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Non-Abelian strings in supersymmetric Yang-Mills: 4D-2D correspondence

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Discovery of non-Abelian strings in supersymmetric Yang-Mills and applications

Beyond supersymmetry
DUAL MEISSNER EFFECT (Nambu-‘t Hooft-Mandelstam, ~1975)
The Meissner effect: 1930s, 1960s

Superconductor of the 2nd kind

Cooper pair condensate

Abrikosov (ANO) vortex (flux tube)

DUAL MEISSNER EFFECT (Nambu-‘t Hooft-Mandelstam, ~1975)
Qualitative explanation of color confinement: Dual Meissner effect:

Hanany, Strassler, Zaffaroni ’97

SW=Abelian strings, “wrong” confinement...
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- Nambu

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SW=Abelian strings, “wrong” confinement...
Qualitative explanation of color confinement: Dual Meissner effect:

- 't Hooft, 1976
- Mandelstam
- Nambu

★ Non-Abelian theory, but Abelian flux tube

★ Hanany, Strassler, Zaffaroni ‘97  ➡️ SW=Abelian strings, “wrong” confinement...
"...[monopoles] turn to develop a non-zero vacuum expectation value. Since they carry color-magnetic charges, the vacuum will behave like a superconductor for color-magnetic charges. What does that mean? Remember that in ordinary electric superconductors, magnetic charges are connected by magnetic vortex lines ... We now have the opposite: it is the color charges that are connected by color-electric flux tubes."

G. 't Hooft (1976)
\[ N=2 \Rightarrow \text{add the second gluino + add a scalar gluon } \varphi^a \text{ (a complex scalar field in the adjoint)} \]

\[ V(\varphi^a) = |\varepsilon^{abc} \varphi^b \varphi^c|^2 \]

In the vacuum \( \varphi^3 \neq 0 \) while \( \varphi^1 = \varphi^2 = 0 \) \Rightarrow

\[ \text{SU}(2)_{\text{gauge}} \rightarrow \text{U}(1) \Rightarrow \]

Georgi–Glashow model

\Rightarrow \text{ 't Hooft–Polyakov monopoles}

If \( |\varphi^3| \gg \Lambda \), then monopoles are very heavy!
First demonstration of the dual Meissner effect: Seiberg & Witten, 1994

- gluons + complex scalar superpartner
- two gluinos
- Georgi-Glashow model built in
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\( SU(2) \rightarrow U(1), \) monopoles

analytic continuation
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\[ \text{SU}(2) \rightarrow \text{U}(1), \text{monopoles} \]

Monopoles become light if \( |\varphi^3| \lesssim \Lambda \) $\Rightarrow$ At two points, massless!

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$N=1$ deform. forces $M$ condensation

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$U(1)$ broken, electric flux tube formed

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Dynamical Abelization ... dual Abrikosov string

analytic continuation
Non-Abelian Strings, 2003 $\rightarrow$ Now
“Non-Abelian” string is formed if all non-Abelian degrees of freedom participate in dynamics at the scale of string formation.

2003: Hanany, Tong
Auzzi et al.
Yung + M.S.

classically gapless excitation

\[ \text{SU}(2)/\text{U}(1) = \text{CP}(1) \sim \text{O}(3) \text{ sigma model} \]
Prototype model

\[
S = \int d^4x \left\{ \frac{1}{4g_2^2} (F_{\mu \nu}^a)^2 + \frac{1}{4g_1^2} (F_{\mu \nu})^2 + \frac{1}{g_2^2} |D_\mu a^a|^2 \right. \\
+ \left. \text{Tr} (\nabla_\mu \Phi)^\dagger (\nabla^\mu \Phi) + \frac{g_2^2}{2} \left[ \text{Tr} (\Phi^\dagger T^a \Phi) \right]^2 + \frac{g_1^2}{8} \left[ \text{Tr} (\Phi^\dagger \Phi) - N\xi \right]^2 \right. \\
+ \left. \frac{1}{2} \text{Tr} \left| a^a T^a \Phi + \Phi \sqrt{2M} \right|^2 + \frac{i \theta}{32\pi^2} F^a_{\mu \nu} \tilde{F}^a_{\mu \nu} \right\}, \\
\Phi = \left( \begin{array}{c} \phi^{11} \\ \phi^{12} \\ \phi^{21} \\ \phi^{22} \end{array} \right), \\
M = \left( \begin{array}{cc} m & 0 \\ 0 & -m \end{array} \right)
\]

Basic idea:
- Color-flavor locking in the bulk → Global symmetry G;
- G is broken down to H on the given string;
- G/H coset; G/H sigma model on the world sheet.

\[\Phi = \sqrt{\xi} \times I\]
Prototype model

\[
S = \int d^4x \left\{ \frac{1}{4g_2^2} (F^a_{\mu\nu})^2 + \frac{1}{4g_1^2} (F_{\mu\nu})^2 + \frac{1}{g_2^2} |D_\mu a^a|^2 \\
+ \text{Tr} (\nabla_\mu \Phi)^\dagger (\nabla^\mu \Phi) + \frac{g_2^2}{2} \left[ \text{Tr} (\Phi^\dagger T^a \Phi) \right]^2 + \frac{g_1^2}{8} \left[ \text{Tr} (\Phi^\dagger \Phi) - N\xi \right]^2 \\
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\]

\[
\Phi = \begin{pmatrix} \phi^{11} & \phi^{12} \\ \phi^{21} & \phi^{22} \end{pmatrix}
\]

\[
M = \begin{pmatrix} m & 0 \\ 0 & -m \end{pmatrix}
\]

U(2) gauge group, 2 flavors of (scalar) quarks
SU(2) Gluons A^a_\mu + U(1) photon + gluinos+ photino

Basic idea:
- Color-flavor locking in the bulk \(\rightarrow\) Global symmetry \(G\);
- \(G\) is broken down to \(H\) on the given string;
- \(G/H\) coset; \(G/H\) sigma model on the world sheet.
**ANO strings are there because of U(1)!**

**New strings:**

\[
\pi_1(U(1) \times SU(2)) \text{ nontrivial due to } \mathbb{Z}_2 \text{ center of } SU(2)
\]

\[
\sqrt{\xi} e^{i\alpha} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
\]

**ANO**

\[
T = 4\pi \xi
\]

\[
\sqrt{\xi} \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & 1 \end{pmatrix}
\]

**Non-Abelian**

\[
T_{U(1) \pm T^3_{SU(2)}} = 2\pi \xi
\]

\[SU(2)/U(1) \leftrightarrow \text{orientational moduli; } O(3) \sigma \text{ model}\]
ANO strings are there because of U(1)!

New strings:

\[ \pi_1(\text{SU}(2) \times \text{U}(1)) = \mathbb{Z}_2: \text{rotate by } \pi \text{ around 3-d axis in SU}(2) \]

\[ \rightarrow -1; \text{ another -1 rotate by } \pi \text{ in U}(1) \]

\[ \pi_1(\text{U}(1) \times \text{SU}(2)) \text{ nontrivial due to } \mathbb{Z}_2 \text{ center of SU}(2) \]

\[ \text{ANO} \quad \sqrt{\xi} e^{i \alpha} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \]

\[ T=4\pi \xi \]

\[ \text{Non-Abelian} \quad \sqrt{\xi} \begin{pmatrix} e^{i \alpha} & 0 \\ 0 & 1 \end{pmatrix} \]

\[ T_U(1) \pm T^3_{\text{SU}(2)} \]

\[ T=2\pi \xi \]

SU(2)/U(1) \leftrightarrow \text{orientational moduli; O}(3) \sigma \text{ model}
CP(1) model with twisted mass

\[
S = \int d^2x \left\{ \frac{2}{g^2} \frac{\partial_\mu \bar{\phi} \partial^\mu \phi - (\Delta m)^2 \bar{\phi} \phi}{(1 + \bar{\phi} \phi)^2} + \text{fermions} \right\}
\]
Evolution in dimensionless parameter $m^2/\xi$
The 't Hooft–Polyakov monopole

$\xi = 0$
$\Delta m \neq 0$

Almost free monopole

$\xi \neq 0$
$|\Delta m| \gg \xi^{1/2}$

Confined monopole, quasiclassical regime

$\Lambda_{CP(1)} \ll |\Delta m| \ll \xi^{1/2}$

Confined monopole, highly quantum regime

$\xi \ll m \ll \xi^{1/2}$

$\Lambda_{CP(1)}^{-1}$

$\Delta m \rightarrow 0$
Kinks are confined in 4D (attached to strings).

Kinks are confined in 2D:

Kink = Confined Monopole

4D ↔ 2D Correspondence

World-sheet theory ↔ strongly coupled bulk theory inside

Dewar flask
1) Confined monopoles in dense QCD

**Color Superconductivity (CSC)**

- QCD at high density $\rightarrow$ Fermi surface, weak-coupling
- Attractive channel $\rightarrow$ Cooper instability

$[3]_C \times [3]_C = [6]_S + [\overline{3}]_A$

\[
(\tau_a)_{ij}(\tau_a)_{kl} = \frac{2}{3}(\tau_S)_{ik}(\tau_S)_{lj} - \frac{4}{3}(\tau_A)_{ik}(\tau_A)_{lj}
\]

"diquark condensate"

Neutron stars?
At large $\mu$ QCD is in the \textbf{CFL} phase. Diquark condensate

3 colors and 3 flavors

$$\Phi^{kC} \sim \varepsilon_{ijk} \varepsilon_{ABC} \left( \psi^i A \psi^j B \alpha + \bar{\psi}^{\bar{i} A} \bar{\psi}^{\bar{j} B} \right)$$

At $T \rightarrow T_c$ gap fluctuations become important.

Chiral fluctuations ($\pi$-mesons) are considered less important

$$S = \int d^4x \left\{ \frac{1}{4g^2} (F_{\mu\nu}^a)^2 + 3 \text{Tr} (D_0 \Phi)\dagger (D_0 \Phi) \right.$$ $$+ \left. \text{Tr} (D_i \Phi)\dagger (D_i \Phi) + V(\Phi) \right\}$$

with the potential

$$V(\Phi) = -m_0^2 \text{Tr} (\Phi\dagger \Phi) + \lambda \left( \left[ \text{Tr} (\Phi\dagger \Phi) \right]^2 + \text{Tr} \left[ (\Phi\dagger \Phi)^2 \right] \right)$$
\[ \Phi_{\text{vac}} = v \text{ diag } \{1, 1, 1\} \]

where

\[ v^2 = \frac{m_0^2}{8 \lambda} = \frac{4 \pi^2}{3} \frac{T_c - T}{T_c} \mu^2 \]

The symmetry breaking pattern

\[ SU(3)_C \times SU(3)_F \times U(1)_B \rightarrow SU(3)_{C+F} \]

9 symmetries are broken.
8 are eaten by Higgs mechanism.
One Goldstone boson associated with broken U(1)_B.

\[ \text{Broken} \rightarrow SU(2) \times U(1) \rightarrow CP(2) \text{ model on the string w.-s. !} \]
Low-energy excitations (gapless modes)

\[ \Delta H_{GL} = \frac{T}{2}(\partial_z x_{\text{perp}} \partial_z x_{\text{perp}}) + h.d. \]

**Kelvin modes or Kelvons**
- 2 NG gapless modes in relat.
- 1 NG gapless mode in non-rel.

**E_{\text{excit}} \ll m_\gamma \sim \eta**

Perpendicular plane

**Nambu-Goto \rightarrow String Theory**

**E_{\text{str}} = TL + C/L**

Counts # of gapless modes!
\[ G = U(1)_p \times SO_S(3) \times SO_L(3) \]

In the ground state \[ U_p(1) \times SO_S(3) \times SO_L(3) \rightarrow H_B = SO(3)_{s+l} \]

Hence, contrived NG modes in the bulk!
Amending Abrikosov to non-Abelian

\[ ΔH_{GL} = D_kϕ^+D_kϕ + λ(ϕ^+ϕ-η^2)^2 \quad \Rightarrow \quad \text{time derivatives can be rel. or non-relat.} \]

\[ ΔH_{NA} = \partial_k n^i \partial_k n^i + (-μ^2+ϕ^+ϕ)n^i n^i + β(n^i n^i)^2 + \text{time derivatives} \]

\[ \text{with } η^2>μ^2 \]

★ In ground state \( ϕ^+ϕ_{\text{gr.st.}} = η^2 \), hence the mass term of \( n^i = η^2-μ^2 >0 \) and \( O(3) \) is unbroken

★★ Inside Abrikosov \( ϕ^+ϕ_{\text{gr.st.}} = 0 \) hence the mass term of \( n^i = -μ^2 <0 \) and \( O(3) \) is broken down to \( O(2) \), while \( n^i n^i = μ^2/2β \)

★★★ Classically \( O(3) \) sigma model on vortex, 2 gapless interacting modes
Conclusions

★ Non-Abelian strings in N=1 SUSY → heterotic CP(n-1) models on string; poorly explored.

★ ★ Unexpected applications in condensed matter (not explored).