



Dynamical symmetry breaking in nonperturbative string theory

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String theory as quantum gravity

Fundamental requirements of string theory:

- quantum gravity:
 - consistent quantum (field) theory
 - determines space–time itself
 - reduced to the Einstein gravity
 - four–dimensional Minkowski space
(including its signature?),
standard model of universe
- unified theory:
 - **standard model** of particle physics

Perturbative string as a candidate

- includes both gauge boson (open string), and graviton (closed string)
- (perturbatively) consistent quantum theory
- nonrenormalizability of gravity
→ extended object



(almost) **unique ultimate theory**
respecting quantum theory
(not Einstein gravity)

Virtues of perturbative string

essence: **local scale invariance** on WS

- st-duality

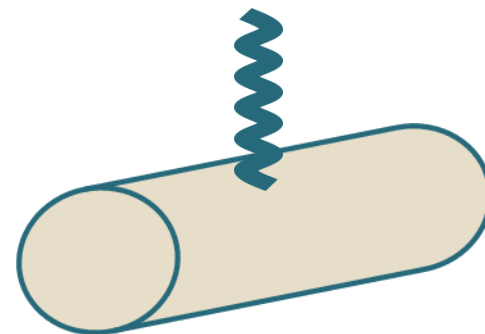
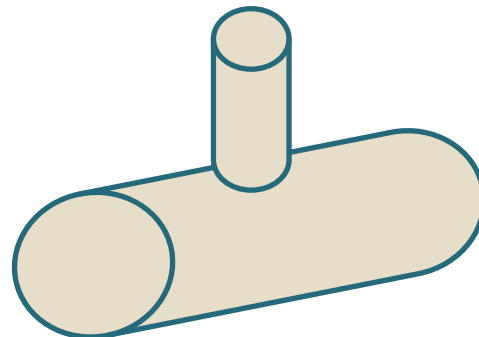
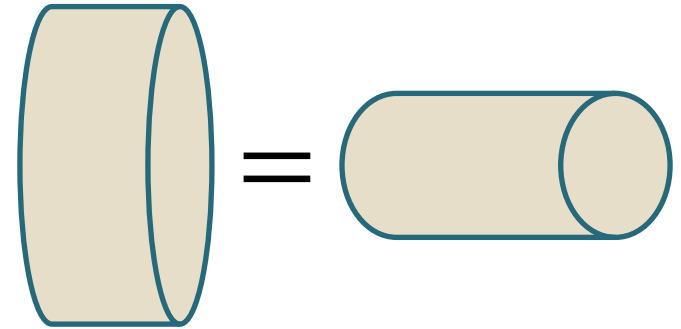
- UV-finiteness

- perturbative QG

- graviton in the spectrum (cf. QCD string)

- unification of external field & spectrum

- desirable as unified theory



Faults of perturbative string

- local scale invariance

- background EOM (Einstein eq.)

$$S = -T \int d^2\sigma \sqrt{-h} h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu}(X)$$

→ essentially perturbation theory
on-shell (1st quantization)

- anomaly → ten dimensions (critical dim.)

- infinite # of perturbative vacua

- serious as quantum gravity

→ nonpert. formulation:

abandon local scale inv, even WS picture!

Nonpert. effect seen by perturbative string

- large order behavior

$$F(g_s^2) = \sum_{n=-1}^{\infty} C_n g_s^{2n}, \quad C_n \sim (2n)! C^{-2n} \quad (n \rightarrow \infty)$$

cf. field theory: $C_n \sim n! C^{-2n} \quad (n \rightarrow \infty)$

suggests nonpert. effect $F \sim e^{-C/g_s}$

in spite of closed SFT action [Shenker 1991]

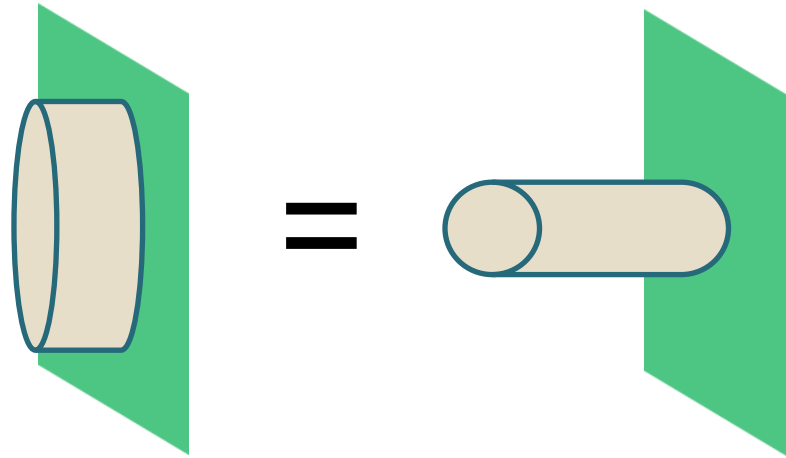
$$S[\Psi] = \frac{1}{g_s^2} \int (\Psi K \Psi + \Psi^3 + \Psi^4 \dots)$$

cf. $S = -\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} \rightarrow F_{\text{inst}} \sim e^{-C/g^2}$

D-branes as nonpert. effect (1)

- classical solution of closed SFT (soliton)
- black brane solution at low energy
- Dirichlet b.c. for **open string**
- **st-duality**

→ source of closed string



D-branes as nonpert. effect (2)

- dynamics: open string oscillation

[Polchinski 1995]

- action: open string disk amplitude



- tension $\sim C/g_s$
- result of st-duality:

large order behavior, nonpert. effect.:
reflects difference from particle theory

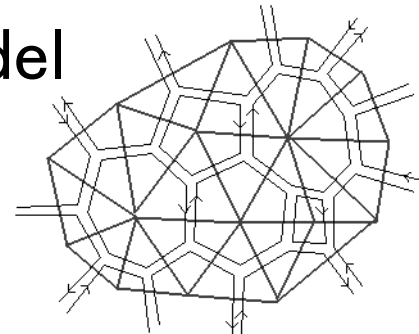
Toward nonpert. formulation

- nonpert. effect does not always suggest nonpert. formulation itself
(D-brane: within perturbation theory)
- existence of D-particle, D-instanton
 - “quantization” of soliton
 - tractable thanks to **st-duality**:
(a kind of) gauge theory (not BH dynamics)
- string duality:
F-string \Leftrightarrow D-brane (as SG \Leftrightarrow MT)
in some string theory,
condensation of string \sim D-brane

Candidates so far

- discretization of WS \rightarrow matrix model

$$S = N \text{tr} \left(\frac{1}{2} \phi^2 + \frac{g}{4} \phi^4 \right)$$



critical pt.: + large- N lim. (**double scaling lim.**)

\rightarrow smooth surface (ϕ : technical tool for WS)

- D-brane matrix model:

Yang-Mills type gauge theory (target sp. pic.)

$$S = \int d^{p+1}x \left(-\frac{1}{4g^2} \text{tr} (F_{\mu\nu} F^{\mu\nu} + D_\mu \phi^i D^\mu \phi^i + \dots) \right)$$

$A_\mu(x)$: gauge field on brane, $\phi^i(x)$: position

Virtues & faults: WS discretization

- virtues

- reproduce all order perturbation theory

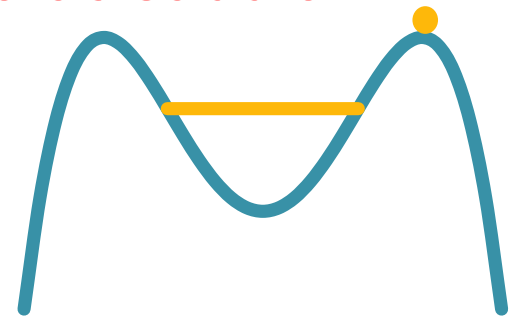
- nonpert. effect \sim eigenvalue at saddle

- \sim tunneling of eigenvalue

- WS boundary

$$e^{-C/g_s}$$

nonpert. exact free energy (Painlevé eq.)



- faults

- available only in $D \leq 2$ (noncritical string)

- no space-time SUSY

Virtues & faults: D-brane matrix model

- virtues

- captures D-brane dynamics (@ low energy)

- many examples:

- force, scattering, decay, recombination...

- concrete example as (classical) gravity:

- AdS/CFT: \exists region where closed string can be described by gauge theory (st-duality)

- target sp. symm. (sometimes nontrivially)

- faults

- F-string DOF, scattering amp., st-duality

Why D-brane MM can contain gravity

D-brane MM \leftarrow quantization of **massless open string DOF** \rightarrow should be valid only for D-brane dynamics @ low energy

far from full nonpert. closed string theory

e.g.

IIB matrix model \cdots D(-1)-brane (0d) [IKKT 1997]

Matrix theory \cdots D0-brane (1d MQM) [BFSS 1997]

Matrix string th. \cdots D-string (2d SYM) [DVV 1997]

AdS/CFT (strongest) \cdots D3-brane (4d SYM)
[Maldacena 1998]

Key: symmetries (1)

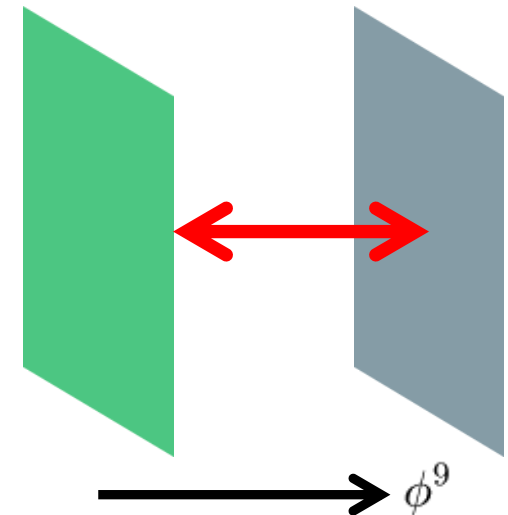
- SUSY

e.g. IIBMM

- maximal SUSY: graviton multiplet
- “emergent” 10d gravity
(remnant of st-duality)

$$\phi^i = \left(\begin{array}{c|c} \text{green} & \text{int} \\ \hline \text{int} & \text{blue} \end{array} \right)$$

$$\phi^9 = \left(\begin{array}{c|c} 0 & \text{int} \\ \hline \text{int} & r \end{array} \right)$$



one-loop in gauge th. in background $\rightarrow \frac{1}{r^6}$

Key: symmetries (2)

- isometry & conformal symmetry
 - AdS₅/CFT₄: SO(4,2)
 - other symm. ((broken) chiral symm. ...)
- ensure validity low energy effective action (w/ small # of parameters)
- (Note: even if symmetries do not agree, some quantities can be well described by their peculiarity (universality, BPS, ...))
- usually, only very special closed string can be reproduced ...

(Maybe) Promising candidates

- IIBMM

- D(-1)-brane low energy effective action

- WS formulation of Schild gauge

$$S_n = - \int d^2\sigma e \left[\frac{1}{e^n} \left\{ -\frac{T^2}{2} \underbrace{(\epsilon^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu)^2}_{2 \det_{\alpha\beta}(\partial_\alpha X^\mu \partial_\beta X_\mu)} \right\}^{\frac{n}{2}} + n - 1 \right]$$

$$n = 1 : \quad \text{NG} \quad S = -m \int ds \sqrt{-\dot{X}^2}$$

$$n = 2 : \quad \text{Schild} \quad S = -\frac{1}{2} \int ds \left(\frac{1}{e} (-m^2 \dot{X}^2) + e \right)$$

$$\text{gauge fix: } \rightarrow S = -\frac{T^2}{2} \int d^2\sigma \{X^\mu, X^\nu\}^2$$

$$\rightarrow S = \beta \text{tr}_N [A^\mu, A^\nu]^2 : \text{0d SYM}$$

0-dim. SYM can be regarded as

- D(-1)-brane low energy effective action
- matrix reg. of F-string in Schild gauge

- $c = 1$ matrix reloaded

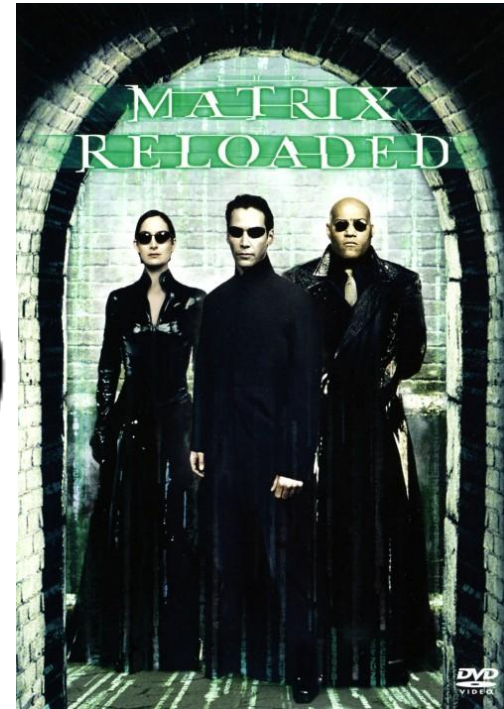
[McGreevy Verlinde 2003]

- MQM: WS discretization
of $D = 2$ string

$$S = \beta \int dt \operatorname{tr} \left(\frac{1}{2} \dot{U}(t)^2 - V(U(t)) \right)$$

with $\beta \rightarrow \beta_c, \quad N \rightarrow \infty$

- LEEE of unstable D0-brane



Strings from Tachyons: The $c = 1$ Matrix Reloaded

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Abstract

We propose a new interpretation of the $c=1$ matrix model as the world-line theory of N unstable D-particles, in which the hermitian matrix is provided by the non-abelian open string tachyon. For D-branes in 1+1-d string theory, we find a direct quantitative match between the closed string emission due to a rolling tachyon and that due to a rolling eigenvalue in the matrix model. We explain the origin of the double-scaling limit, and interpret it as an extreme representative of a large equivalence class of dual theories. Finally, we define a concrete decoupling limit of unstable D-particles in IIB string theory that reduces to the $c=1$ matrix model, suggesting that 1+1-d string theory represents the near-horizon limit of an ultra-dense gas of IIB D-particles.

Should perturbative string be realized?

- D-brane MMs (SYMs): difficult to reproduce pert. string amplitude
 - they must realize F-string, st-duality?
they should contain 10d gravity?
recall def. of quantum gravity:
well-defined quantum th. + 4-dim. EH @ LE
(cf. LGT: allows weak coupling exp.)

desirable:

- $1/N$ -expansion = perturbative exp. of F-string
- double scaling limit = nonperturbative string theory
(cf. noncritical string)
- SUSY DW MM ← new e.g. of “desirable” one

Dynamical symmetry breaking in nonpert. string theory

- in “desirable” scenario,
 - $1/N$ -expansion: perturbative (w.r.t. g_s) exp. symm. (in particular SUSY) is inevitable
 - double scaling limit: nonpert. string theory
 - two standard models @ LE
 - no symmetries!
- ∴ MM or SYM with
symm. preserved @ all orders in $1/N$ -exp.
but gets broken in the DSL

Examples of DSB (1)

- rotational symm. in IIBMM: 0-dim. SYM

$$S = \frac{1}{g^2} \text{tr} \left(-\frac{1}{4} [A_\mu, A_\nu]^2 + \frac{1}{2} \bar{\psi} \Gamma^\mu [A_\mu, \psi] \right)$$

manifest SO(10) symm.

eigenvalues of A_μ : space-time pts

→ originally 10-dim. flat sp.

- mean field approx.
(adding $M_\mu \text{tr}(A_\mu A_\mu)$)

[Nishimura Sugino 2001]

[川河黒松篠 2002]

[Kawai Aoyama 2006] etc.

→ $T_{\mu\nu} = \left\langle \frac{1}{N} \text{tr}(A_\mu A_\nu) \right\rangle$: 10 eigenvalues

→ four-dim. space “emerges” **dynamically**

Example of DSB (2)

- **SUSY DW MM:** [T.K. Sugino 2013]
[Endres T.K. Sugino Suzuki]

$$S = N \text{tr} \left(\frac{1}{2} B^2 + iB(\phi^2 - \mu^2) + \bar{\psi}(\phi\psi + \psi\phi) \right)$$

w/ nilpotent SUSY' s

we have recently shown that:

- in $N \rightarrow \infty$ this MM reproduces several types of tree level two-point functions of 2D type IIA superstring theory: reproduce perturbative string (although in 0-dim.!!)
- SUSY' s are broken in the DSL via an isolated eigenvalue as $\sim e^{-C/g_s}$

Virtues & faults of our model

- virtues
 - reproduces tree level string amplitude
→ expected st-duality
 - SUSY is preserved at all orders in $1/N$ -exp.
 - first e.g. of **dynamical & nonperturbative breaking of SUSY in string theory**
- faults
 - **loses both WS picture & D-brane picture**
 - poor target space and fields
breaking of nilpotent SUSY is significant?