Walking signals
in Nf=8 QCD on the lattice

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Plan of the Talk:

1. Introduction
2. Lattice study of Nf=8 QCD
   - Chiral Perturbation Theory (ChPT)
   - Finite Size Hyperscaling (FSHS)
3. Summary

♠ Nf=8 is the candidate of the walking behavior.

Walking signals in Nf=8 QCD on the lattice
Yasumichi Aoki, Tatsumi Aoyama, Masafumi Kurachi, Toshihide Maskawa, Kei-ichi Nagai, Hiroshi Ohki, Akihiro Shibata, Koichi Yamawaki, Takeshi Yamazaki, Feb 27, 2013,
1. Introduction

- LQCD with many fermions
- BSM (quark mass, FCNC, ..., model building)
- \( \Rightarrow \) Candidate of the walking technicolor (WTC)
Requirements for the successful WTC theory

- **spontaneous chiral symmetry breaking**
- **running coupling “walks”** = slowly changing with \( \mu \) → nearly conformal
- **large mass anomalous dimension**: \( \gamma_m \sim 1 \)
- **light scalar 0^{++}**
  - with input \( F_\pi = 246 / \sqrt{N} \) GeV (N: # weak doublet in techni-sector)
    - to reproduce \( W^\pm \) mass
    - typical QCD like theory: \( M_{\text{Had}} >> F_\pi \) (ex.: QCD: \( m_\rho / f_\pi \sim 8 \))
  - Naive TC: \( M_{\text{Had}} > 1,000 \) GeV
  - \( 0^{++} \) is a special case: pseudo Nambu-Goldstone boson of scale inv.
    - → is it really so?
conformal window and walking coupling
- non-Abelian gauge theory with $N_f$ massless fermions -

- Walking Technicolor could be realized just below the conformal window

- crucial information: $N_f^{\text{crit}}$ & mass anomalous dimension around $N_f^{\text{crit}}$
Many flavor QCD
⇒ Candidate of walking/conformal

Our investigation in $N_f=12$ (Ohki’s talk)
⇒ consistent with the conformal with $\gamma=0.4--0.5$.

not favor as WTC (model building)

Thus, we investigate $N_f=8$ QCD.
strong coupling dynamics and non-perturbative

Lattice simulation of $N_f=8$ QCD
Lattice studies of Nf=8:

Y. Iwasaki et al. (’92, ’04) [pioneering work] → conformal

K. Ishikawa et al. (’12, ’13) → conformal

(Yukawa-type correlator in Nf=7 and 16)

A. Cheng et al. (’13) → large $\gamma_m$ over a wide range

of energy scales (slow-running?)

A. Deuzeman et al. (’08), K. Miura et al. (’12)

→ $S\chi$ SB, but near the conformal edge

(thermodynamics)

Z. Fodor et al. (’09) → $S\chi$ SB

T. Appelquist et al. (’09) → no IRFP

LatKMI(’13) → In this talk (walking?)
What is the signal of walking?

Scenario of Walking Dynamics:
Case-1: probe $m_f \ll m_D \to S \chi$ SB-like
Case-2: probe $m_f \gg m_D \to$ conformal-like

\[ \alpha(\mu) \]

Schematic picture of the gauge coupling

\[ m_f \quad m_D \quad m_f \quad \Lambda_{QCD} \]

FIG. 1. Schematic two-loop/ladder picture of the gauge coupling of the massless large $N_f$ QCD as a walking gauge theory in the $S\chi$SB phase near the conformal window. $m_D$ is the dynamical mass of the fermion generated by the $S\chi$SB. The effects of the bare mass of the fermion $m_f$ would be qualitatively different depending on the cases: Case 1: $m_f \ll m_D$ (red dotted line) well described by ChPT, and Case 2: $m_f \gg m_D$ (blue dotted line) well described by the hyper scaling.

$\Rightarrow$ Spectrum?

$S\chi$ SB and/or conformal in some $m_f$ region?
2. Lattice Study of Nf=8 case

*Walking signals in Nf=8 QCD on the lattice*

Yasumichi Aoki, Tatsumi Aoyama, Masafumi Kurachi, Toshihide Maskawa, Kei-ichi Nagai, Hiroshi Ohki, Akihiro Shibata, Koichi Yamawaki, Takeshi Yamazaki, Feb 27, 2013,
Published in *Phys.Rev. D87* (2013) 094511,
e-Print: [arXiv:1302.6859](https://arxiv.org/abs/1302.6859) [hep-lat].
Simulation for $\textbf{Nf=8}$ (same setup with Nf=12)

**lattice action** (Hybrid Monte-Carlo simulation)

- Tree-level Symanzik gauge action
- Highly Improved Staggered Quarks = **HISQ**
  (without tadpole improvement and mass correction in Naik term)

★ **parameter set**

- $\beta \equiv \frac{6}{g^2} = 3.8$, $V = L^3 \times T$, $T/L = 4/3$ fixed.

<table>
<thead>
<tr>
<th>$V$</th>
<th>$12^3 \times 16$</th>
<th>$18^3 \times 24$</th>
<th>$24^3 \times 32$</th>
<th>$30^3 \times 40$</th>
<th>$36^3 \times 48$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mf</td>
<td>0.01~0.16</td>
<td>0.04~0.1</td>
<td>0.02~0.1</td>
<td>0.02~0.07</td>
<td>0.015~0.03</td>
</tr>
</tbody>
</table>

★ Measurements (P+AP method $\Rightarrow$ double size in T-dir.)

★ **about 1000 trajectories after thermalization**

- $M_\pi$, $F_\pi$, $M_\rho$, chiral condensate
- analysis for $M_\pi L > 6$ (to cut the finite size effect)
$F_{\pi}/M_{\pi}$ for $N_f=12, 8$ and $4$ (flat or divergent in $\chi$-limit?)

$M_{\rho}/M_{\pi}$ for $N_f=12, 8$ and $4$ (flat or divergent in $\chi$-limit?)
Nf=8 $\Rightarrow$ spontaneous chiral symmetry breaking? (S $\chi$ SB)

**Chiral Perturbation Theory (ChPT)**

In S $\chi$ SB; \[ M_\pi^2 = C_1^\pi m_f + C_2^\pi m_f^2 + \cdots, \quad F_\pi = F + C_1^F m_f + C_2^F m_f^2 + \cdots, \]

$\Rightarrow$ Polynomial fit in small mf region

We regard the data on the largest volume at each mf as the ones on the infinite volume. (Backup figs.)

We don’t discuss the chiral log behavior in this talk.

However, we discussed the chiral log in the published paper.
$F\pi$ vs $mf$ on various Lattice sizes

for the ChPT fit (to cut the size effect)

$F\pi$ data on the largest volume at each $mf$
$F_\pi$

Quadratic fit in $mf=[0.015, 0.04]$
Linear fit in $mf=[0.015, 0.04]$

In wider fit range, $\chi^2$/dof becomes worse.

$F_\pi = 0.0310(13) + 1.4(1) \, mf - 2.6(1.7) \, mf^2 \, (\chi^2$/dof=0.46)

$F_\pi = 0.0329(3) + 1.24(1) \, mf \hspace{1cm} (\chi^2$/dof=1.4)
$M_\rho$

Quadratic fit in $mf=[0.015, 0.04]$  
Linear fit in $mf=[0.015, 0.04]$

$N_f=8$ ($F_\pi$ and $M_\rho$) in small $mf$ is consistent with ChPT.

\[ M_\rho = 0.168(32) + 7.6(4.1) \, mf - 1.2(73.4) \, mf^2 \]  \[ (\chi^2/\text{dof}=0.0017) \]

\[ M_\rho = 0.169(13) + 7.6(5) \, mf \]  \[ (\chi^2/\text{dof}=0.0010) \]
Chiral condensate (direct and indirect calc.)

direct: \[ \text{Tr}[D_{\text{HISQ}}^{-1}(x,x)]/4 \]

indirect: \[ \Sigma = F_{\pi}^2 M_{\pi}^2/(4mf) \]

based on the GMOR relation

In chiral limit \( \text{(quadratic fit in } 0.015 \leq mf \leq 0.04) \)

\[ \langle \bar{\psi} \psi \rangle \bigg|_{mf \to 0} = 0.00052(5), \quad \Sigma \bigg|_{mf \to 0} = 0.00059(13). \quad F^2 \cdot \left( \frac{M_{\pi}^2}{4mf} \right) \bigg|_{mf \to 0} = 0.00050(3) \]
Summary-1, ChPT analysis

- The quadratic fit was done in $0.015 \leq mf \leq 0.04$.
- $N_f=8$ is consistent with $S\chi$ SB in the small $mf$ region.
- $F_\pi > 0$, $M_\rho > 0$, Condensate $> 0$, $M_\pi = 0$ in the $\chi$-limit.
- In the $\chi$-limit, $F_\pi = 0.0310(13)$, $M_\rho/(F_\pi/\sqrt{2}) = 7.7(1.5)$.
- The expansion parameter $\chi = O(1)$ of ChPT in the smallest $mf$ (self-consistent), in contrast to $N_f=12$.
- $\Rightarrow$ simple $S\chi$ SB phase?

$$\chi = N_f \left( \frac{M_\pi}{4\pi F} \right)^2$$
$F_\pi$ vs mf:
How is in the region mf \(\geq 0.05\)?

Polynomial-like behavior
(ChPT-like)

Power-like behavior?
(Remnant of conformality?)
Finite size Hyperscaling analysis
(critical phenomena in conformal transition)

\[ LM_H = \mathcal{F}_H(x), \quad LF_H = \mathcal{G}_F(x) \]

\[ x \equiv L m^{1/1+\gamma} \quad \text{(universal } \gamma \text{)} \]

DeGrand, Del Debbio et al.

\[ LM_H = \mathcal{F}_H(x), \quad LF_H = \mathcal{G}_F(x) \]

information at (approximate) IRFP

\[ \alpha(\mu) \]

If the coupling is in slow-running, ...

\[ m_f \quad m_D \quad m_f \]

\[ \Lambda_{\text{QCD}} \quad \mu \]

Dig it out!

The hyperscaling with mass corrections is needed,
from the lesson of Schwinger-Dyson analysis (our paper, ’12).
Finite size Hyperscaling analysis

Comformal $\rightarrow$ Finite size Hyperscaling behavior with universal $\gamma$

(Critical exponent obtained from the finite volume setup)

$$LM_H = \mathcal{F}_H(x), LF_H = \mathcal{G}_F(x)$$

$$x \equiv L m^{1/1+\gamma}$$

data aligned $\rightarrow$ different from Nf=4

$\gamma (F_\pi L) \sim 1.0, \gamma (M_\pi L) \sim 0.6, \gamma (M_\rho L) \sim 0.8$

different from Nf=12
Simultaneous fit of Hyperscaling with mass corrections

\[ \xi_H = C_0^H + C_1^H X + C_2^H L \mu f^\alpha, \]

in the middle region of \( mf \geq 0.05 \) and \( \xi_\pi (=M_\pi L) \geq 8 \)
(Schwinger-Dyson eq. with large mass)

\[ \xi_H (=M_H L) \text{ vs } mf \text{ (not } X): \alpha = 1 \text{ fixed (example)} \]

Fig. 5. Simultaneous FSHS fit in \( \xi_\pi \) (left), \( \xi_F \) (center) and \( \xi_\rho \) (right) with \( \alpha = 1 \). The filled symbols are included in the fit, but the open symbols are omitted. The fitted region is \( m_f \geq 0.05 \) and \( \xi_\pi \geq 8 \). The solid curve is the fit result. For a comparison, the simultaneous fit result without correction terms is also plotted by the dashed curve, whose \( \chi^2 / \text{dof} = 83 \).

In this case, the mass correction works well with \( r = 0.874(25), \chi^2 / \text{dof} = 0.75, \text{ dof} = 32 \).

- From various trials of this analysis: \( r = 0.78 - 0.93 \sim 1 \)
Summary-2, Finite-size Hyperscaling analysis

- In the region of $mf \geq 0.05$, hyperscaling is seen. (different from Nf=4)

- non-universal $\gamma$ for each observable in the finite-size hyperscaling (different from Nf=12)

- Lesson from the Schwinger-Dyson analysis with the (large) mass.

- Simultaneous fit of hyperscaling with mass correction gives the universal $\gamma = 0.78 - 0.93 \sim 1$. [requirement for the successful walking technicolor.]

- Nf=8 has the “remnant” of the conformality in the middle range of mf.
Summary

- SU(3) gauge theories with 4, 12 and 8 HISQ quarks.
- $N_f=8$; consistent with $S\chi$SB in the small mass region of our simulation and the remnant of the conformality in the middle region of $m_f$ with $\gamma \sim 1$. (In contrast to $N_f=4$ and 12 cases.)

$N_f=8 \rightarrow$ Candidate of Walking dynamics

In Progress:

- Simulation on larger volumes at lighter masses
- Finite Size Effect (due to the difficulty to take $V=\infty$)
- Lattice spacing dependence (Enhancement) $\leftarrow$ many $\beta$
- Spectroscopy ($M_{\text{glueball}}, M_{\text{"scalar"}}, M_{\text{baryon}}, M_{\text{meson}}, F_\rho/\sigma, S$-param. etc.)
- String tension
- $M_{\text{"flavor-singlet light scalar"}} \Rightarrow 125$GeV? ($\rightarrow$ Yamazaki's talk; next)
Thank you
Backup
KMI computer,

- non GPU nodes
  - 148 nodes
  - 2x Xeon 3.3 GHz
  - 24 TFlops (peak)
- GPU nodes
  - 23 nodes
  - 3x Tesla M2050
  - 39 TFlops (peak)
Size dependence of $M_\pi$ and $F_\pi$ at $\beta = 3.8$

FIG. 7. $F_\pi$ (left), $M_\pi$ (center) and $M_\rho$ (right) as functions of $L$.

On the larger volume, there is not (or very tiny) size dependence.

We use the data on the largest volume at each $mf$. 
HISQ with Nf=8:

effective mass for the lowest mf(=0.015) on the largest size(L=36)

\[ M_{\pi}^{PS} = M_{\pi}^{SC}, M_{\rho}^{PV} = M_{\rho}^{VT} \]
→ good flavor symmetry

FIG. 2 (color online). Effective masses of PS meson, \( M_{\pi}^{eff} \), at \( L = 36 \). Triangles and other symbols denote results from point sink correlators with random wall source and corner wall source, respectively. Fit results with error band obtained from random wall source correlator are also plotted by solid lines.

FIG. 4 (color online). Comparisons of \( M_{\pi} \) and \( M_{SC} \), and of \( M_{\rho(PV)} \) and \( M_{\rho(VT)} \) as a function of \( m_f \) with largest volume data at each \( m_f \).
F\(\pi\) (left panel) and M\(\rho\) (right panel):

quadratic and linear fit in 0.015 \(\leq\) mf \(\leq\) 0.04

power function fit (critical phenomena) in 0.05 \(\leq\) mf.

\[ N_f=8 \quad \beta=3.8 \]

non-universal:
\[ \gamma(F\pi) \sim 1.0, \quad \gamma(M\rho) \sim 0.8 \]

different from \(N_f=4\) and 12
Simultaneous fit of hyperscaling with mass corrections

\[ \xi_H = C_0^H + C_1^H X + C_2^H Lm_f^\alpha. \]

(same method with Nf=12)

Hyperscaling? in the middle region of mf (mf \(\geq 0.05\) and \(\xi_\pi \approx M_{\pi L} \approx 8\))

The mass corrections might be needed, as done in Nf=12,
from the lesson in SD analysis.

\[ \gamma = 0.9130(76), \chi^2/\text{dof} = 1.73, \text{dof} = 33 \]

\[ \gamma = 0.874(25), \chi^2/\text{dof} = 0.75, \text{dof} = 32 \]

\[ \gamma = 0.775(56), \chi^2/\text{dof} = 0.93, \text{dof} = 32 \]

\( \Rightarrow \) good \( \chi^2/\text{dof} \), but unclear which \( \alpha \) is better.
Comparison with $N_f = 4$

: trial of hyperscaling in $F \pi L$ and $M_\rho L$ (in $S \chi$ SB)

$\beta = 3.7, \gamma = 0.0$

$\beta = 3.7, \gamma = 1.0$

$\beta = 3.7, \gamma = 2.0$

no scaling in $0 < \gamma < 2$