Strongly coupled near- or conformal systems with fundamental fermions

SCGT12 Workshop

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Outline & Summary

- Why are these calculations difficult?
  - Conformal systems are conformal in the chiral limit only

    → Simulations at finite mass entangle conformal & chirally broken behavior
  - Lattice simulations are often forced to strong coupling

    → Spurious fixed points can change the scaling behavior!
- SU(3) gauge system with 12 flavors
  - MCRG with improved gauge action

    → Emergence of an IRFP (A.H. 1106.5293)
  - The phase structure at zero and finite temperature

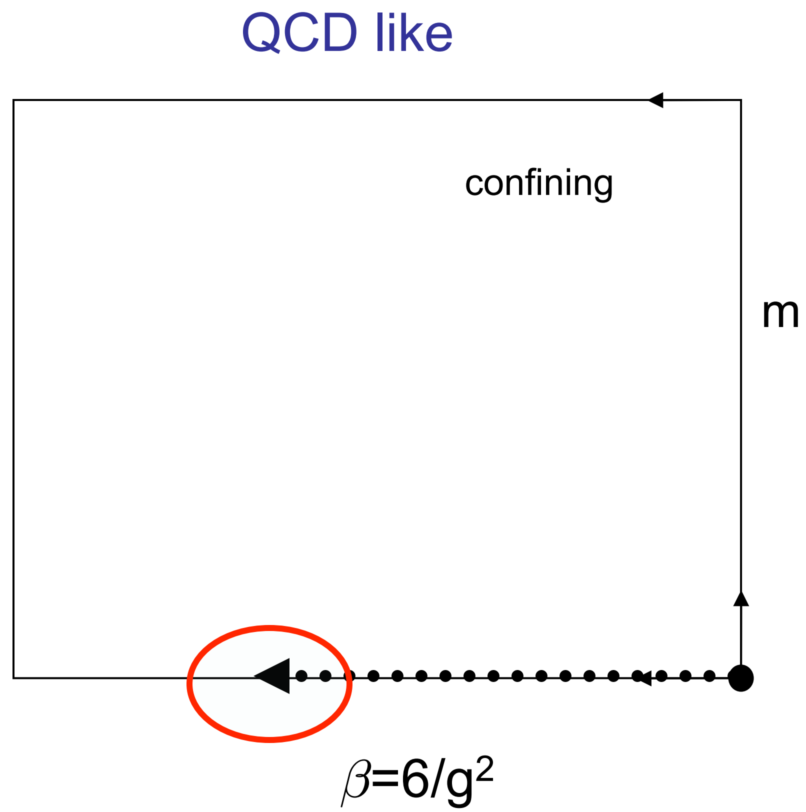
    →Novel phase with new symmetry breaking pattern

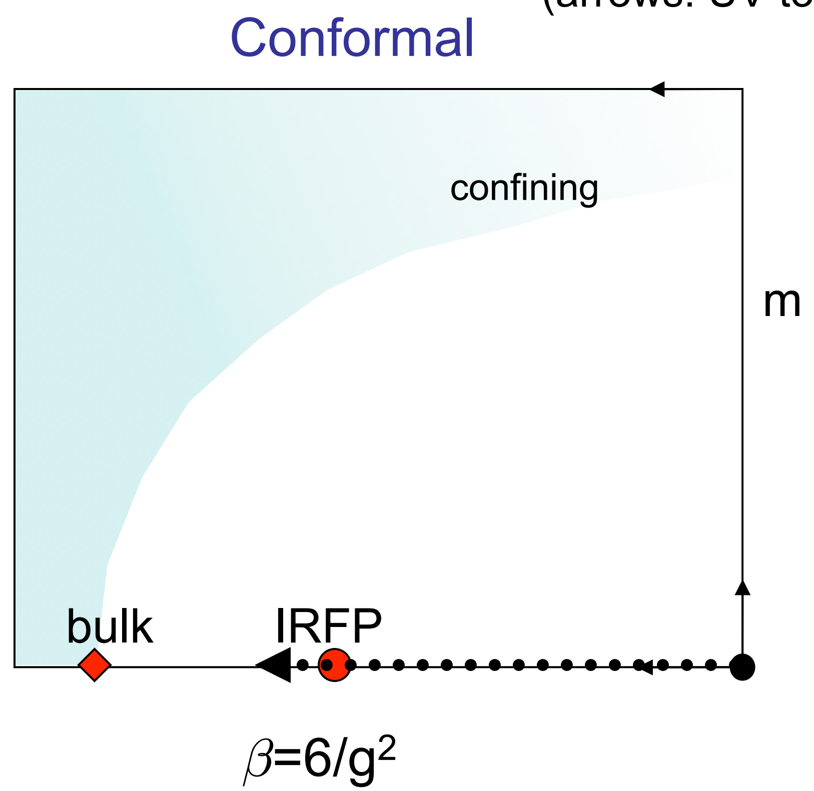
    (A. Cheng, A.H., D. Schaich)
- SU(3) gauge system with 8 flavors
  - Novel phase is present – implications for simulations & IR behavior

    (A. Cheng, A.H., D. Schaich, G. Petropoulos, in preparation)

How can we distinguish QCD-like and conformal systems?

(arrows: UV to IR)



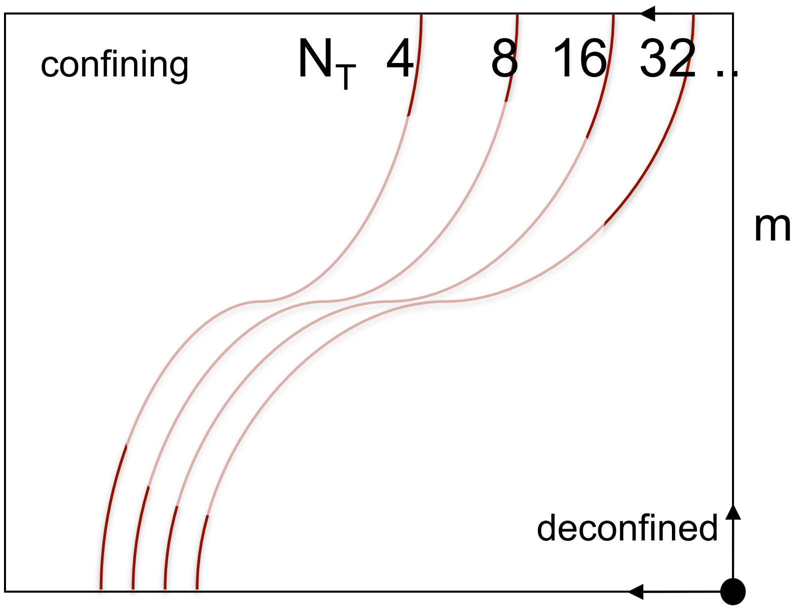


Lattice simulations can connect the perturbative FP and strong coupling

- Found IRFP ? Done ✔
- No IRFP? Show that it is confining before a bulk transition is reached

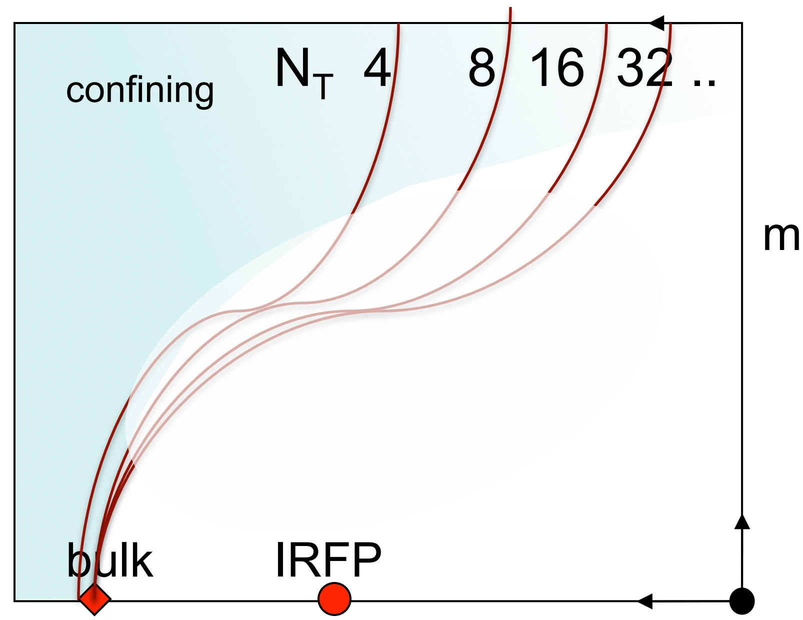
Finite temperature and bulk phase transitions

QCD like



$\beta_c \to \infty$ as $N_T \to \infty$

Conformal



$\beta_c \to \beta_{bulk}$ as $N_T \to \infty$

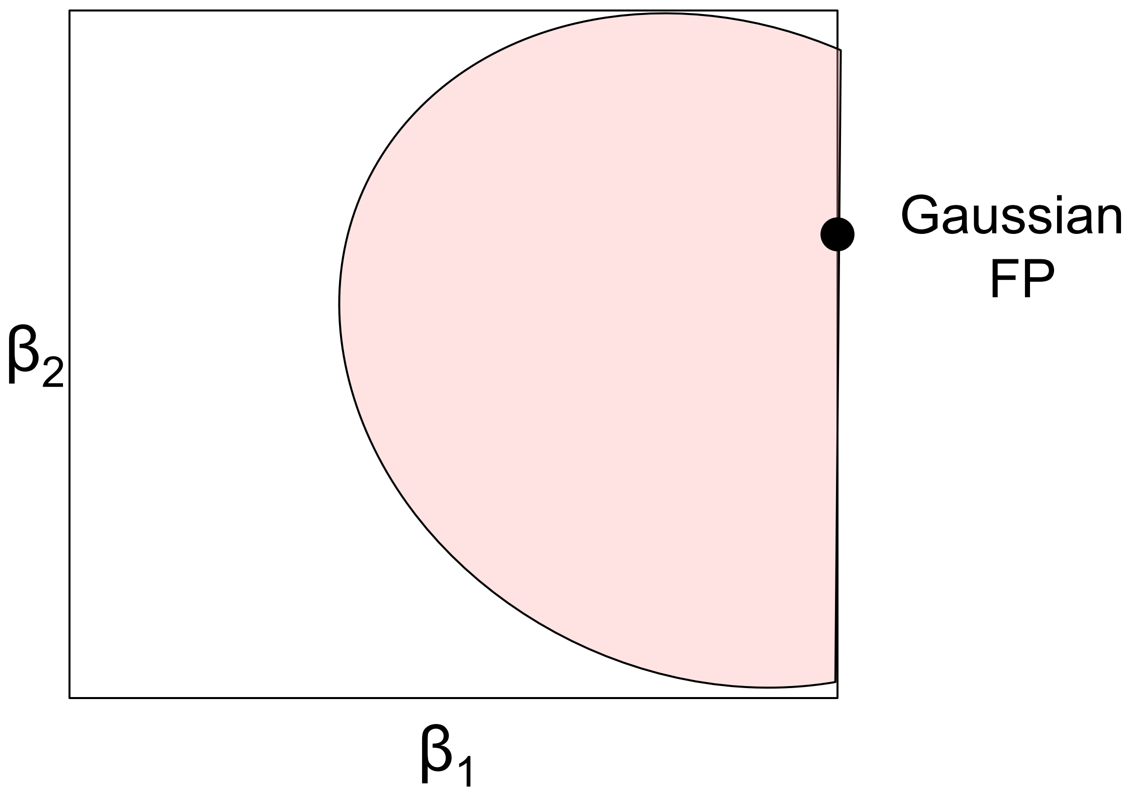
In a conformal system

- finite temperature transitions run into a bulk (T=0) transition
- β_{bulk} separates strong coupling (confining) and weak coupling (conformal) phases

Fixed points and their basin of attraction

In QCD like systems continuum limit is defined at the Gaussian UVFP

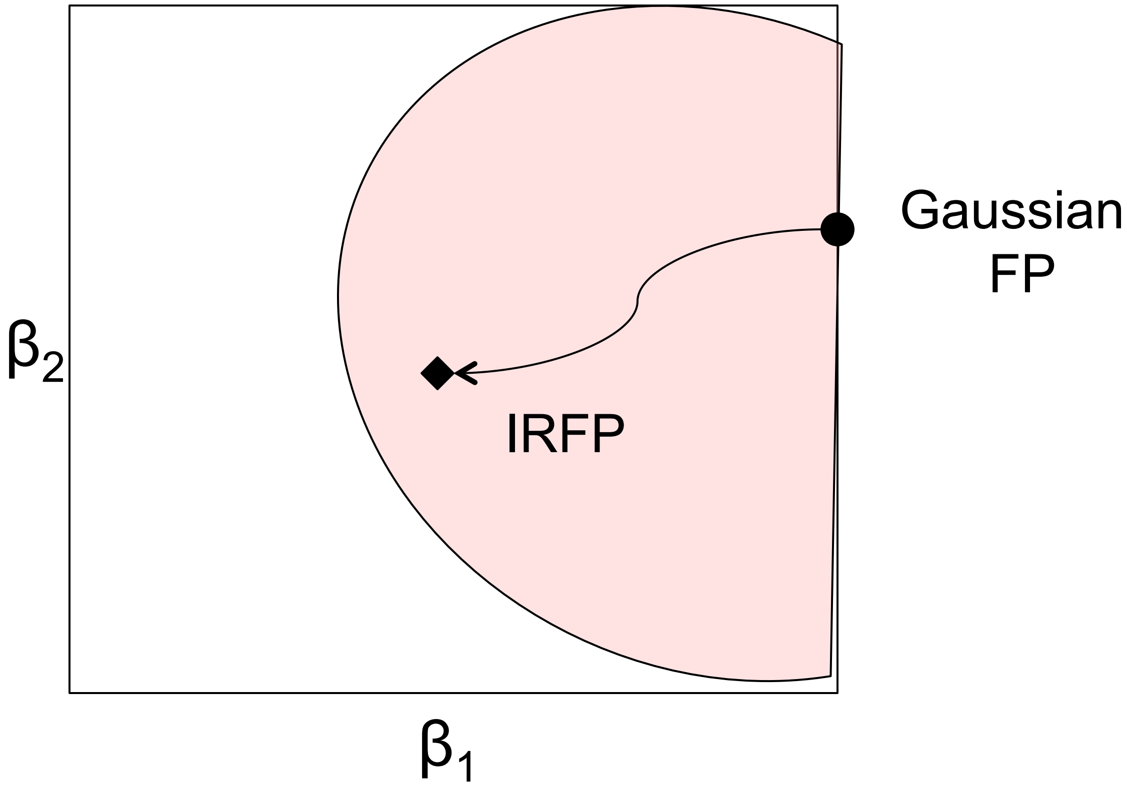
Continuum scaling is expected in the basin of attraction of G-FP



Fixed points and their basin of attraction

In conformal systems there is a new IRFP

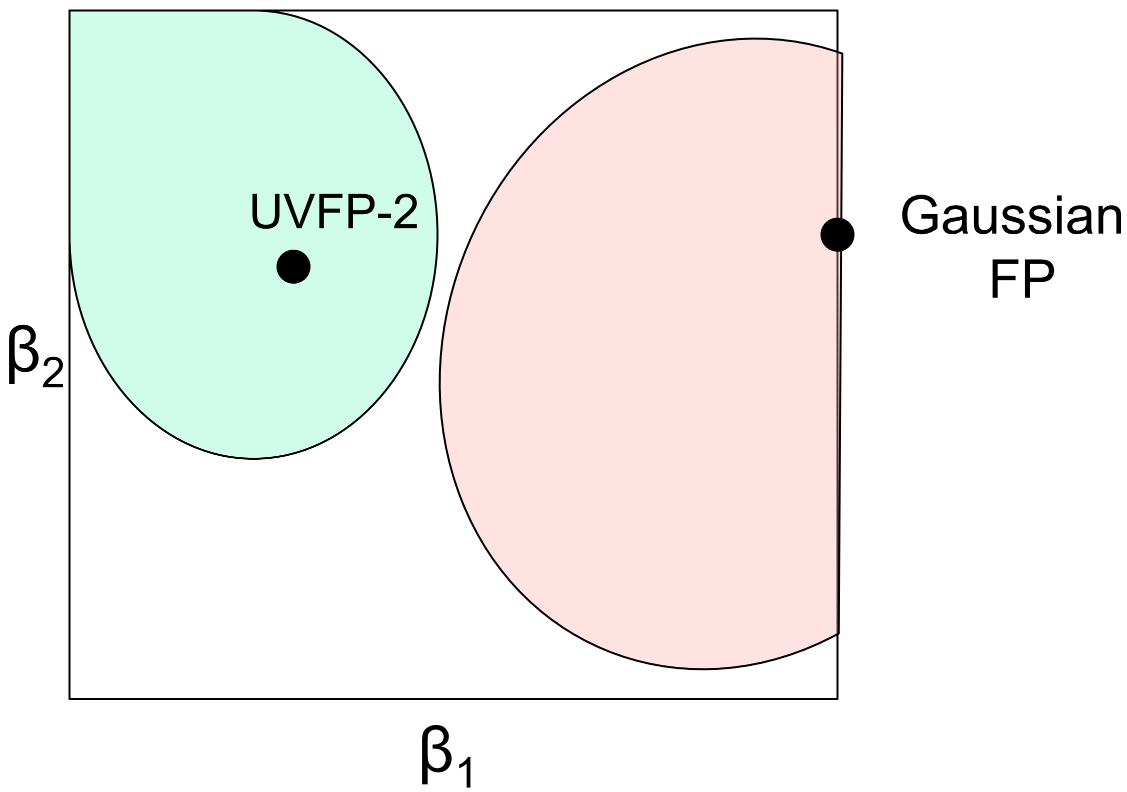
- asymptotically free around G-FP,
- the conformal behavior in the infrared around the IRFP



Fixed points and their basin of attraction

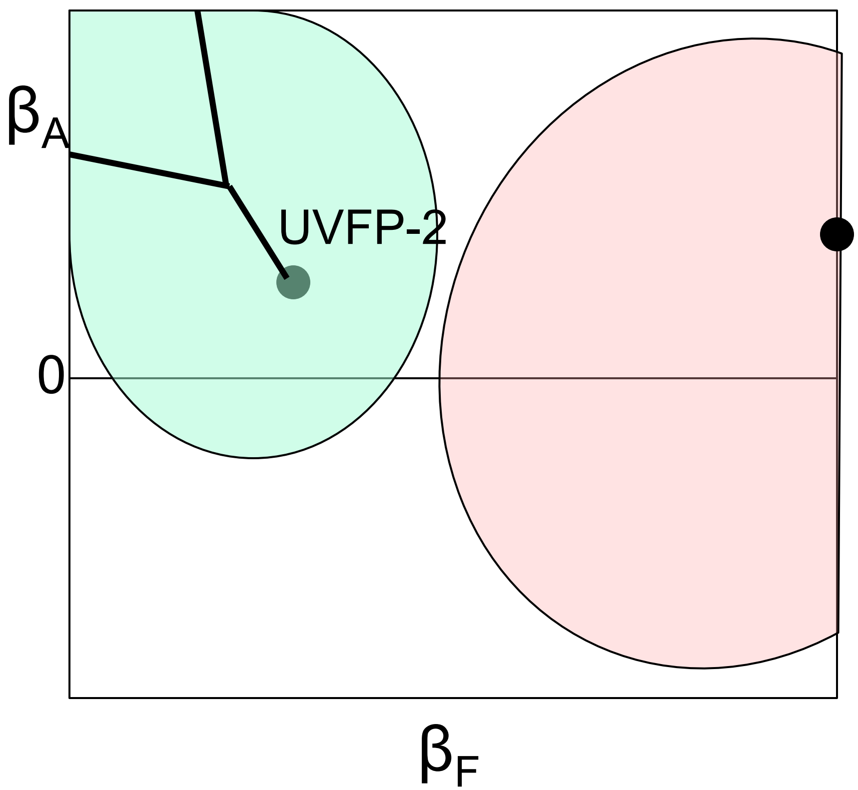
If there are two UV fixed points, continuum limit can be defined at both.

The basin of attractions are exclusive, stay in one or the other to get desired continuum scaling!



Fixed points and their basin of attraction

Pure gauge SU(2), SU(3) has this structure in the fundamental-adjoint plaquette plane: 1st order transitions ending in a 2^{nd} order endpoint

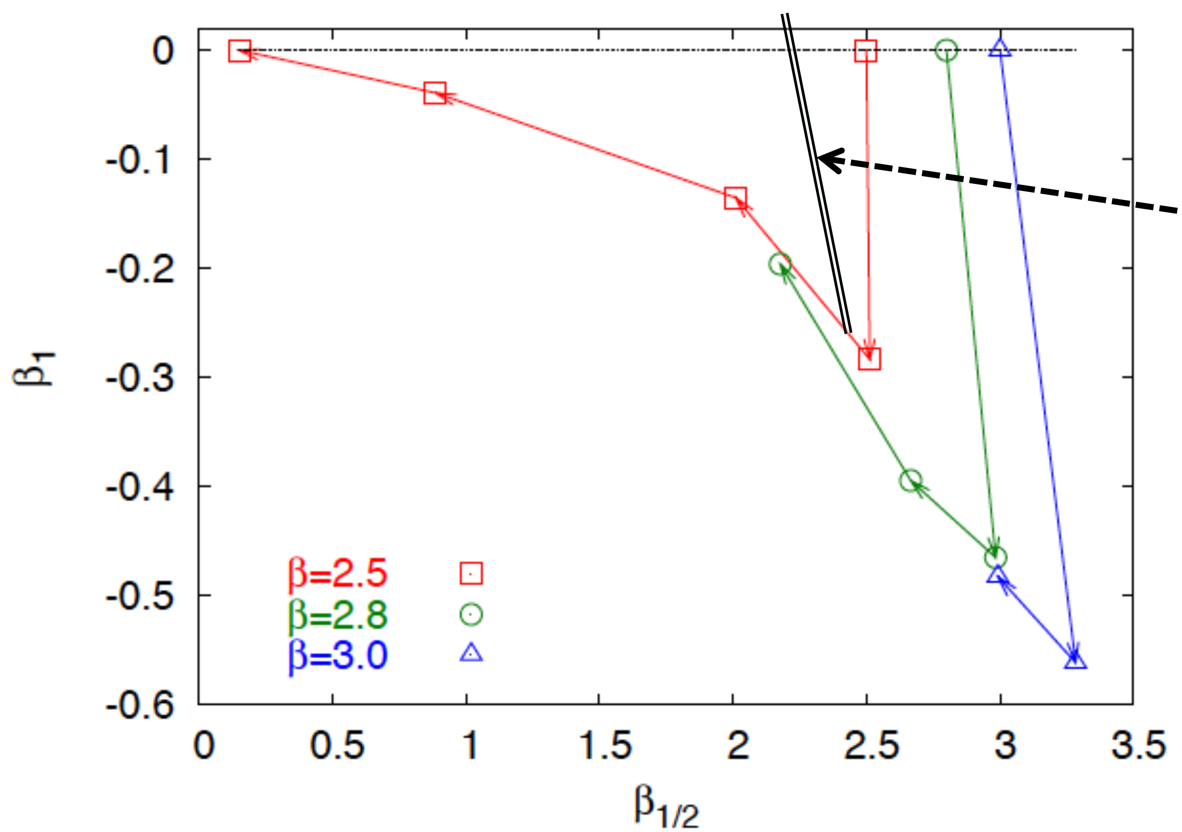


Gaussian FP

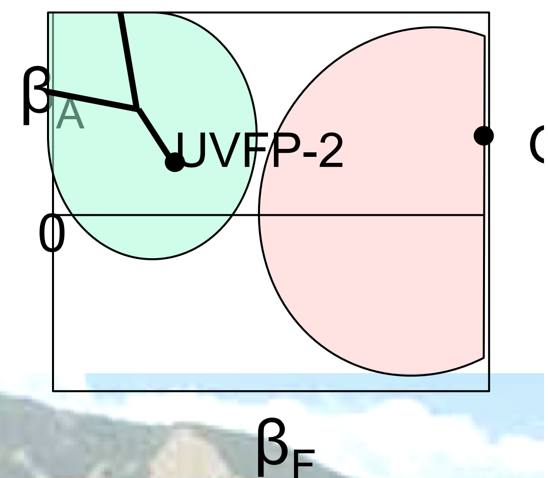
Between the basins of attraction is a no-man's land

RG flow in the fundamental-adjoint plane

RG flow in pure gauge SU(2) Tomboulis,Velitski (hep-lat/0702015)
The flow runs away from the first order line/end point:



Extension of the first order line

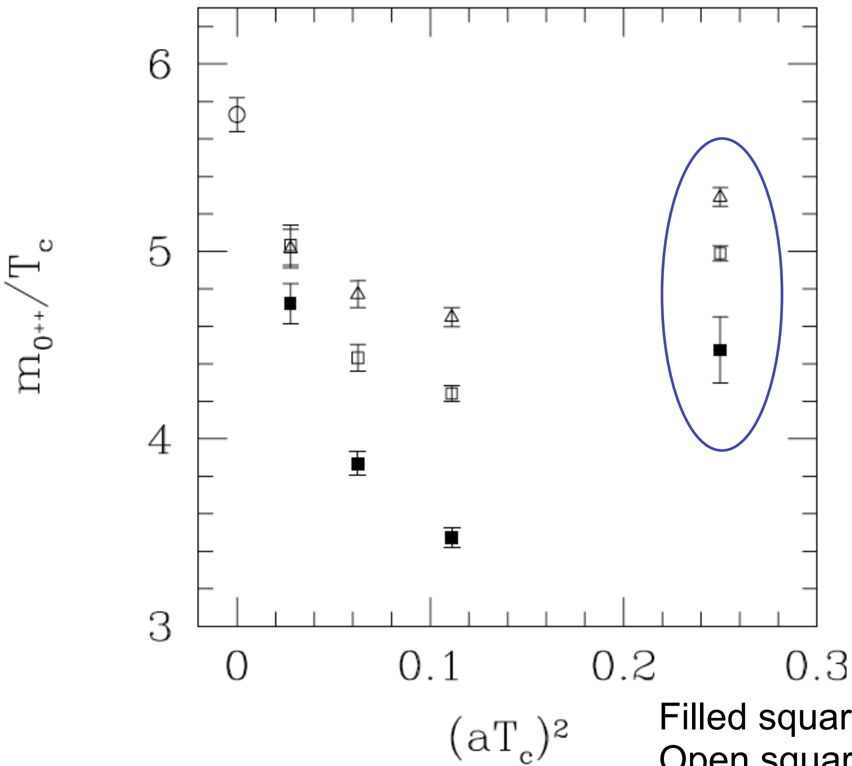


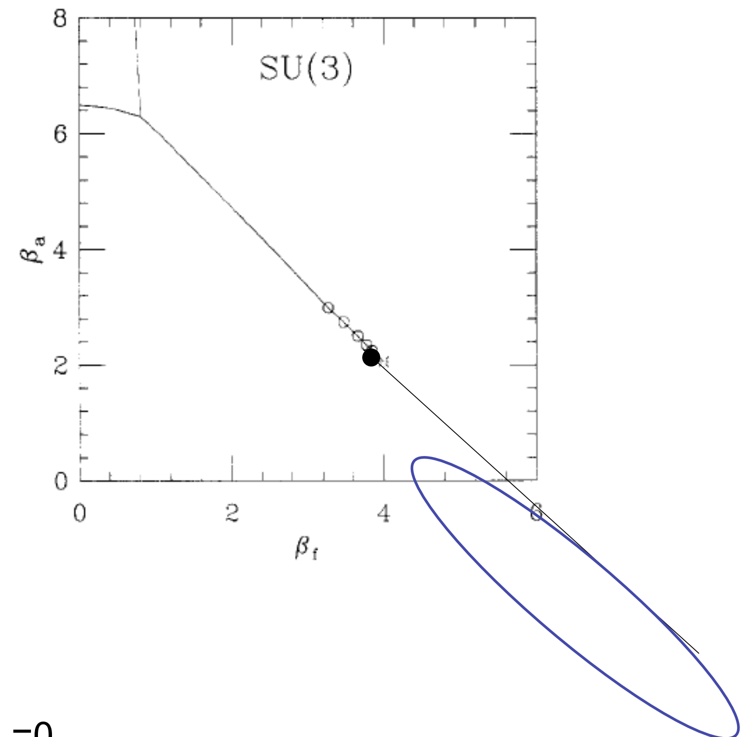
Scaling in the fundamental-adjoint gauge action

SU(3) pure gauge theory

Hasenbusch,Necco JHEP08(2004)005:

Scaling test of the glueball, T_c and r_0 at β_A=0, - 2.0, - 4.0 shows scaling breaks down on the strong coupling side

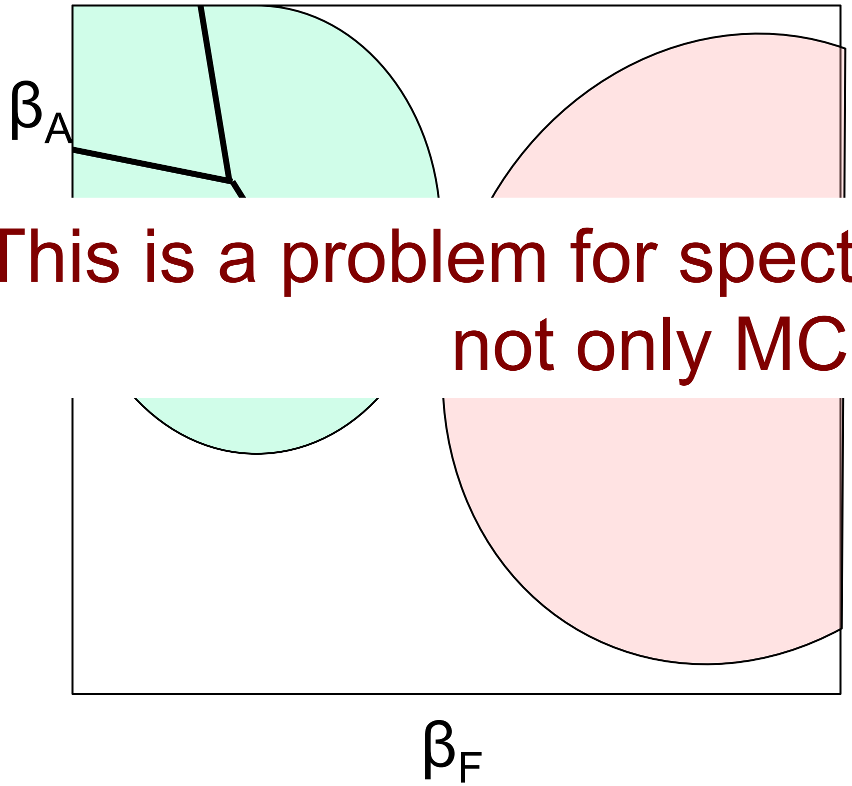




Filled squares: β_A=0
Open squares: β_A=-2.0
Triangles : β_A=-4.0

Is UVFP-2 a problem?

- Not for QCD simulations, those are on the weak coupling side.
- BSM models are strongly coupled and simulations can end up in the wrong FP basin



This is a problem for spectral studies as well, not only MCRG!

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  - MCRG with improved gauge action
    - → Emergence of an IRFP (A.H. 1106.5293)
  - The phase structure at zero and finite temperature
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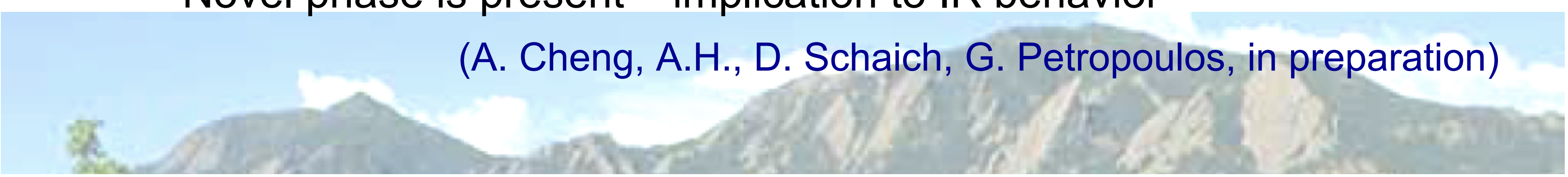
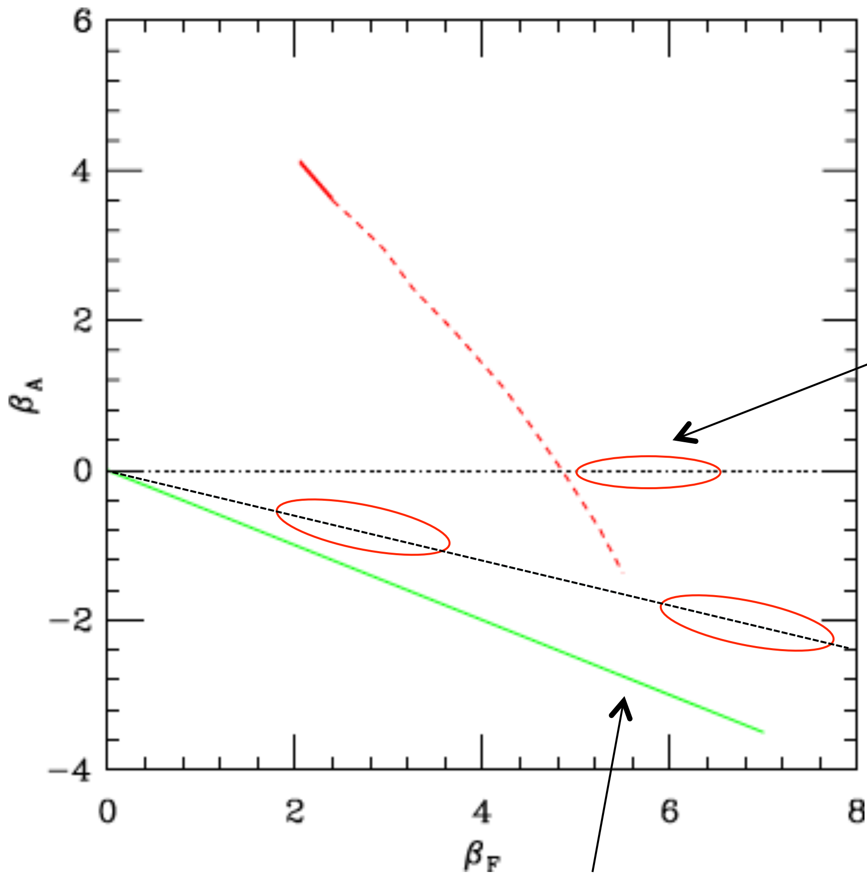


(A. Cheng, A.H., D. Schaich, G. Petropoulos, in preparation)

SU(3) gauge with N_f=12 fundamental flavors

- Controversial system, likely very close to the conformal window.
- I use nHYP staggered fermions (very good taste restoration) with fundamental+adjoint plaquette gauge action
- Fermion masses are tiny
  - In MCRG studies the simulations can be considered to be in the chiral limit
  - In phase structure studies we investigate finite volume/mass scaling

SU(3) gauge with N_f=12 fundamental flavors

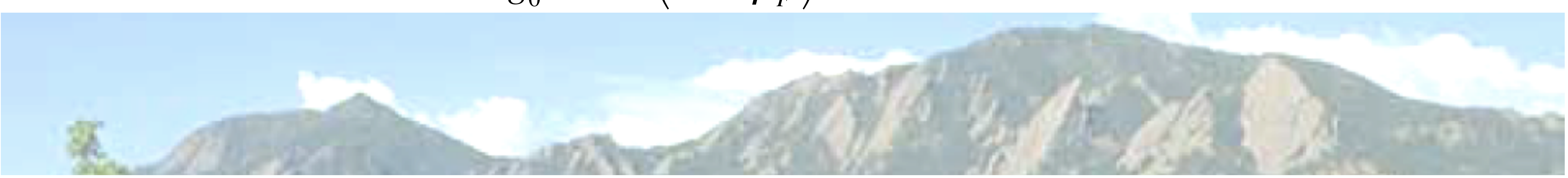


MCRG at β_A=0 (1004.1004)
remained inconclusive
(too close to crossover)

This work: β_A/β_F=-0.25

$\beta_F = 12/g^2$

$$\frac{2N_c}{g_0^2} = \beta_F\left(1+2\frac{\beta_A}{\beta_F}\right) = 0$$



The step scaling function around a UVFP

The *bare* differential step scaling function $s_b(\beta)$

$$s_b(\beta) = \beta - \beta' \quad \text{where} \quad \xi(\beta) = \xi(\beta')/2 \qquad (\beta = 2N_c/g_0^2)$$

ξ is the correlation length defined by some physical mass

$s_b(\beta)$ is the projection of the RG flow to a lower dimensional coupling space

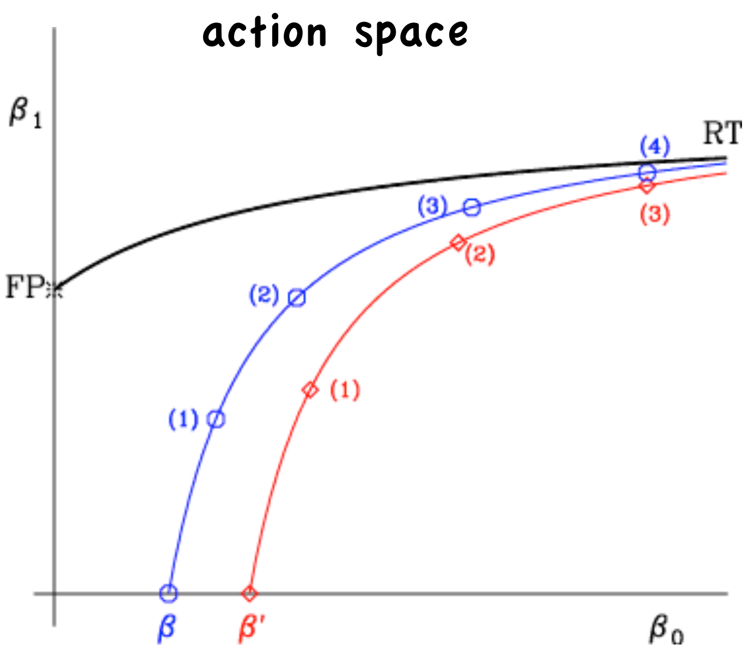
$s_b(\beta)$ *has the opposite sign of the RG* β *function*

The step scaling function & MCRG

If blocked actions match:

$$S(\beta^{(n)}) = S(\beta'^{(n-1)}) \rightarrow \xi(\beta)=2\xi(\beta') \rightarrow s_b(\beta) = \beta-\beta'$$

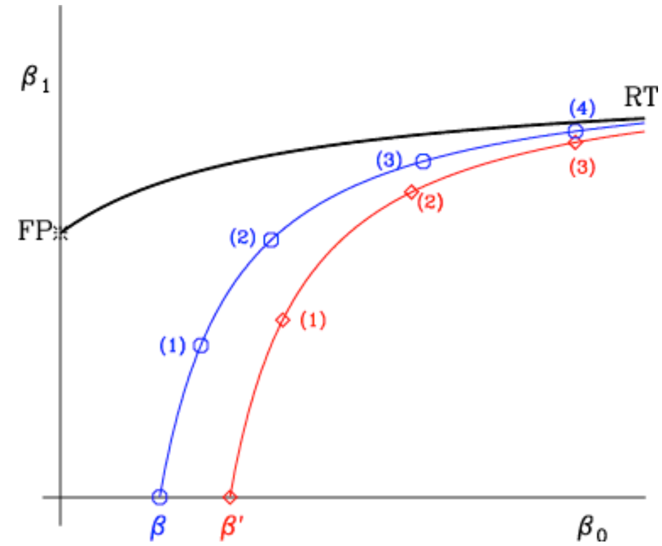
MCRG finds (β,β') pairs by matching blocked lattice actions



Two actions are identical if all operator expectations values agree

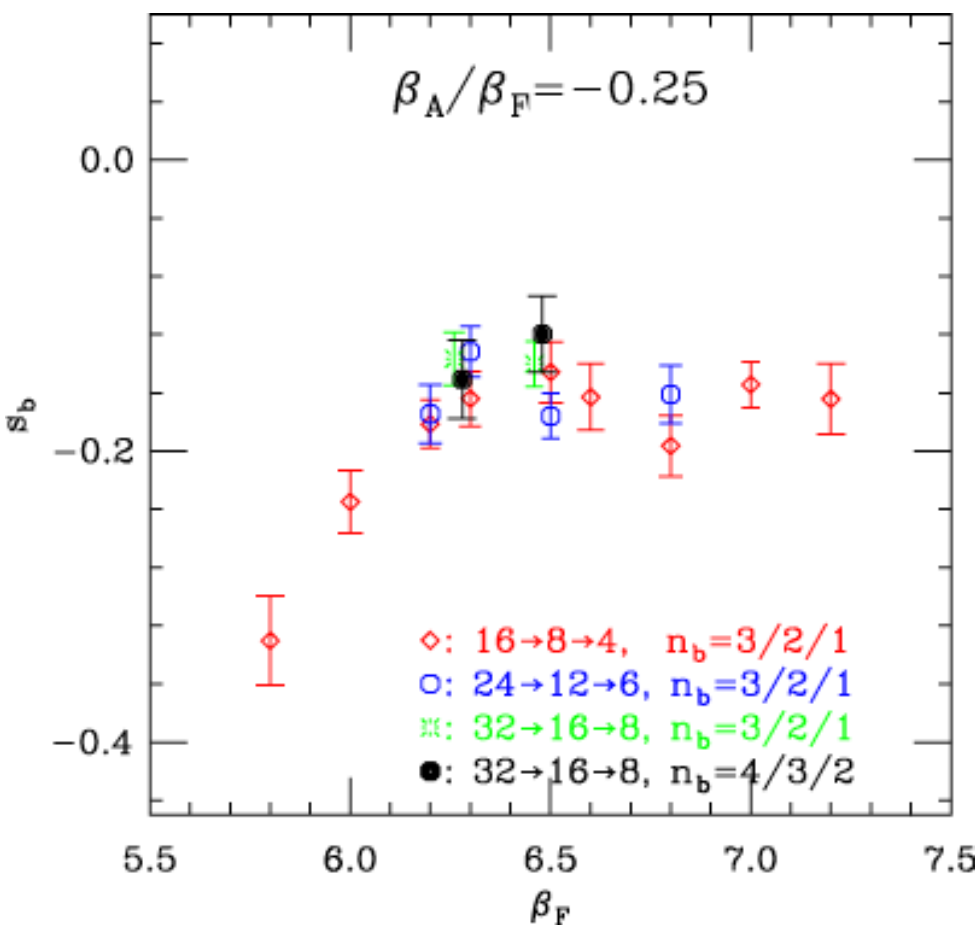
Match operators (local expectation values) after several blocking steps

Where all the bodies and bombs are buried (and what to do with them)



- The RG flow might not reach the renormalized trajectory
    - Improved blocking is essential
    - Compare different blocking levels
- The blocked lattices are small, finite volume effects are significant
    - Careful matching on identical volumes helps;
    - Compare different volumes
- Spurious lattice fixed points can effect the results
    - Check the phase structure

The step scaling function $N_f = 12$



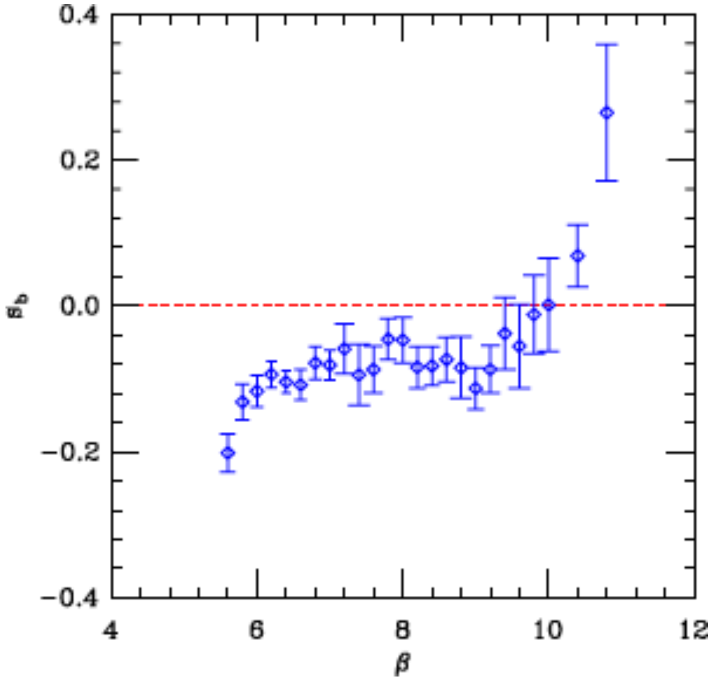
Results from many blocking levels, many volumes are all consistent.

- At $\beta_F=\infty$ the step scaling function $s_b>0$
- In the investigated β range it is negative

→ There has to be an IRFP (around/above β=11.0)

→ Indicates a conformal system

The step scaling function – a different action



With β_A/β_F = -0.15 the IRFP is closer
And MCRG can find it

(16→8 →4 matching)

Summary of MCRG matching

MCRG:

- Optimized, volume-matched MCRG gives consistent results for $\Delta\beta$ (the step scaling function)
- For Nf=12 fermions, SU(3) gauge the step scaling function is consistently negative, indicating an IRFP and conformal dynamics

Phase diagram studies

N_f=12 (and 8) flavors, SU(3) gauge + nHYP' fermions

(arXiv:1111.2317, A. Cheng, A.H., D. Schaich)

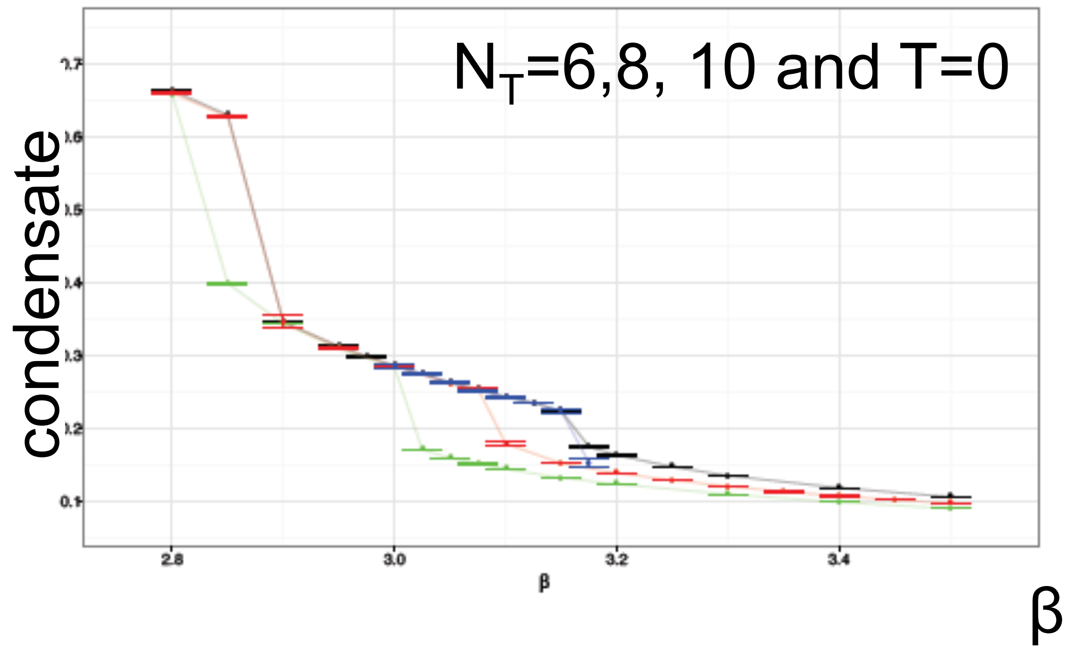
The action:

- Fundamental-adjoint gauge : $\beta_A/\beta_F = -0.25$
- nHYP projection has numerical problems when the smeared link develops near-zero eigenvalues
  - small tweak of the HYP parameters can fix that! $(\alpha_1,\alpha_2,\alpha_3)=(0.40,0.50,0.50)$ will do the trick
  - This action is 10-15 times faster than HISQ and can be pushed to stronger couplings

Previous results on the phase stucture N_f=12

Groningen-INFN group found 2 first order transitions (2010,2011)

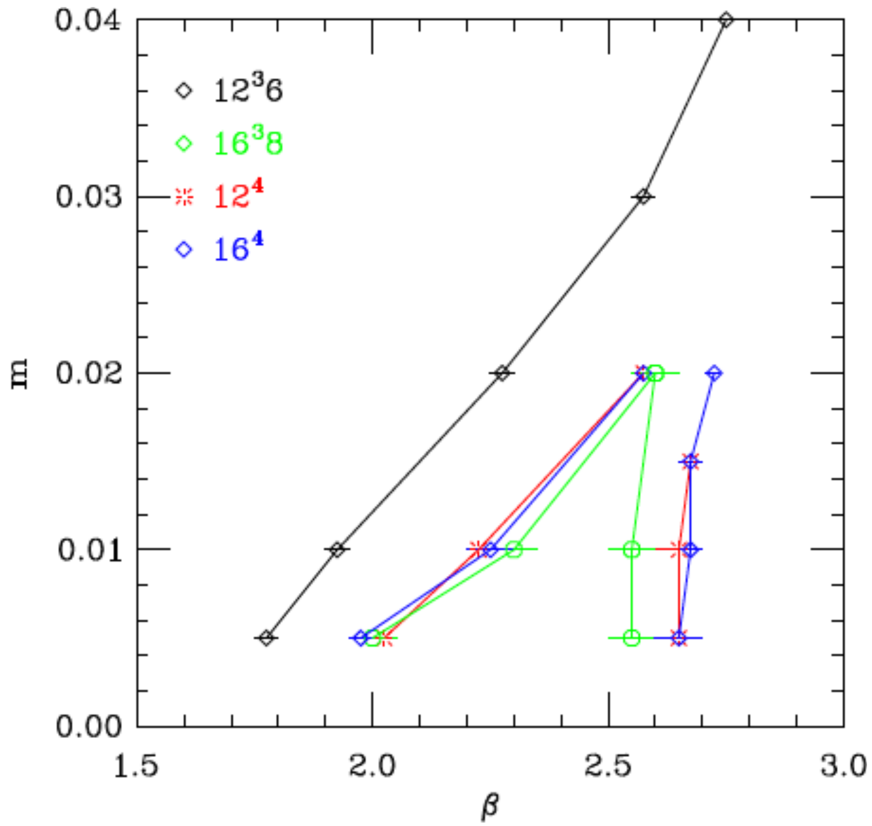
m=0.025 (tree level imp. staggered)



Both sets of transitions are converging to bulk transitions

Phase diagram β-m plane N_f=12

Our results are consistent with the phase diagram of Deuzeman et al
We use different action at several (small) mass values, volumes

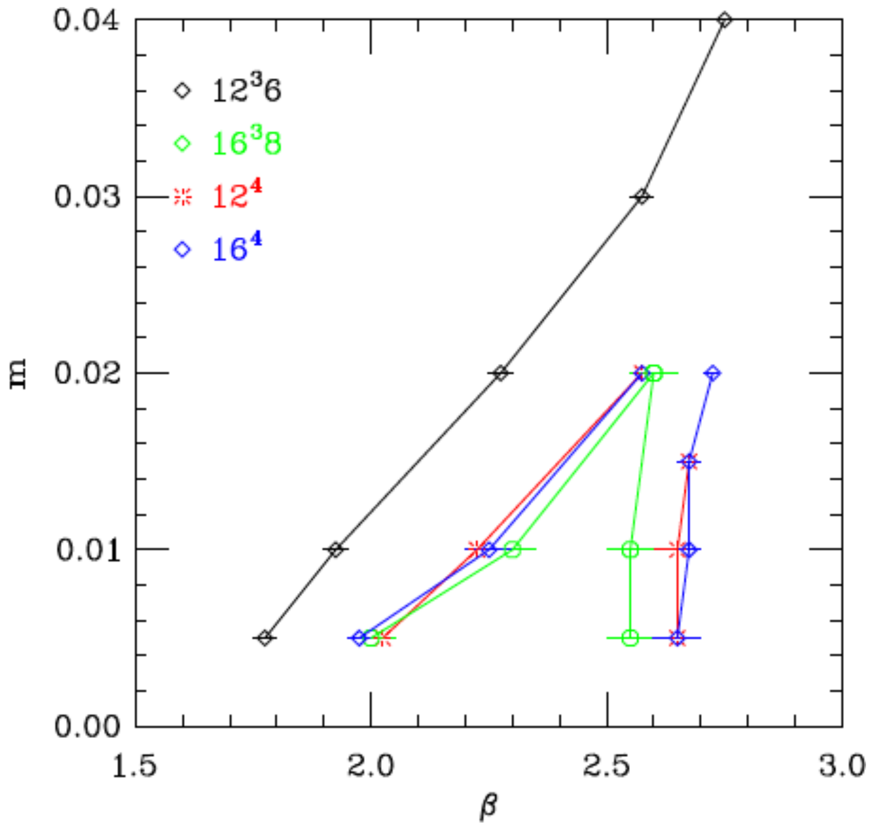


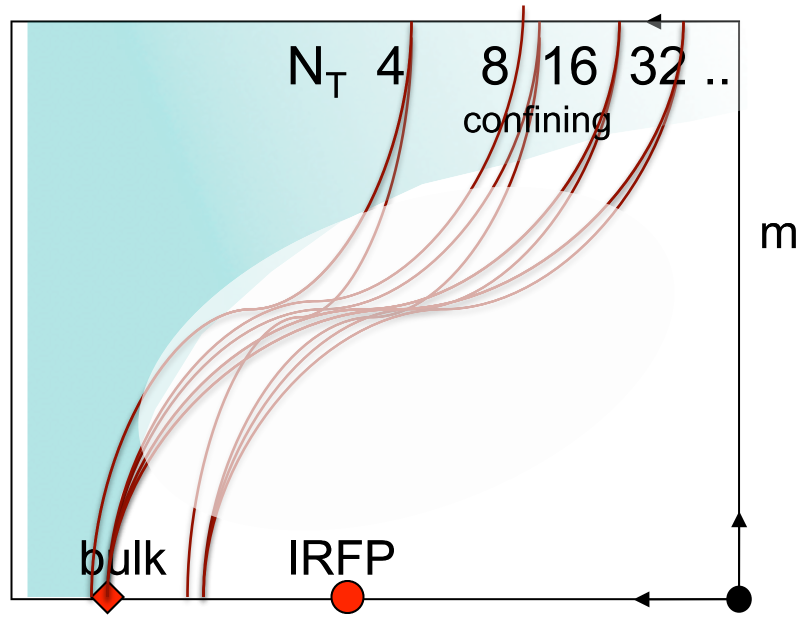
Finite temperature phase transitions converge to zero temperature "bulk" transitions

This is robust property of N_f = 12 staggered fermions

First order transitions at small mass turning into crossover

Phase diagram β-m plane N_f=12





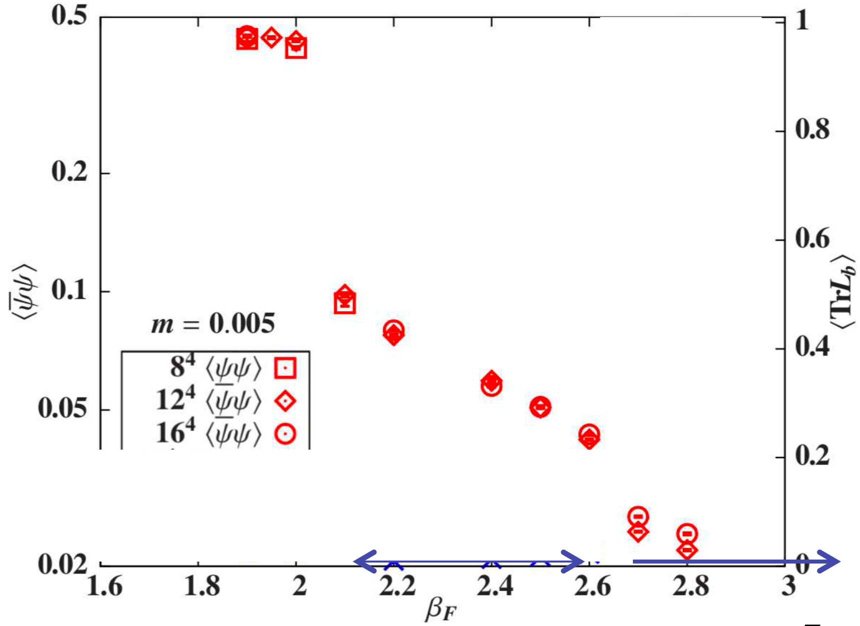
$\beta_c \rightarrow \beta_{bulk}$ as $N_T \rightarrow \infty$

It is as if the finite T transitions fissioned into two transitions before converging to bulk

Or maybe not….

Phase diagram N_f=12

What are the three phases?



<ψψ>~ 10 m <ψψ>~ 5m

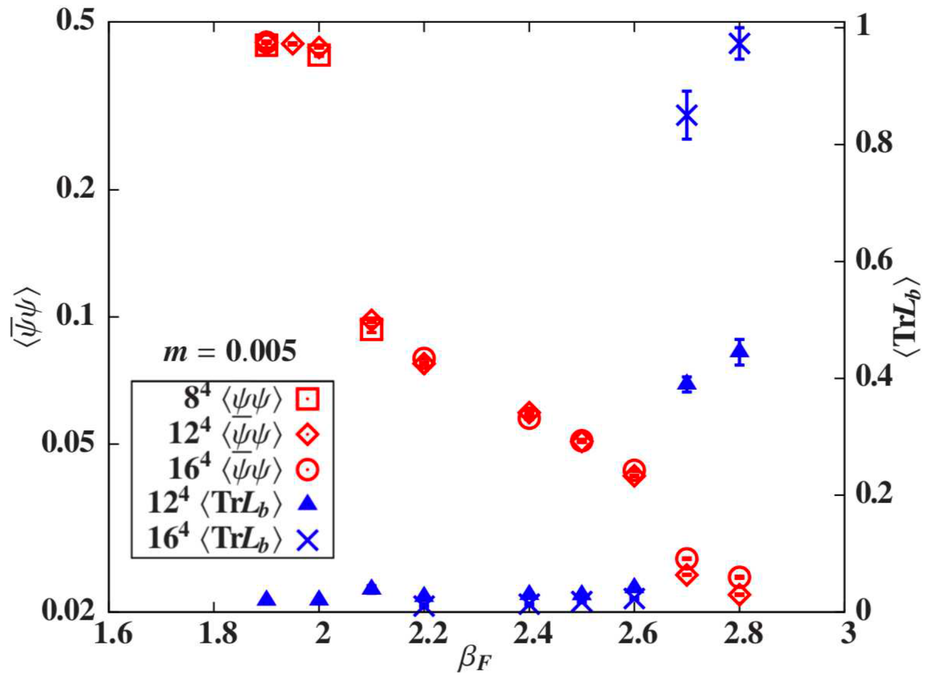
Chiral condensate extrapolates to zero in the chiral limit on the weak coupling side of the “big” jump

→ Chiral restoring transition (more on that later!)

Is it deconfining?

Phase diagram

Is it deconfining? Polyakov line is very noisy but the blocked Poly line is sensitive:



Blocked Poly line is measured on RG blocked lattices:

- improved Poly line

or

- Poly line on renormalized trajectory, after removing UV fluctuations

The blocked Polyakov line sees the “weak” transition strongly but hardly changes at the “strong” transition

The intermediate phase

possibly only a lattice artifact (bordered by 1^{st} order phase transitions)

- it is not partial restoration of taste symmetry
- it does not go away with increasing volume
- It cannot extend to $g^2=0$
- Staggered version of Aoki phase?

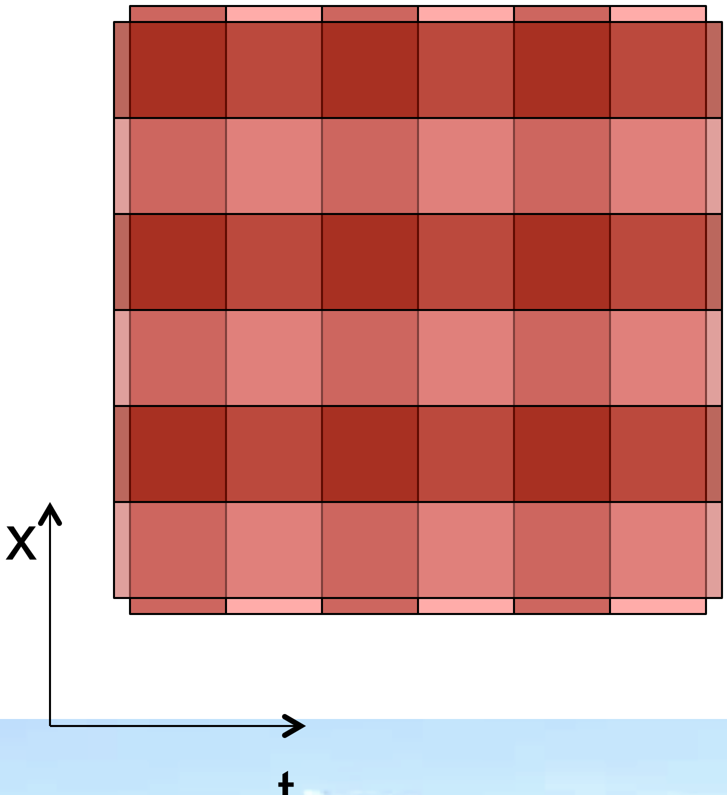
In the following I explore the properties of the IM phase (at $\beta=2.6$ $m=0.005$) and contrast it with the weak coupling phase ($\beta=2.7$, $m=0.005$)

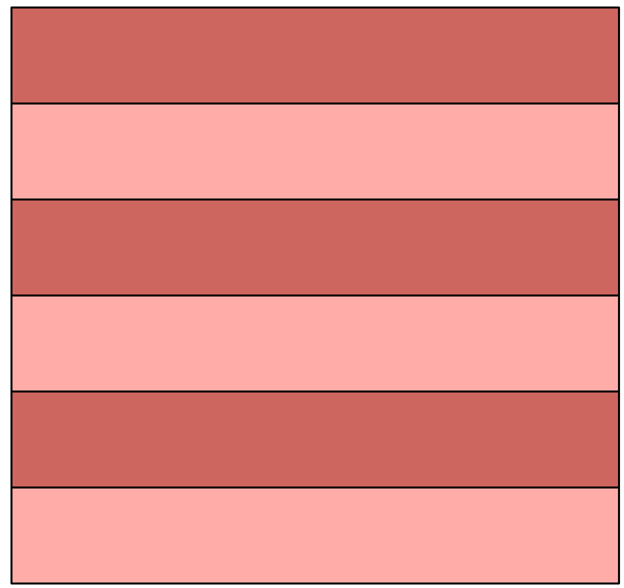
New symmetry breaking pattern

single-site shift symmetry (S^4) is broken

$$\chi_n \rightarrow \xi_\mu(n)\chi_{n+\mu} \;,\; U_{n,\mu} \rightarrow U_{n+\mu,\mu}$$

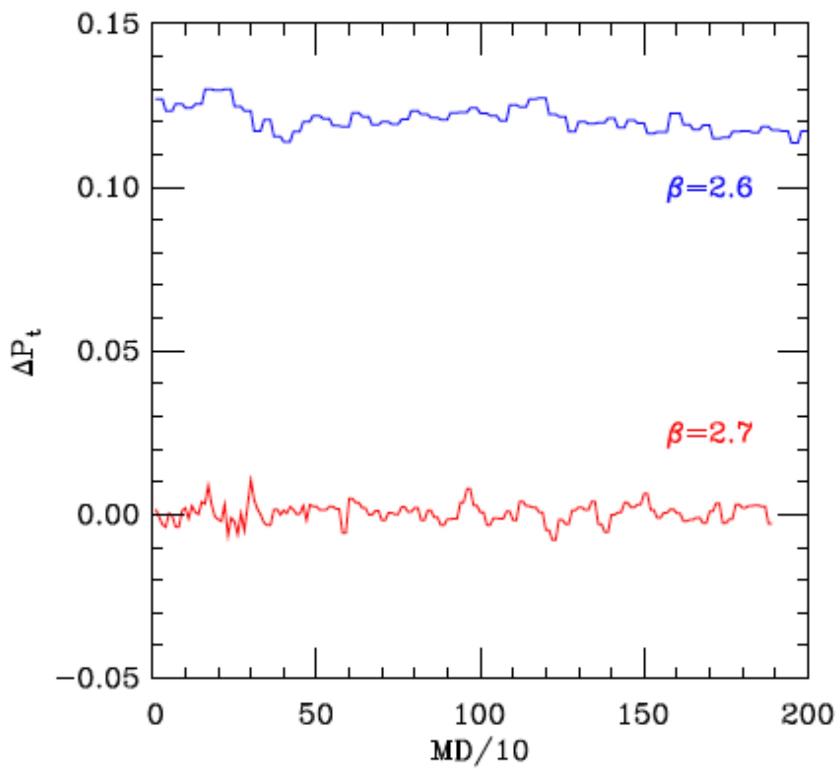
Plaquette expectation value is "striped"





New symmetry breaking pattern

Order parameter I: $\Delta P_\mu = \square_n - \square_{n+\mu}$



β = 2.6 IM phase

β=2.7 weak coupling phase

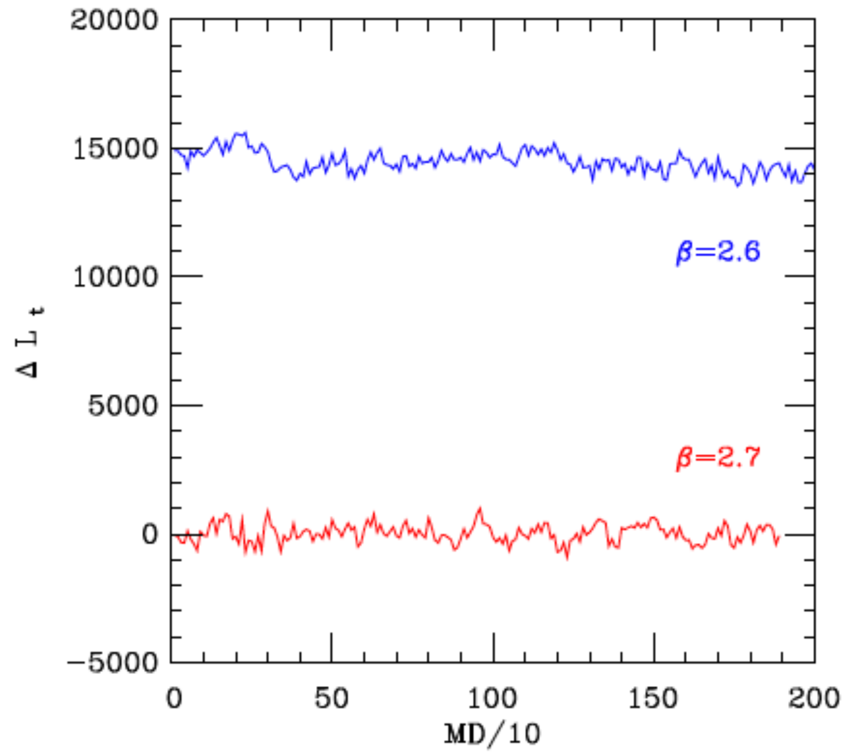
New symmetry breaking pattern

Order parameter II:

S^4 breaking occurs at the fermionic level: staggered link

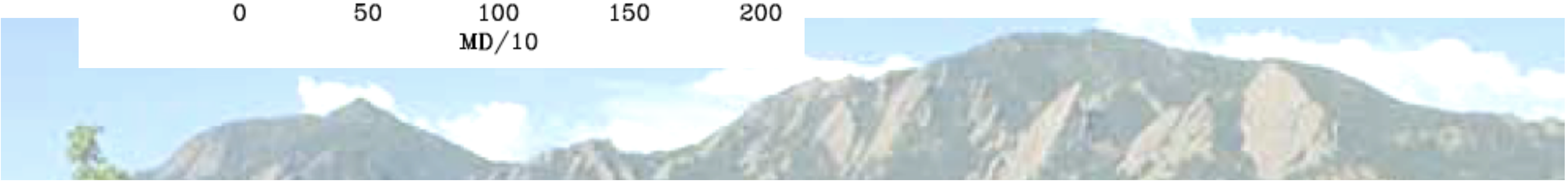
$$\Delta L_\mu = \bar{\psi}_n U_\mu(n)\psi_{n+\mu} - \bar{\psi}_{n+\mu} U_\mu(n+\mu)\psi_{n+2\mu}$$

link in μ shifted in μ



β=2.6 IM phase

β=2.7 weak phase

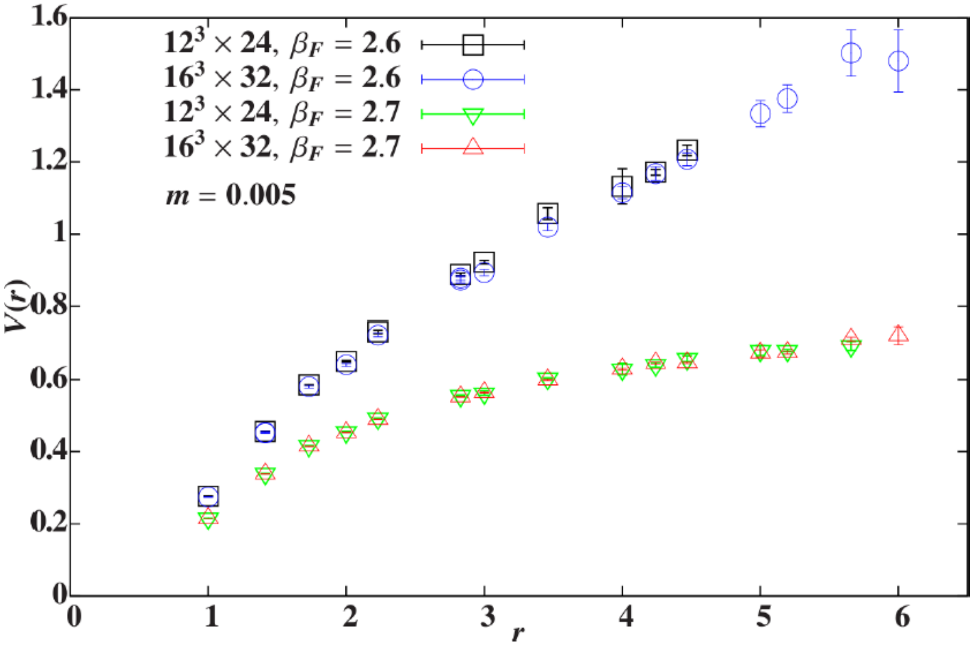


New symmetry breaking pattern

- Single-site shift symmetry is exact in the action.
- When S^4 is broken, the phase has to be separated by a “real” phase transition
- The S^4 broken phase cannot go away with the volume
- S^4 is related to taste so this could be a special taste breaking
- Both ΔP and ΔL are order parameters

Confinement in SSTB phase

- Polyakov line is small
- Static potential on 12^3,16^3 volumes (no volume dependence!) shows a linear term: r_0=2.1 – 2.7, $\sqrt{\sigma}$ =0.40 -- 0.48

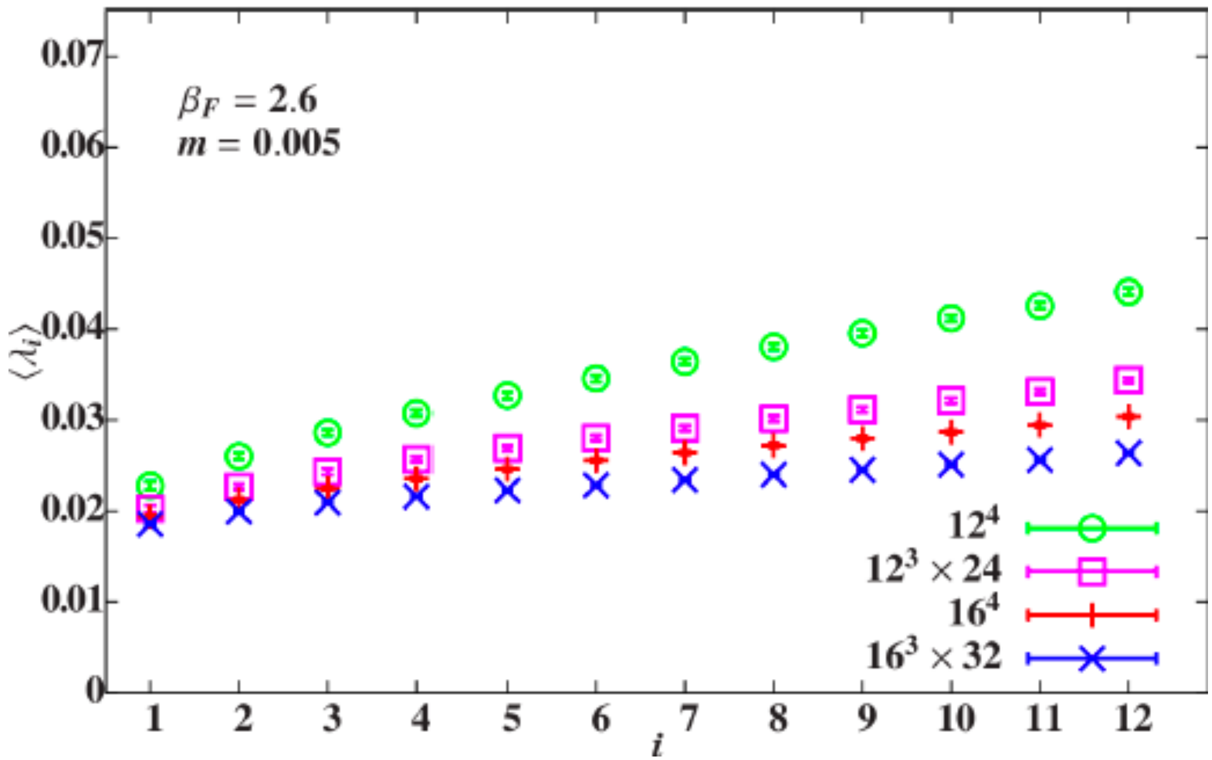


β=2.6 – IM phase

β=2.7 – weak coupling

Dirac eigenvalue spectrum in SSTB phase

Chiral properties: Dirac eigenvalue spectrum on different volumes



β=2.6 – IM phase

Dirac eigenvalue spectrum

- RMT predictions require knowledge of dynamics (N_f, chiral breaking, etc)
- Simple volume scaling is more general. In the chiral limit

$\rho(\lambda) \sim (\lambda - \lambda_0)^{\alpha}$ implies

$$\int_m^n \rho(\lambda) d\lambda = \frac{n-m}{V} + O(1/V^2)$$

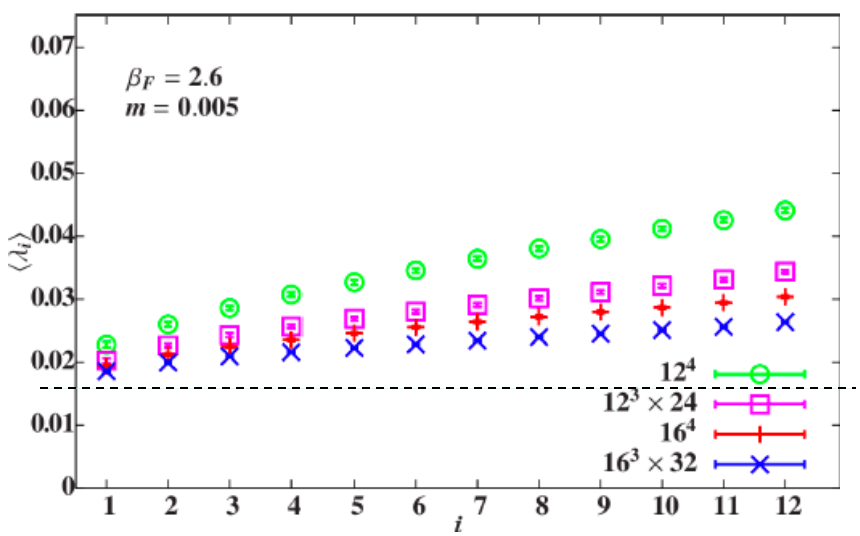
$$\lambda_n - \lambda_0 \sim \left(\frac{n - x_0}{V} \right)^{1/(\alpha+1)}, \qquad \frac{D}{\alpha+1} = y_m$$

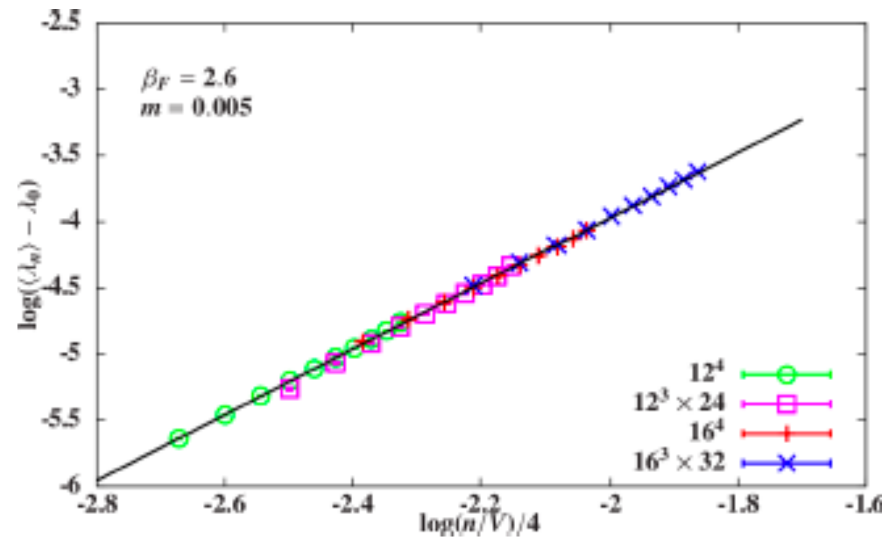
- Chirally broken systems: $\alpha = 0$, $\lambda_0 = 0$
- Free field $\alpha = 3$
- $\lambda_0 \neq 0$: soft edge (**very** chiral symmetric)

Dirac eigenvalue spectrum

Fit: in S^4B phase (β=2.6):
soft edge with α=0.6(1), λ_0=0.0175 (RMT prediction: α=1/2)

Very chiral symmetric!



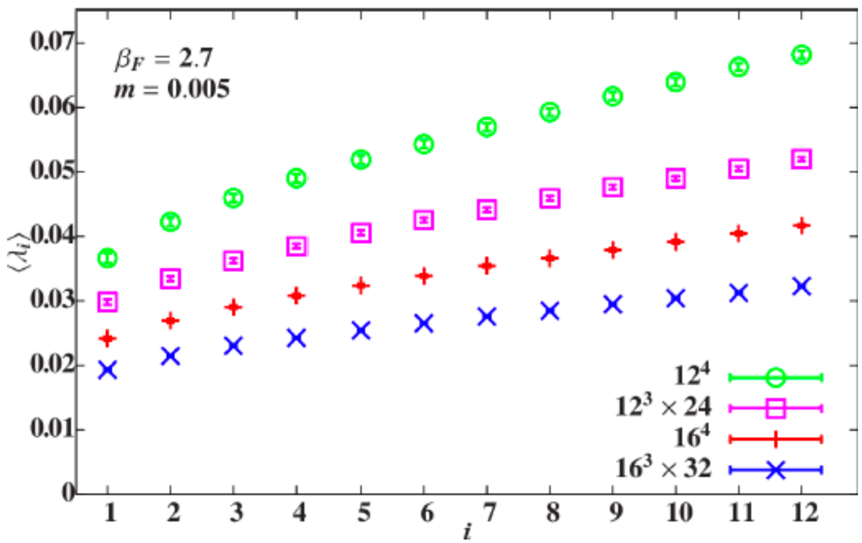


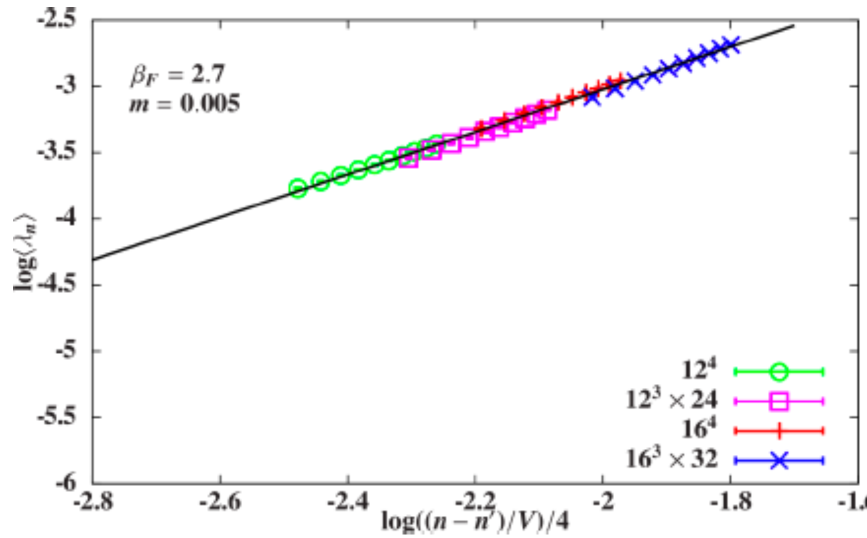
$\log(\lambda_n - \lambda_0) \sim \log(n/V)/(\alpha+1)$

Weak coupling phase for contrast:

Fit in weak coupling (β=2.7) phase

α=1.5, λ_0=0 → γ_m=0.61(5) anomalous dim.



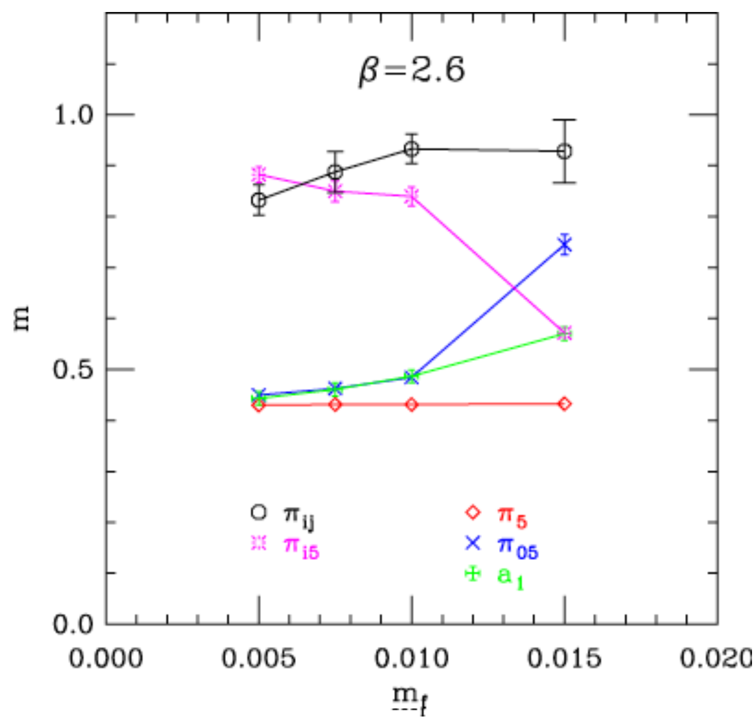


$\log(\lambda_n) \sim \log((n-x_0)/V)/(\alpha+1)$

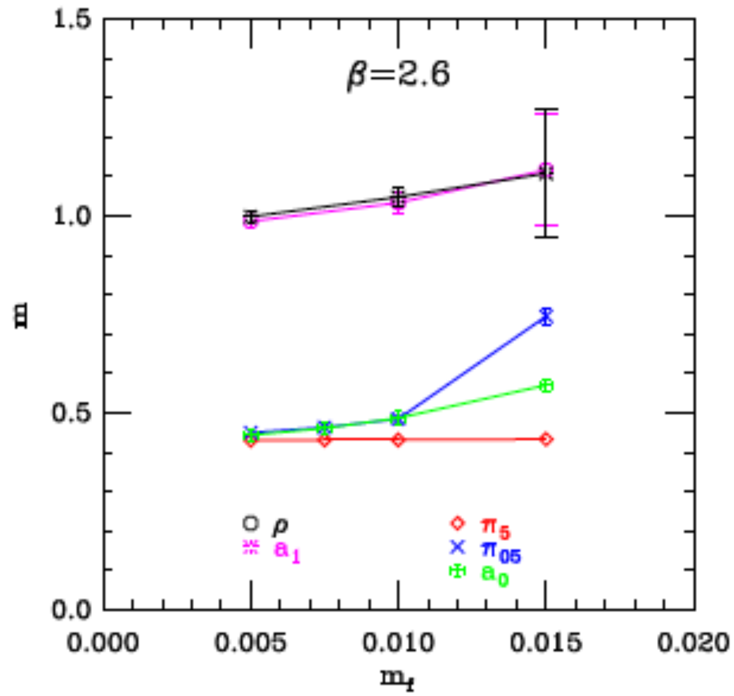
Consistent results at smaller masses, larger β

Intermediate phase:

The meson spectrum is parity degenerate (chiral symm.)
shows no finite volume effect



On-site pions + scalar

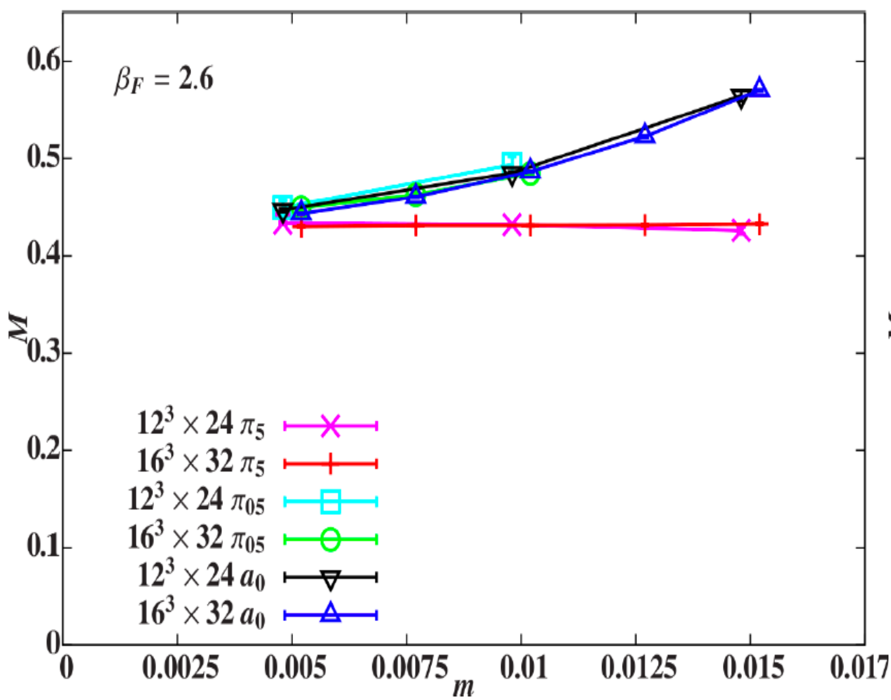


Rho + axial vector

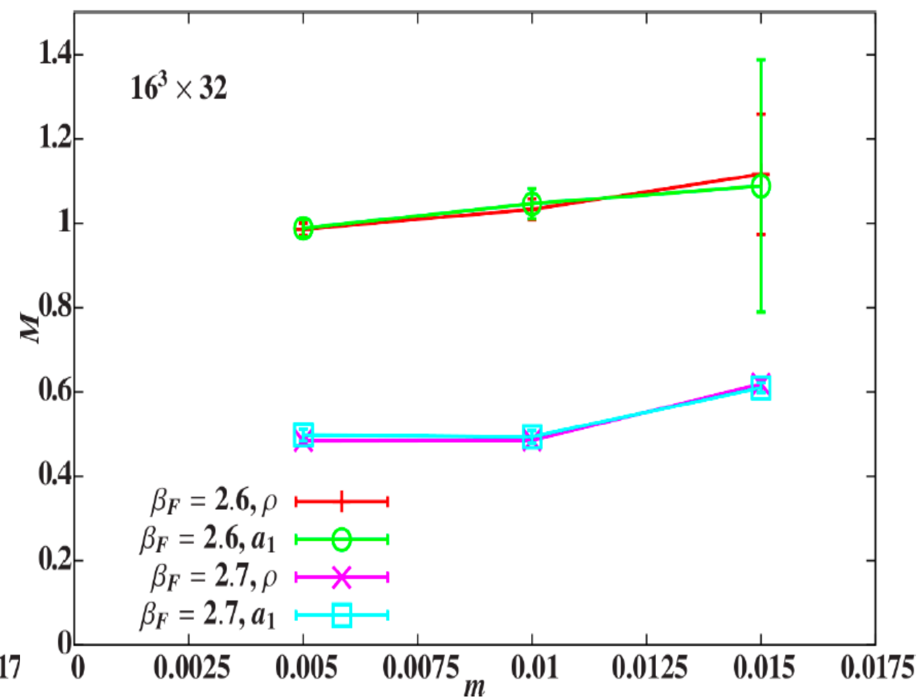
1-link pions

Intermediate phase:

The meson spectrum is parity degenerate (chiral symm.)
Shows no finite volume effects in S^{4}b phase

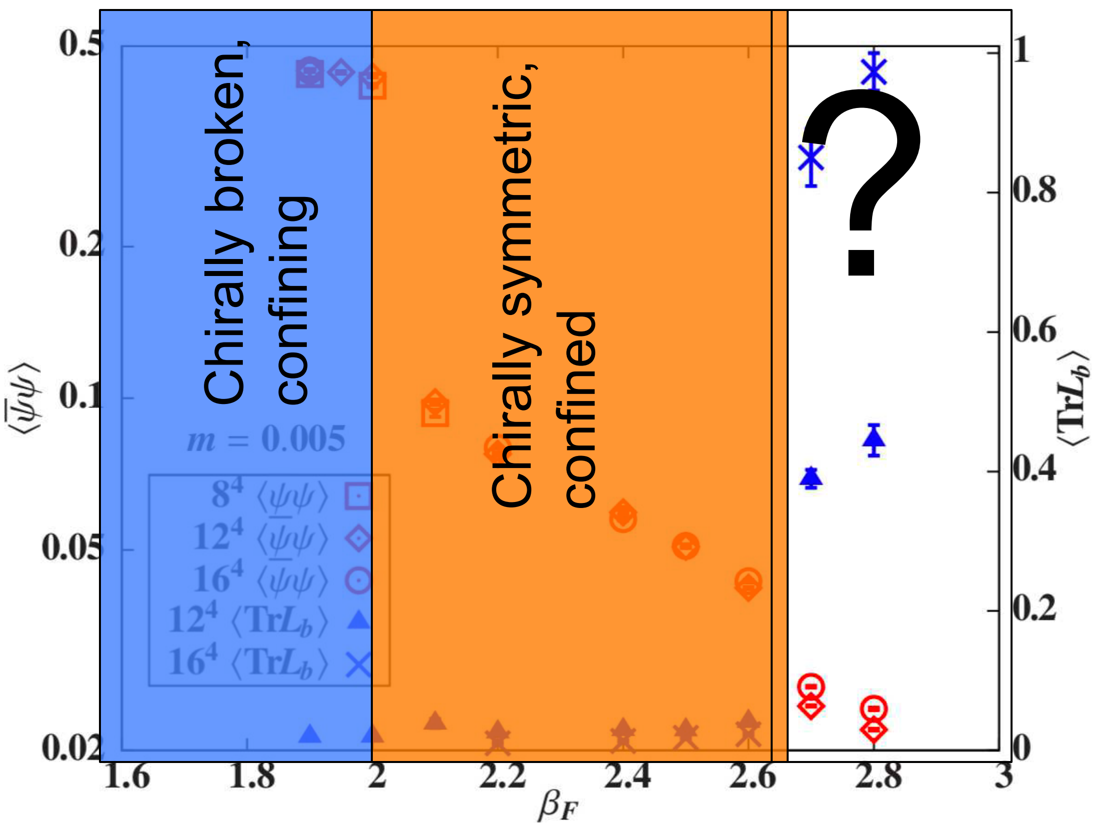


On-site pions + scalar
This is not a Goldstone!



Rho + axial vector

Phase diagram



Intermediate S^4b phase

It is a strange phase.

- It is not consistent with the perturbative $g^2 = 0$ fixed point
- Chirally symmetric as shown by
  - Dirac spectrum
  - Meson spectrum
- Confining with a small correlation length

Chirally symmetric & confining phase is nor supposed to exist at all

- There is no continuum limit here (1st order transitions)
- Lattice might generate new relevant interactions

Does it exist in any other system than $N_f = 12$?

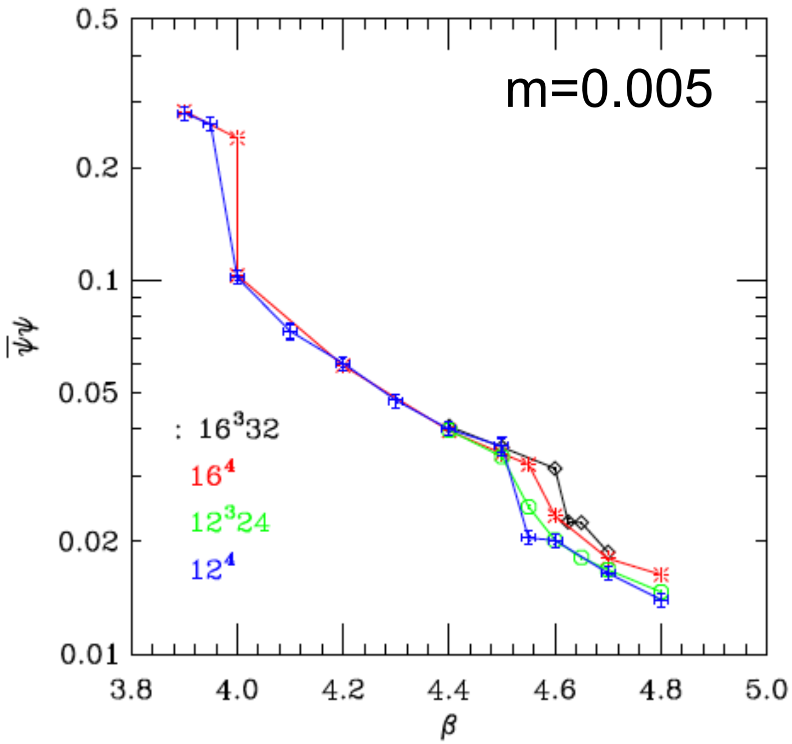
No for $N_f = 4$ ✓

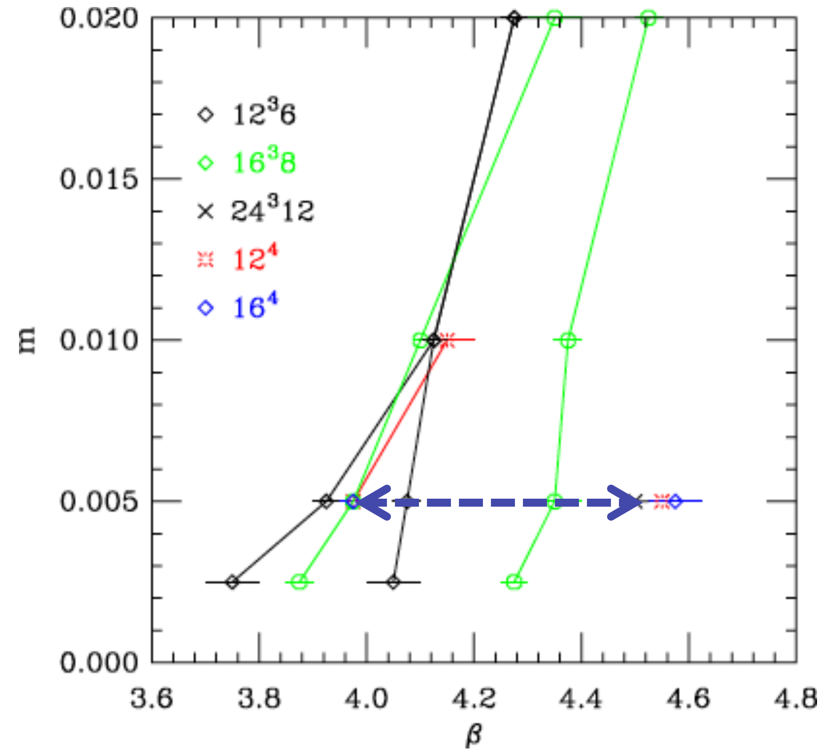
Yes for $N_f = 8,12,16$ ✓

Intermediate phase with N_f = 8

The phase diagram is eerily similar to N_f = 12 :

2 transitions, converging (likely) to bulk ones, intermediate phase

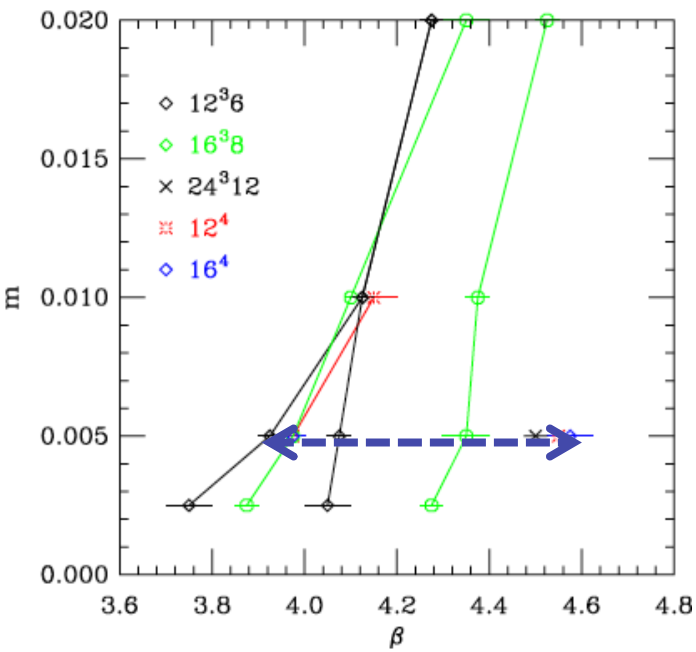




intermediate phase with the same properties as N_f = 12
(S⁴ breaking, soft edge spectrum, confinement)

Intermediate phase with N_f = 8

In the chiral limit there is a phase with S^4 breaking in the infinite volume limit



Speculations

- This phase cannot persist to g^2=0 → it has to be bulk transition
- If N_f=8 is below the conformal window, the SSTB phase is a staggered lattice artifact
- Simulations near $6/g^2$ = 2.3 can pick up scaling of this phase

Conclusion: more questions than answers

- SU(3) gauge with N_f =12 fundamental flavors is the test case of BSM calculations:
  - MCRG indicates an IRFP at relatively weak coupling
  - The phase structure in the strong coupling is complicated
    - There are two sets of phase transitions
    - The intermediate phase
      - exists with many actions
      - is chirally symmetric but confining
      - breaks single-site translational symmetry (taste?)
- N_f = 8, 16 systems show the same S^4b phase; Even if it’s a staggered lattice artifact, it still has to be noticed (and avoided)