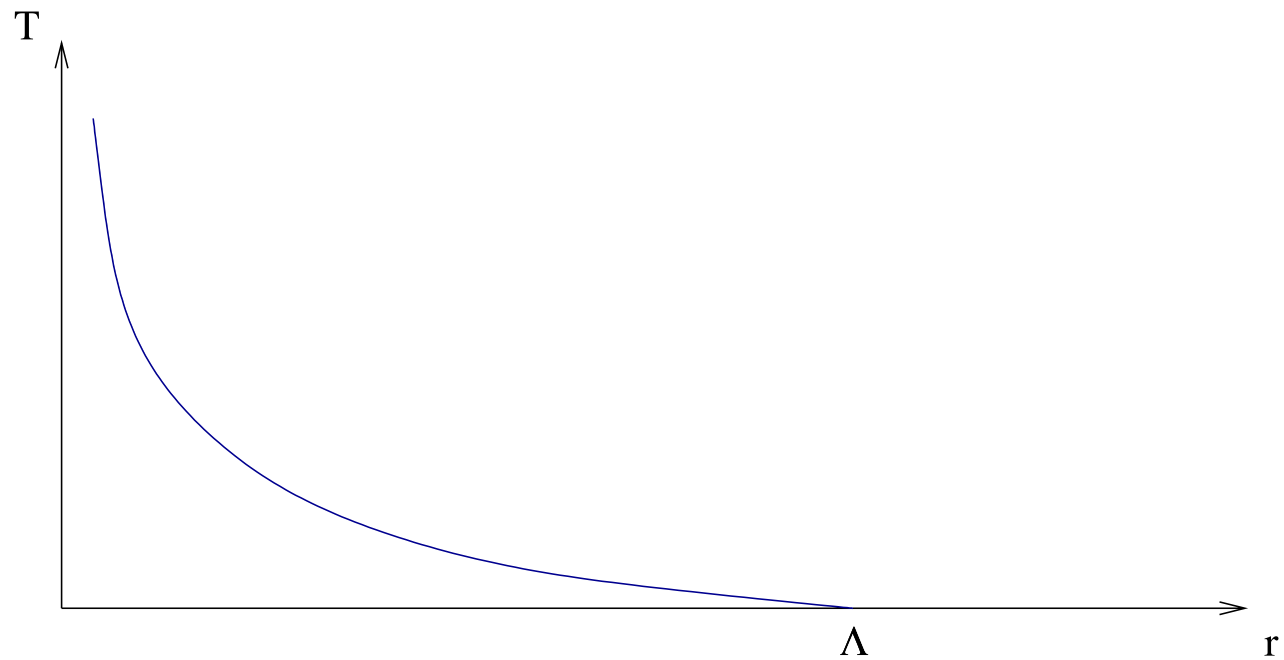
$$\mu \simeq \Lambda \exp(-c/\sqrt{\kappa})$$

Thus, as one approaches the transition it becomes parametrically smaller than the UV cutoff. One can focus on the physics at the scale μ by taking the double scaling limit

$$\Lambda \to \infty; \kappa \to 0$$

with μ held fixed.

For infinite T_{IR} the vacuum solution takes the form



In this case there is no excised region in space. The dynamically generated scale can be thought of as the value of r for which T takes some prescribed value, say T=1.

- The resulting dynamics is very similar to that of soft wall AdS/QCD.

- The ``open string'' metric G_{MN} is approximately AdS; the soft wall is provided by a non-trivial effective dilaton.

- The dynamically generated scale again satisfies Miransky scaling.

Mesons

One can use the bulk description to study the spectrum of small excitations, which give rise to mesons. We will only discuss scalar mesons. One can study vector mesons by including gauge fields in the TDBI action, and pseudoscalars and axial vectors by studying the complex TDBI.

One expands

$$T(x^M) = T_0(r) + y(x^M)$$

and writes the quadratic action for y

$$\mathcal{S}_2 = -\frac{1}{2}\int d^d x dr \sqrt{-G} e^{-\Phi} \left(G^{MN}\partial_M y \partial_N y + m^2(r) y^2\right)$$

where

$$\begin{aligned} e^{-\Phi} &= \frac{V(T_0)}{(1+r^2 T_0'^2)}, \\ m^2(r) &= \frac{\sqrt{1+r^2 T_0'^2}}{V}\left[\frac{\partial^2 V}{\partial T^2}\frac{1}{\sqrt{1+r^2 T_0'^2}} - (d-1)\frac{\partial V}{\partial T}\frac{r T_0'}{\sqrt{1+r^2 T_0'^2}} - \frac{\partial V}{\partial T}\frac{\partial}{\partial r}\left(\frac{r^2 T_0'}{\sqrt{1+r^2 T_0'^2}}\right)\right] \end{aligned}$$

After a change of coordinates

$$G_{MN}dx^M dx^N = h_{\alpha\beta}dx^\alpha dx^\beta = r^2(z)dx^\alpha dx^\beta \eta_{\alpha\beta}$$

$$\frac{dz}{dr} = -\frac{\sqrt{1 + r^2 T_0'^2}}{r^2}.$$

One gets an effective Schroedinger problem

$$-\psi'' + V_{\text{eff}}(z)\psi = \mathcal{M}^2\psi$$

where

$$V_{\text{eff}}(z) = \frac{1}{4}(B')^2 - \frac{1}{2}B'' + r(z)^2 m^2(r(z))$$

$$B(z) = \Phi(z) - (d-1)\ln r(z)$$

If one takes T(r)=0 (the symmetric solution), one finds

$$V_{\text{eff}}(z) = \left(\frac{d^2-1}{4} + m^2\right)\frac{1}{z^2} = -\left(\frac{1}{4} + \kappa\right)\frac{1}{z^2}$$

This is a well known potential in QM. For $\kappa > 0$ and in the presence of a UV cutoff $z \geq z_{UV} = 1/\Lambda$ there are normalizable states with $\mathcal{M}^2 < 0,$ i.e. tachyons.

These tachyons have an obvious interpretation: they signal the instability of the unbroken (T=0) vacuum. Their mass squared is of order $-\mu^2$.

Expanding around the stable vacua above modifies the potential at $z \sim 1/\mu$ and lifts these tachyons. Interestingly, the lowest lying of the mesons is relatively light, since the tachyonic contribution from small z is approximately canceled by that of larger z. This light scalar can be thought of as a pseudo dilaton; it is of interest in the context of technicolor.

The highly excited meson spectrum is in general interesting (for soft wall potentials). E.g. if we take the potential V(T) to behave at large T like

$$V(T) \simeq e^{-\frac{1}{2}\beta T^2}$$

we find the spectrum

$$m_n \sim \mu n^{\frac{1}{2}\left(1-\frac{d}{\beta}\right)}.$$

To study the low lying spectrum one needs to specify the tachyon potential V(T). For $V(T) = (\cos T)^{\alpha}$, which is relevant for the defect fermion system, one finds $(\bar{\mu} \sim \mu/\sqrt{\lambda})$

σ - mesons: $m^2/\overline{\mu}^2 \approx 0.44, 9.65, 26.63, 51.35, 84, \cdots$

vector mesons: $m^2/\bar{\mu}^2 \approx 3.08, 15.12, 34.87, 62.32, 97.46, \cdots$

The anomalously light scalar meson is the techni-dilaton.

In technicolor there is a long-standing debate about the fate of the dilaton near the CPT in QCD. There are two schools of thought:

(1) $m_{\rm td}/m_{\rm meson} \to 0$

(2) $m_{\rm td}/m_{\rm meson} \to \rm const$

as $\mu/\Lambda \to 0.$

We find that (2) is correct.

Intriguingly, in QCD it was argued (by M. Hashimoto and K. Yamawaki) that the mass of the techni-dilaton is smaller than that of the lightest vector meson by a factor of about 2.8. In our system this ratio is about 2.6...

It would be interesting to explore the range which this ratio takes as one changes the potential V(T).

Finite temperature

To introduce finite temperature we replace the AdS background by an AdS black hole. Even if the zero temperature theory is in the broken phase, the symmetry is restored at a finite temperature of order μ . Interestingly, the nature of the phase transition is only sensitive to the behavior of the potential V(T) near the origin.

Parametrizing

$$V(T) = 1 + \frac{1}{2}m^2T^2 + \frac{a}{4}T^4 + \cdots$$

for a below a critical value the transition is first order, while above this value it is second order.

Summary

- Holography provides a useful tool for studying conformal phase transitions. In this approach, one needs to analyze the dynamics of a scalar field in AdS, with mass close to the BF bound.
- The universal physics near the transition is sensitive to the full non-linear Lagrangian for T.

- We took the Lagrangian to have the TDBI form and found that the dynamics is similar to that of hard and soft wall AdS/QCD (depending on the potential V(T)), with a dynamically generated wall.

- We analyzed the spectrum and thermodynamics of these models and found some properties that are different from standard holographic models, such as a second order symmetry restoration transition.

- It would be interesting to analyze in more detail the properties of the lightest scalar meson, which plays the role of the techni-dilaton in technicolor theories.

- It would also be interesting to further study the finite temperature dynamics and look for applications to condensed matter and other systems.

Thank You!