Dynamics with isospin symmetric Higgs boson: the quark mass hierarchy and the LHC data

Volodya Miransky

Western University
The masses of quarks are

\[
\begin{align*}
    m_t &= 171.2^{+2.1}_{-2.1} \text{ GeV}, \\
    m_c &= 1.27^{+0.07}_{-0.11} \text{ GeV}, \\
    m_s &= 104^{+26}_{-34} \text{ MeV}, \\
    m_u &= 1.5 - 3.3 \text{ MeV}, \\
    m_d &= 3.5 - 6.0 \text{ MeV}.
\end{align*}
\]

The quark spectrum is characterized by the following striking features:

1. There is a large hierarchy between quark masses from different families,

\[
\begin{align*}
    m_u/m_t &\sim 10^{-5}, \\
    m_u/m_c &\sim 10^{-3}, \\
    m_c/m_t &\sim 10^{-2}, \\
    m_d/m_b &\sim 10^{-3}, \\
    m_d/m_s &\sim 10^{-2}, \\
    m_s/m_b &\sim 10^{-1}.
\end{align*}
\]

2. The isospin violation is also hierarchical: It is very strong in the third family, strong (although essentially weaker) in the second family, and mild in the first one:

\[
\begin{align*}
    m_t/m_b &\sim 40.8, \\
    m_c/m_s &\sim 11.5, \\
    m_u/m_d &\sim 0.35 - 0.60.
\end{align*}
\]
Isospin symmetric Higgs mechanism

Michio Hashimoto and V.M.:
PRD 81, 055014 (2010); arXiv:0912.4453[hep-ph]

We assume the separation of the dynamics triggering the strong isospin violation in the third and second families from that responsible for the generation of the $W$ and $Z$ masses, i.e., electroweak symmetry breaking (EWSB). The latter could be provided by one of the following known mechanisms:

(a) An elementary Higgs field (or fields).
(b) A modern version of the technicolor (TC) scenario.
(c) Dynamical Higgs mechanism with a Higgs doublet (or doublets) composed of $t'$ and $b'$ quarks of the fourth family.
Our basic assumption is that the dynamics primarily responsible for the EWSB leads to the mass spectrum of quarks with no (or weak) isospin violation. Moreover, we assume that the values of these masses are of the order of the observed masses of the down-type quarks. In the case of an elementary Higgs field (or fields), they are provided by the conventional Yukawa interactions. In the case of the dynamical Higgs mechanism, in order to generate these masses, one should use flavor-changing-neutral (FCN) interactions: the extended technicolor (ETC) in the case of the TC scenario, and the horizontal interactions between the 4th family and the first three ones in the case of the scenario with the fourth family.
Isospin symmetric quark masses

\[ m_0^{(3)} \sim 1\text{GeV}, \quad m_0^{(2)} \sim 100\text{MeV}, \quad m_0^{(1)} \sim 1\text{MeV} \]

\[ \bar{u}^{(i)} \rightarrow H^0 \quad \sim \quad \bar{d}^{(i)} \rightarrow H^0 \]

IS Yukawa interactions

\[ \Lambda^{(i4)} \]

\[ u^{(i)}_L \quad t' \quad u^{(i)}_R \quad \sim \quad d^{(i)}_L \quad b' \quad d^{(i)}_R \]

FCN interactions of the up- and down-quark sectors. Here \( u^{(1,2,3)} = u, c, t \) and \( d^{(1,2,3)} = d, s, b \), respectively. \( \Lambda^{(i4)} \) are masses of exchange vector particles.

\[
m_0^{(i)} \sim \frac{C_2 g^{2}_{t' u^{(i)}}}{4\pi^2} \left( \Lambda^{(4)} \right)^2 m_{t'} \sim \frac{C_2 g^{2}_{b' d^{(i)}}}{4\pi^2} \left( \Lambda^{(4)} \right)^2 m_{b'},
\]

\[
\left( \Lambda^{(14)} \right)^2 \gg \left( \Lambda^{(24)} \right)^2 \gg \left( \Lambda^{(34)} \right)^2 \gg \left( \Lambda^{(4)} \right)^2
\]

\[ \Rightarrow m_0^{(1)} \ll m_0^{(2)} \ll m_0^{(3)} \ll m_{t'} \sim m_{b'}. \]
Top quark as the source of isospin violation in quark mass spectrum

The second (central) stage is introducing horizontal interactions between quarks in the first three families (this stage is essentially the same for all EWSB mechanisms mentioned above.) First, we utilize strong (although subcritical) interactions for the top quark which lead to the observed ratio $\frac{m_t}{m_b} \approx 40.8$.


This is the only source of the isospin violation in the present model.

The second step is introducing the equal strengths horizontal FCN interactions between the $t$ and $c$ quarks and the $b$ and $s$ ones in order to get the observed ratio $m_c/m_s \approx 11.5$ in the second family.
Low energy effective interactions

At energy scales less than the mass $\Lambda^{(3,3)}$ and $\Lambda^{(2,3)}$ of the vector bosons, the corresponding interactions can be presented by the four-fermion Nambu-Jona-Lasinio (NJL) ones. Let us start from the third family. The isospin symmetric mass $m_0^{(3)}$ plays the role of a bare mass with respect to these interactions.

Schwinger-Dyson (SD) equations

\[
\begin{align*}
G_t^{-1} & \equiv \left( \begin{array}{c}
\end{array} \right)^{-1} = S_t^{-1} + g_t \\
G_b^{-1} & \equiv \left( \begin{array}{c}
\end{array} \right)^{-1} = S_b^{-1} + g_b
\end{align*}
\]

$g_t \gg g_b$
The solution of the SD equation for the $t$ quark propagator leads to the following mass $m_t$

$$m_t \simeq \frac{1}{\Delta g_t} m_0^{(3)},$$

where $\Delta g_q$ denotes the difference of the critical coupling and the (normalized) dimensionless NJL one for a $q$ quark, so that

$$\Delta g_t \simeq \frac{m_0^{(3)}}{m_t} \sim 6 \times 10^{-3},$$

where we used $m_t = 171.2$ GeV and $m_0^{(3)} = 1$ GeV. For the bottom quark, it should be $\Delta g_b \sim \mathcal{O}(1)$:

$$\Delta g_b \simeq \frac{m_0^{(3)}}{m_b}.$$
Let us now turn to the generation of the realistic masses for the second family.

\[ m_c = m_0^{(2)} + \eta_{t}^{(23)} m_t, \quad m_s = m_0^{(2)} + \eta_{b}^{(23)} m_b, \]

where \( m_0^{(2)} \sim 100 \text{ MeV} \) is the isospin symmetric mass for the second family, and \( \eta_{t}^{(23)} = \eta_{b}^{(23)} \). They are

\[ \eta_{t(b)}^{(23)} \equiv \frac{C_2 g_{tc(bs)}^2}{4\pi^2} \frac{(\Lambda^{(33)})^2}{(\Lambda^{(23)})^2} \]

for \( \Lambda^{(23)} \gg \Lambda^{(33)} \).
Taking $m_0^{(2)} = 100$ MeV and $\eta_t^{(23)} = \eta_b^{(23)} = 1/100$, we get

\begin{align*}
m_c &= 100 \text{ MeV} + m_t/100 \sim 1 \text{ GeV}, \\
m_s &= 100 \text{ MeV} + m_b/100 \sim 140 \text{ MeV}.
\end{align*}

with $m_b/m_t \approx 1/40$. Let us emphasize that the presence of the isospin symmetric mass $m_0^{(2)} \sim 100$ MeV $\sim m_s$ is crucial here: with $m_0^{(2)} \ll 100$ MeV, the ratio $m_s/m_c$ would be close to $m_b/m_t$.

As to the horizontal FCN gauge bosons which couple to the quarks of the 1st and 2nd families, we assume that they are very heavy,

\begin{align*}
c - u - \Lambda^{(12)}, \
s - d - \Lambda^{(12)},
\end{align*}

with $\Lambda^{(12)} \gtrsim \mathcal{O}(1000 \text{ TeV})$. As a result, their contributions to the masses of the $u$ and $d$ quarks are very small.
One of the signatures of the scenario with subcritical but nearcritical interactions is the appearance of a composite top-Higgs doublet $\Phi_{ht}$ (resonance) composed of the quarks and antiquarks, $\Phi_{ht} \sim \bar{t}_R(t, b)_L$

V. M., PRL 69, 1022 (1992)

The mass of $\Phi_{ht}$ can be estimated via the NJL relation:

$$M_{ht} \sim \Lambda^{(33)} \left( \frac{2\Delta g_t}{\ln \frac{1}{2\Delta g_t}} \right)^{1/2} \sim 0.05\Lambda^{(33)},$$

where we used $\Delta g_t \sim 6 \times 10^{-3}$. 
Quark mass matrices

The Yukawa interactions are

\[-\mathcal{L}_Y = \sum_{i,j} \bar{\psi}_L^{(i)} Y^{ij}_D d_R^{(j)} \Phi_h + \sum_{i,j} \bar{\psi}_L^{(i)} Y^{ij}_U u_R^{(j)} \tilde{\Phi}_h + y_{ht} \bar{\psi}_L^{(3)} t_R \tilde{\Phi}_h,\]

with \( \tilde{\Phi}_h \equiv i \tau_2 \Phi_h^* \), \( \langle \Phi_h \rangle = \left( \begin{array}{c} 0 \\ \frac{v_h}{\sqrt{2}} \end{array} \right) \), \( \langle \Phi_{ht} \rangle = \left( \begin{array}{c} 0 \\ \frac{v_t}{\sqrt{2}} \end{array} \right) \),

\[Y_D \equiv \frac{\sqrt{2}}{v_h} M_D, \quad Y_U \equiv \frac{\sqrt{2}}{v_h} M_U, \quad \text{and} \]

\[M_D = \begin{pmatrix} m_0^{(1)} & \xi_{12} m_0^{(1)} & \xi_{13} m_0^{(1)} \\ \xi_{21} m_0^{(1)} & m_0^{(2)} + \delta \cdot m_b & \xi_{23} m_0^{(2)} \\ \xi_{31} m_0^{(1)} & \xi_{32} m_0^{(2)} & m_0^{(3)} \end{pmatrix}, \quad M_U = \begin{pmatrix} \eta_{11} m_0^{(1)} & \eta_{12} m_0^{(1)} & \eta_{13} m_0^{(1)} \\ \eta_{21} m_0^{(1)} & m_0^{(2)} + \delta \cdot m_t & \eta_{23} m_0^{(2)} \\ \eta_{31} m_0^{(1)} & \eta_{32} m_0^{(2)} & m_0^{(3)} \end{pmatrix}.
\]

\( \Phi_{ht} \) acquires a vacuum expectation value only due to its mixing with \( \Phi_h \). It is responsible for the top mass, \( m_t \approx y_{ht} \frac{v_t}{\sqrt{2}} \). The IS masses \( m_0^{(i)} \) of the order of the masses of the down-type quarks, say, \( m_0^{(3)} \sim 1 \text{ GeV}, \ m_0^{(2)} \sim 100 \text{ MeV}, \) and \( m_0^{(1)} \sim 1 \text{ MeV}. \) The common one-loop factor \( \delta \sim 1/100 \) yields the correct mass hierarchy between \( m_s \) and \( m_c \) via the hierarchy between \( m_b \) and \( m_t \). Also, the off-diagonal coefficients are assumed to be \( \xi_{ij}, \eta_{ij} \sim \mathcal{O}(1) \), with some dynamical mechanism.
The CKM matrix is approximately determined by the down-type quark mass matrix,

\[
V_{\text{CKM}} \approx \begin{pmatrix}
1 - \frac{|\xi_{12}|^2}{2} & \frac{\xi_{12} m_0^{(1)}}{m_0^{(2)}} & \frac{\xi_{13} m_0^{(1)}}{m_0^{(3)}} \\
- \xi_{12}^* \frac{m_0^{(1)}}{m_0^{(2)}} & 1 - \frac{|\xi_{12}|^2}{2} & \frac{\xi_{23} m_0^{(2)}}{m_0^{(3)}} \\
-(\xi_{13} - \xi_{12} \xi_{23}) \frac{m_0^{(1)}}{m_0^{(3)}} & \frac{\xi_{23} m_0^{(2)}}{m_0^{(3)}} & 1
\end{pmatrix}.
\]

For example, with the inputs, \(m_0^{(1)} = 10\text{ MeV}, \ m_0^{(2)} = 68\text{ MeV}, \ m_0^{(3)} = 4.2\text{ GeV}, \ m_t = 173.5\text{ GeV}, \ \delta = 7 \times 10^{-3}, \)
\(\xi_{12} = \xi_{21} = \eta_{12} = \eta_{21} = 2.0, \ \xi_{13} = \xi_{31} = \eta_{13} = \eta_{31} = 1.6, \)
\(\xi_{23} = \xi_{32} = \eta_{23} = \eta_{32} = -2.5, \ \eta_{11} = \frac{1}{4}, \) we obtain
\(m_d = 4.9\text{ MeV}, \ m_s = 95\text{ MeV}, \ m_b = 4.2\text{ GeV}, \)
\(m_u = 2.2\text{ MeV}, \ m_c = 1.3\text{ GeV}, \ |V_{ud}| \simeq |V_{cs}| = 0.975, \)
\(|V_{tb}| \simeq 1, \ |V_{us}| \simeq |V_{cd}| = 0.22, \ |V_{cb}| = 0.041, \ |V_{ts}| = 0.039, \)
\(|V_{ub}| = 0.0042, \ |V_{td}| = 0.013. \)

These values fairly agree with the PDG ones.
The IS Higgs doublet $\Phi_h$, which is mainly responsible for EWSB and couples to the top and bottom in the isospin symmetric way. The origin of its compositeness (if any) is not specified.

The top-Higgs doublet $\Phi_{ht}$, which is required to obtain the correct top mass.

While the neutral top-Higgs $h_t$ has a large top-Yukawa coupling, the IS Higgs neutral boson $h$ does not, $y_t \simeq y_b \sim 10^{-2}$.

On the other hand, the $hWW^*$ and $hZZ^*$ coupling constants are close to those in the SM. We identify $h$ with the 125 GeV $h$ boson discovered at the ATLAS and CMS experiments.
Best-fit Higgs mass $m_H$: 126.0 ± 0.4 (stat) ± 0.4 (syst) GeV

- $\sigma/\sigma_{SM} = 0.88 \pm 0.21$
Decay modes $h \rightarrow \gamma \gamma$, $h \rightarrow Z \gamma$, $h \rightarrow WW^*$, $h \rightarrow ZZ^*$

Diagrams contributing to $h \rightarrow \gamma \gamma$
In the SM, the W-loop contribution to $H \rightarrow \gamma\gamma$ is dominant, while the top-loop effect is destructive against the W-loop,

$$
\Gamma(H \rightarrow \gamma\gamma) = \frac{\sqrt{2}G_F\alpha^2m_H^3}{256\pi^3} \left| A_1(\tau_W) + N_c Q_t^2 A_{1/2}(\tau_t) \right|^2,
$$

$\tau_W \equiv \frac{m_H^2}{4m_W^2}$, $\tau_t \equiv \frac{m_H^2}{4m_t^2}$. The numerical values of $A_1$ and $A_2$ are $A_1(\tau_W) = -8.32$, $A_{1/2}(\tau_t) = 1.38$ for $m_W = 80.385$ GeV, $m_t = 173.5$ GeV, and $m_H = 125$ GeV. On the other hand, in the IS Higgs model, $y_t$ between the top and the IS Higgs $h$ is as small as $y_b$. The top contribution is strongly suppressed. The partial decay width of $h \rightarrow \gamma\gamma$ is thus enhanced without changing essentially $h \rightarrow ZZ^*$ and $h \rightarrow WW^*$. For $y_t \simeq y_b \simeq 10^{-2}$, they read

$$
\frac{\Gamma^{IS}(h \rightarrow \gamma\gamma)}{\Gamma^{SM}(H \rightarrow \gamma\gamma)} \simeq 1.56,
$$

$$
\frac{\Gamma^{IS}(h \rightarrow WW^*)}{\Gamma^{SM}(H \rightarrow WW^*)} = \frac{\Gamma^{IS}(h \rightarrow ZZ^*)}{\Gamma^{SM}(H \rightarrow ZZ^*)} = \left(\frac{v_h}{v}\right)^2 \simeq 0.96.
$$

These values agree well with the ATLAS and CMS data. Here using the Pagels-Stokar formula the vacuum expectation value of the top-Higgs $h_t$ is estimated as $v_t \simeq 50$ GeV, and that of IS Higgs $h$ follows from $v^2 = v_h^2 + v_t^2$ with $v = 246$ GeV. It is important that the values of these ratios are not very sensitive to the value of $v_t$: for $v_t = 40 - 100$ GeV, the suppression factor in $WW^*$ and $ZZ^*$ decays is $0.97 - 0.84$ and the enhancement factor in $\gamma\gamma$ decay is $1.58 - 1.37$. 
For the decay mode of $h \rightarrow Z\gamma$, the model yields

$$\frac{\Gamma^{IS}(h \rightarrow Z\gamma)}{\Gamma^{SM}(H \rightarrow Z\gamma)} \approx 1.07$$

(the LHC data concerning this decay channel has not yet been reported).
IS boson $h$ production

Because $y_t \simeq y_b \sim 10^{-2}$, the gluon fusion process $gg \rightarrow h$ is now in trouble. The presence of new chargeless colored particles, considered in

R. Boughezal and F. Petriello, Phys. Rev. D 81, 114033 (2010);

can help to resolve this problem.

$$\mathcal{L} \supset \mathcal{L}_s = \frac{1}{2} (D_\mu S)^2 - \frac{1}{2} m_{0,s}^2 S^2 - \frac{\lambda_S}{4} S^4 - \frac{\lambda_{hS}}{2} S^2 \Phi_h^\dagger \Phi_h,$$

$$\Phi_h = \begin{pmatrix} \omega^\dagger \\ \frac{1}{\sqrt{2}} (v_h + h + iz_0) \end{pmatrix}, \text{ where } \omega^\pm \text{ and } z_0 \text{ are components eaten by } W^\pm \text{ and } Z, \text{ and real scalar } S \text{ is assigned to the } (8, 1)_0 \text{ representation of } SU(3)_c \times SU(2)_W \times U(1)_Y. \text{ The mass of } S \text{ is } M_s^2 = m_{0S}^2 + \frac{\lambda_{hS}}{2} v_h^2 > 0.$$
Gluon fusion $gg \rightarrow h$

With $\lambda_{hS} \simeq 2$, $M_S = 150 - 400$ GeV and $m^2_{0S} \ll M_S^2$, one obtains

$$\frac{\sigma(gg \rightarrow h)}{\sigma^{SM}(gg \rightarrow H)} \sim 1.$$
Conclusion

The model with an IS Higgs boson yields not only an explanation of the ATLAS and CMS data, including the enhanced diphoton Higgs decay rate, but also makes several predictions. The most important of them is that the value of the top-Yukawa coupling $h - t - \bar{t}$ should be close to the bottom-Yukawa one. Another prediction relates to the decay mode $h \rightarrow Z\gamma$, which unlike $h \rightarrow \gamma\gamma$ is enhanced only slightly, $\Gamma^{\text{IS}}(h \rightarrow Z\gamma) = 1.07 \times \Gamma^{\text{SM}}(H \rightarrow Z\gamma)$. Last but not least, the LHC might potentially discover the top-Higgs resonance $h_t$, if lucky.