Neutrino Spectral Density at Electroweak Scale Temperature

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References

The Spectral Density $\rho(\omega, p)$ is responsible for a physical content of the system.
The standard model (SM) seems to fail to explain the observed baryon asymmetry of the universe (BAU): $\eta_B = (n_B - \bar{n}_B)/n_γ \approx 6.1 \times 10^{-10}$.

An extension of the SM by adding right-handed Majorana neutrinos ($N_R$) may have a chance to account for the BAU (Fukugita et al. ('86)): A decay of $N_R$ (e.g. Fig.) generates a net lepton number, which are partially converted into the baryon number via sphaleron process in the electroweak (EW) phase trans. (Kuzmin et al. ('85), Klinkhamer et al. ('84), Arnold et al. ('87)).

If the mass difference between two $N_R$s is in the order of their CP-violating decay width, the CP asymmetry is dynamically enhanced (Pilaftsis ('97)), and the leptogenesis in the EW scale can be relevant (Pilaftsis et al. ('05)).
Decay of Right-Handed Neutrino: Typical Example

\[ N_R \rightarrow \text{Right-Handed Neutrino} \]
\[ L^T = (\nu, l) \]
\[ \Phi = \text{Higgs Doublet} \]

**Figure:** Plumacher et.al. PRD (2010).
Im \Sigma_{NR} = \Im \Im \Phi = \text{Higgs Doublet} \quad N_R = \text{Right-Handed Neutrino} \quad L^T = (\nu, l)
If the standard-model leptons have non-trivial spectral properties in EW scale plasma, the lepton number creation via the $N_R$ decay will be significantly modified.
In QED and QCD at extremely high $T$, the Hard Thermal-Loop (HTL) approx. indicates that a probe fermion interacting with thermally excited gauge bosons and anti-fermions admits a collective excitation mode (See, The text book by LeBellac).

In the neutrino dispersion relation in the electroweak scale plasma, the existence of a novel branch in the ultrasoft-energy region has been indicated by using the HTL and the unitary gauge fixing (Boyanovsky (’05)).

The gauge invariance and the strength of the ultrasoft mode should be investigated.
When the probe massless fermion interacts with the massive mesonic mode in plasma (an effective description of QGP), the spectral density of the fermion shows a three-peak structure with an ultrasoft mode. (Kitazawa et.al. ('05-'06), Harada et.al. ('08), w.o. HTL).

In particular, when the fermion is coupled with the massive vectorial meson, the gauge independent nature of the three-peak structure has been confirmed (Satow et.al. ('10)) by using the Stueckelberg formalism.
Partly motivated by the low energy scale (resonant) leptogenesis, we investigate the spectral properties of the standard model neutrinos at finite $T$ around the electroweak scale.

We utilize the technology developed in the QGP physics (Kitazawa et.al. ('06), Satow et.al. ('10)), which leads to the followings advantages:

- the use of $R_\xi$ gauge (Fujikawa et.al. ('72)): Necessary to investigate the gauge invariance of the spectral density.
- Without recourse to the HTL approximation: Necessary to evaluate the strength of the collective modes.

We discuss the implications of the neutrino spectral properties to the leptogenesis in the EW scale.
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Massless Lepton Sector:

\[ \mathcal{L}_L = \sum_{i=e, \mu, \tau} \left[ (\bar{\nu}^i, \bar{l}^i_L) i \partial \left( \nu^i_L \right) + \bar{l}^i_R i \partial \nu^i_L \right] + \left[ W^\dagger_{\mu} J^\mu_{W} + J^\mu_{W} W_{\mu} + Z_{\mu} J^\mu_{Z} + A_{\mu}^{\text{EM}} J^\mu_{\text{EM}} \right], \]  

(3)

Gauge Boson Propagator in \( R_\xi \) Gauge:

\[ G_{\mu\nu}(q, T) = -\frac{(g_{\mu\nu} - q_{\mu} q_{\nu}/M_{W,Z}^2(T))}{q^2 - M_{W,Z}^2(T)} + \frac{q_{\mu} q_{\nu}/M_{W,Z}^2(T)}{q^2 - \xi M_{W,Z}^2(T)}, \]  

(4)
Neutrino Self-Energy and Spectral Density

- We evaluate the one-loop self-energy diagrams at finite $T$ in the imaginary time formalism: Matsubara Sum $\Rightarrow$ Analytic Continuation ($i\omega_m \rightarrow \omega + i\eta$) $\Rightarrow$ Retarded Self-Energy $\Sigma^{(\nu)}_{\text{ret}}(p, \omega; T)$.

- For the massless left-handed neutrinos, the finite-$T$ effects are solely encoded in the coefficients in the decomposition

$$\Sigma^{(\nu)}_{\text{ret}}(p, \omega; T) = \sum_{s=\pm} \mathcal{P}_R \Lambda_s, p \gamma^0 \mathcal{P}_L \Sigma^{(\nu)}_s(|p|, \omega; T),$$

$$\mathcal{P}_L/R = (1 \mp \gamma_5)/2, \quad \Lambda_{\pm, p} = (1 \pm \gamma^0 \gamma \cdot p/|p|)/2.$$  

- For the spectral density, similarly,

$$\rho^{(\nu)}(p, \omega; T) = \sum_{s=\pm} \mathcal{P}_R \Lambda_s, p \gamma^0 \mathcal{P}_L \rho^{(\nu)}_s(|p|, \omega; T)$$

$$\rho_{\pm}^{(\nu)}(|p|, \omega; T) = \frac{-\text{Im} \Sigma^{(\nu)}_{\pm}(|p|, \omega; T)/\pi}{\left\{\omega - |p| \mp \text{Re} \Sigma^{(\nu)}_{\pm}(|p|, \omega; T)\right\}^2 + \left\{\text{Im} \Sigma^{(\nu)}_{\pm}(|p|, \omega; T)\right\}^2}.$$
Higgs Effective Potential:

\[ V_{\text{eff}} = -\frac{\mu_0^2}{2} \left[ 1 - \frac{T^2}{4\mu_0^2 / (2\lambda + 3g^2/4 + g'^2/4)} \right] v^2(T) + \frac{\lambda}{4} v^4(T), \]  

(7)

The potential has been derived in the $R_\xi$ gauge by taking account of the leading order of the high $T$ expansion for thermal one-loop effects of the Higgs and gauge bosons in addition to the tree-level Higgs potential.

The $\xi$ dependences cancel out between the Nambu-Goldstone modes and the ghost contributions.
The effective potential leads to the second-order phase transition. Note that in reality the possibility of the strong first-order transition has been ruled out within the standard model.
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\[ T/v_0 = 0.2, \quad T/M_W(T) \simeq 0.63, \quad (8) \]

\[ \omega - |p| - \text{Re} \Sigma^+(\omega, |p|, T) = 0 \]
\[ T/v_0 = 0.42, \quad T/M_W(T) \simeq 1.45, \] (9)

\[ \omega - |p| - \text{Re} \Sigma_+(\omega, |p|, T) = 0 \]
Intermediate Temperature Region II

\[ T/v_0 = 0.5, \quad T/M_W(T) \simeq 1.83, \quad (10) \]

\[ \omega - |p| - \text{Re } \Sigma_+(\omega, |p|, T) = 0 \]
High Temperature Region

\[ T/v_0 = 0.8, \quad T/M_W(T) \approx 4.9, \quad (11) \]

\[ \omega - |p| - \text{Re} \Sigma_+(\omega, |p|, T) = 0 \]
Spectral Density at \((|p|/v_0, T/v_0) = (0.02, 0.5)\)
Landau Damping (Fig.) makes the imaginary part being finite in the spacelike region:

\[
\text{Im} \Sigma^{(\nu)}_+ \ni \int_k \delta \left[ \omega + |k| - \sqrt{|p - k|^2 + M_W^2} \right] \\
\times \left[ N_F(1 + N_B) + N_B(1 - N_F) \right] \cdot [\cdots]. \tag{12}
\]
Landau Damping Suppression

For a small external momentum \((\omega, p)\) and a not small \(M_W\), the Landau Damping becomes kinematically limited in the loop integral \( \int_k \):

\[
G(x) = \int_x^\infty dx \ x N_B(x) = \sum_{n=1}^\infty \frac{e^{-nx}}{n^2} \left[ 1 + nx \right],
\]

\[
x = \frac{\omega^2 - |p|^2 - M_W^2}{2 T (\omega - |p|)} > 0. \tag{13}
\]

Here, the HTL limit \( T \gg M_W, \omega, |p| \) leads to \( x \to 0 \) and \( G(x) \to \zeta(2) \).

The condition \( T \sim M_W \) and the resultant finite \( x \) leads to \( G(x) < \zeta(2) \) and thereby the suppression of the Landau damping.
Self-Energy at Three-Peak Region

\[ \frac{T}{v_0} = 0.5, \quad \frac{|p|}{v_0} = 0.02. \]  

(14)

In the right panel, the crossing points corresponds to the solutions of

\[ \omega - |p| - \text{Re } \Sigma_+ (\omega, |p|, T) = 0. \]
Spectral Density at \( \left( \frac{|p|}{v_0}, \frac{T}{v_0} \right) = (0.02, 0.5) \)
\[ \frac{T}{v_0} = 0.5 , \quad \frac{|p|}{v_0} = 0.02 . \] (15)
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- **$\xi$ Dependence of Three-Peak Spectral Density**
  
- **Graphical Representation**

- **Equation**
  
- **Parameters**

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"Neutrino Spectral Density at Electroweak Scale Temperature"
The net baryon number $N_b$ is produced in the sphaleron process when the changing rate of $N_b$ is larger than the expanding rate of the universe,

$$\left| \frac{1}{N_b} \frac{dN_b}{dt} \right| \geq H(T) ,$$

where,

$$H(T) = 1.66 \sqrt{N_{\text{dof}}} \frac{T^2}{M_{\text{PL}}} \simeq T^2 \times 1.41 \times 10^{-18} \text{ (GeV)} ,$$

$$\frac{1}{N_b} \frac{dN_b}{dt} = -1023 \cdot g^7 v(T) \exp \left[ -1.89 \frac{4\pi v(T)}{gT} \right] ,$$

and we obtain

$$T \geq T_* \simeq 160 \text{ GeV} , \quad T_*/v_0 \simeq 0.65 .$$
Neutrino Spectral Density around $T = T_*$

![Graph showing the Neutrino Spectral Density at Electroweak Scale Temperature](image-url)
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We have investigated the spectral properties of standard-model left-handed neutrinos at finite $T$ around the electroweak scale in a way where the gauge invariance is manifestly checked ($R_\xi$ gauge).

The spectral density of SM neutrino has the three-peak structure with the ultrasoft mode with a physical significance when $T/M_{W,Z} \gtrsim 1$.

The collective excitation which involves the ultrasoft mode can appear at temperature comparable to $T_\ast$. Thus, the three-peak collective modes could affect the leptogenesis at $T \gtrsim T_\ast$, and therefore, the baryogenesis.
It is desirable to estimate how large the effects of two-loop or higher-order diagrams are on the neutrino spectral density.

The present formulation should be extended to include the bare fermion mass effects.

It is interesting to evaluate the lepton number creation with respect to the non-trivial spectral properties of the neutrinos as well as the charged leptons by adopting the explicit model of the leptogenesis.

The spectral density of the right-handed neutrinos?

Thanks for Your Attention!