Neutrino Spectral Density at Electroweak Scale Temperature

K. Miura[A], Y. Hidaka[B], D. Satow[B], and T. Kunihiro[C]

KMI[A], LNF-INFN[A], RIKEN[B], Kyoto Univ.[C]

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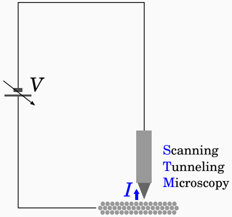
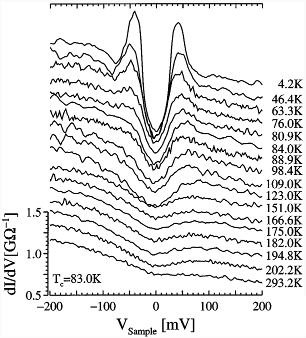
References

K. Miura, Y. Hidaka, D. Satow, and T. Kunihiro, arXiv:1306.1701 [hep-ph].

Spectral Density Example: Superconductivity





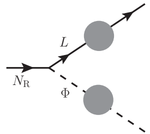
Slightly Underdoped Bi2212
STM Superconductor Scan

Renner et. al.,
Phys.Rev.Lett., 80
149, (2008).

$$\frac{dI}{dV} \sim \frac{d}{dV}\left[\int_{\epsilon_f}^{\omega=\epsilon_f+eV} d\omega \ \mathrm{Dos}(\omega)\right] \sim \mathrm{Dos}(\omega) \sim \int_p \rho(\omega, p) \ , \qquad (1)$$

The Spectral Density $\rho(\omega, p)$ is responsible for
a PHISICAL CONTENTS of the system.

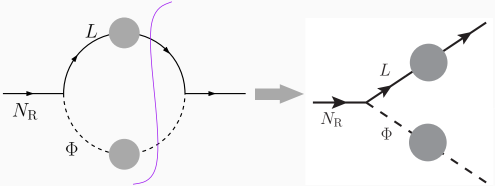
Leptogenesis, Particularly in Low Energy Scale



$N_R =$ Right-Handed Neutrino
$L^T = (\nu, l)$
$\Phi =$ Higgs Doublet

- The standard model (SM) seems to fail to explain the observed baryon asymmetry of the universe (BAU): $\eta_B = (n_B - \bar{n}_B)/n_\gamma \simeq 6.1 \times 10^{-10}$.
- An extension of the SM by adding right-handed Majorana neutrinos (N_R) may have a chance to account for the BAU (Fukugita et.al. ('86)): A decay of N_R (e.g. Fig.) generates a net lepton number, which are partially converted into the baryon number via sphaleron process in the electroweak (EW) phase trans. (Kuzmin et.al. ('85), Klinkhamer et.al.('84), Arnold et.al. ('87)).
- If the mass difference between two N_Rs is in the order of their CP-violating decay width, the CP asymmetry is dynamically enhanced (Pilaftsis ('97)), and the leptogenesis in the EW scale can be relevant (Pilaftsis et.al. ('05)).

Decay of Right-Handed Neutrino: Typical Example



$N_R =$ Right-Handed Neutrino

$L^T = (\nu, l)$

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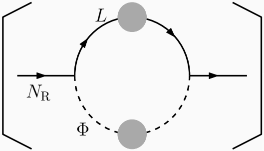
Figure: Plumacher et.al. PRD (2010).

Imaginary Self-Energy of Right-Handed Neutrino Comes into Play!

$$\mathrm{Im}\,\Sigma_{\mathrm{N_R}} = \mathrm{Im}$$



$N_R =$ Right-Handed Neutrino

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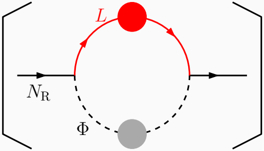
Spectral Density of Leptons in Leptogenesis

$$\mathrm{Im}\,\Sigma_{\mathrm{N_R}} = \mathrm{Im}$$



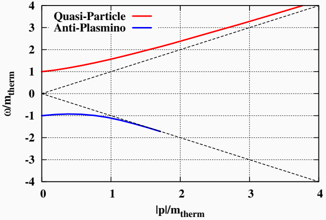
$N_R =$ Right-Handed Neutrino
$L^T = (\nu, l)$
$\Phi =$ Higgs Doublet

$$\tilde{G}_{\mathrm{Lepton}}(\mathbf{k}, i\omega_n; T) = \int_{-\infty}^{\infty} d\omega \, \frac{\rho^{(\nu,l)}_{\mathrm{Left}}(\mathbf{k}, \omega; T)}{\omega + i\omega_n} \, . \quad (2)$$

If the standard-model leptons have non-trivial spectral properties in EW scale plasma, the lepton number creation via the N_{R} decay will be significantly modified.

Spectral Property of Finite T Gauge Theory

- In QED and QCD at extremely high T, the Hard Thermal-Loop (HTL) approx. indicates that a probe fermion interacting with thermally excited gauge bosons and anti-fermions admits a collective excitation mode (See, The text book by LeBellac).



- In the neutrino dispersion relation in the electroweak scale plasma, the existence of a novel branch in the ultrasoft-energy region has been indicated by using the HTL and the unitary gauge fixing (Boyanovsky ('05)).

The gauge invariance and the strength of the ultrasoft mode should be investigated.

QGP Physics

- When the probe massless fermion interacts with the massive mesonic mode in plasma (an effective discription of QGP), the spectral density of the fermion shows a three-peak structure with a ultrasoft mode. (Kitazawa et.al. ('05-'06), Harada et.al. ('08), w.o. HTL).
- In particular, when the fermion is coupled with the massive *vectorial* meson, the gauge independent nature of the three-peak structure has been confirmed (Satow et.al. ('10)) by using the Stueckelberg formalism.

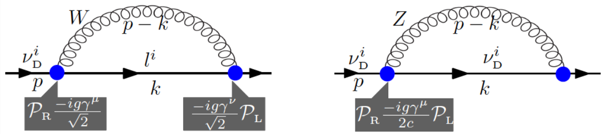
Goal: From QGP to Electroweak Plasma

- Partly motivated by the low energy scale (resonant) leptogenesis, we investigate the spectral properties of the standard model neutrinos at finite T around the electroweak scale.
- We utilize the technology developed in the QGP physics (Kitazawa et.al. ('06), Satow et.al. ('10)), which leads to the followings advantages:
  - the use of R_ξ gauge (Fujikawa et.al. ('72)): Necessary to investigate the gauge invariance of the spectral density.
  - Without recourse to the HTL approximation: Necessary to evaluate the strength of the collective modes.
- We discuss the implications of the neutrino spectral properties to the leptogenesis in the EW scale.

Table of Contents

Table of Contents

Lagrangian and Feyman Diagram



- Massless Lepton Sector:

$$\mathcal{L}_{\mathrm{L}} = \sum_{i=e,\mu,\tau}\left[(\bar{\nu}^i, \bar{l}^i_{\mathrm{L}}) i\partial\!\!\!/ \begin{pmatrix}\nu^i \\ l^i_{\mathrm{L}}\end{pmatrix} + \bar{l}^i_{\mathrm{R}} i\partial\!\!\!/ l^i_{\mathrm{R}}\right] + \left[W^\dagger_\mu J^\mu_{\mathrm{W}} + J^{\mu\dagger}_{\mathrm{W}} W_\mu + Z_\mu J^\mu_{\mathrm{Z}} + A^{\mathrm{EM}}_\mu J^\mu_{\mathrm{EM}}\right], \tag{3}$$

- Gauge Boson Propagator in R_ξ Gauge:

$$G_{\mu\nu}(q, T) = -\frac{\left(g_{\mu\nu} - q_\mu q_\nu / M^2_{\mathrm{W,Z}}(T)\right)}{q^2 - M^2_{\mathrm{W,Z}}(T)} + \frac{q_\mu q_\nu / M^2_{\mathrm{W,Z}}(T)}{q^2 - \xi M^2_{\mathrm{W,Z}}(T)}, \tag{4}$$

Neutrino Self-Energy and Spectral Density

- We evaluate the one-loop self-energy diagrams at finite T in the imaginary time formalism: Matsubara Sum $\Rightarrow$ Analytic Continuation $(i\omega_m \to \omega + i\eta)$ $\Rightarrow$ Retarded Self-Energy $\Sigma^{(\nu)}_{\mathrm{ret}}(\mathbf{p}, \omega; T)$.
- For the massless left-handed neutrinos, the finite-T effects are solely encoded in the coefficients in the decomposition

$$\Sigma^{(\nu)}_{\mathrm{ret}}(\mathbf{p}, \omega; T) = \sum_{s=\pm} \left[\mathcal{P}_{\mathrm{R}} \Lambda_{s,\mathbf{p}} \gamma^0 \mathcal{P}_{\mathrm{L}}\right] \Sigma^{(\nu)}_s(|\mathbf{p}|, \omega; T) \; , \tag{5}$$

$$\mathcal{P}_{\mathrm{L/R}} = (1 \mp \gamma_5)/2 \; , \quad \Lambda_{\pm,\mathbf{p}} = (1 \pm \gamma^0 \boldsymbol{\gamma} \cdot \mathbf{p}/|\mathbf{p}|)/2 \; . \tag{6}$$

- For the spectral density, similarly,

$$\rho^{(\nu)}(\mathbf{p}, \omega; T) = \sum_{s=\pm} \left[\mathcal{P}_{\mathrm{R}} \Lambda_{s,\mathbf{p}} \gamma^0 \mathcal{P}_{\mathrm{L}}\right] \rho^{(\nu)}_s(|\mathbf{p}|, \omega; T)$$

$$\rho^{(\nu)}_\pm(|\mathbf{p}|, \omega; T) = \frac{-\mathrm{Im}\; \Sigma^{(\nu)}_\pm(|\mathbf{p}|, \omega; T)/\pi}{\{\omega - |\mathbf{p}| \mp \mathrm{Re}\Sigma^{(\nu)}_\pm(|\mathbf{p}|, \omega; T)\}^2 + \{\mathrm{Im}\Sigma^{(\nu)}_\pm(|\mathbf{p}|, \omega; T)\}^2} \; .$$

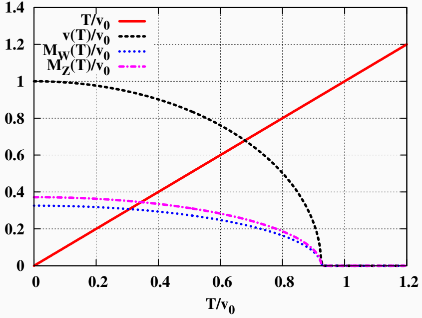
Higgs Effective Potential I

- Higgs Effective Potential:

$$V_{\rm eff} = -\frac{\mu_0^2}{2}\left[1 - \frac{T^2}{4\mu_0^2/(2\lambda + 3g^2/4 + g'^{\,2}/4)}\right] v^2(T) + \frac{\lambda}{4} v^4(T)\ , \qquad (7)$$

- The potential has been derived in the R_ξ gauge by taking account of the leading order of the high T expansion for thermal one-loop effects of the Higgs and gauge bosons in addition to the tree-level Higgs potential.
- The ξ dependences cancel out between the Nambu-Goldstone modes and the ghost contributions.

Higgs Effective Potential II

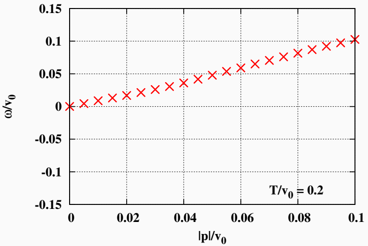


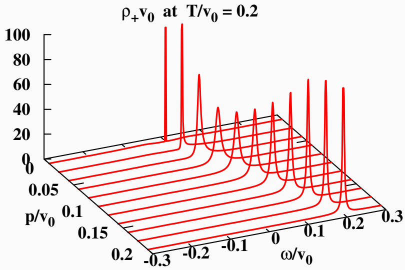
The effective potential leads to the second-order phase transition. Note that in reality the possibility of the strong first-order transition has been ruled out within the standard model.

Table of Contents

Low Temperature Region

$$T/v_0 = 0.2\ , \quad T/M_W(T) \simeq 0.63\ , \tag{8}$$

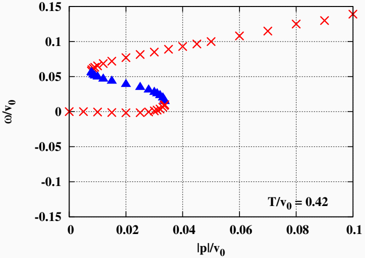


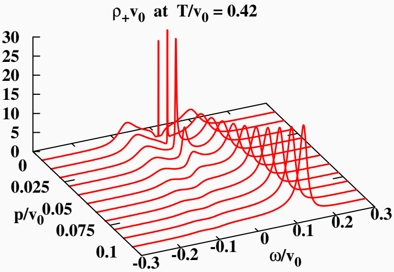


$$\omega - |\mathbf{p}| - \mathrm{Re}\ \Sigma_+(\omega, |\mathbf{p}|, T) = 0$$

Intermediate Temperature Region I

$$T/v_0 = 0.42\ , \quad T/M_W(T) \simeq 1.45\ , \tag{9}$$

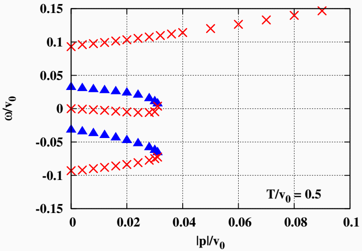


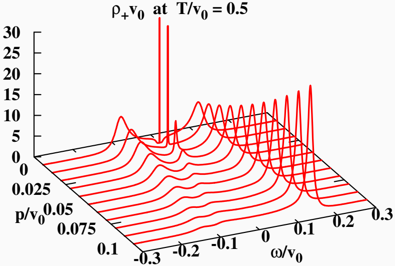


$$\omega - |\mathbf{p}| - \mathrm{Re}\ \Sigma_+(\omega, |\mathbf{p}|, T) = 0$$

Intermediate Temperature Region II

$$T/v_0 = 0.5 \ , \quad T/M_W(T) \simeq 1.83 \ , \tag{10}$$

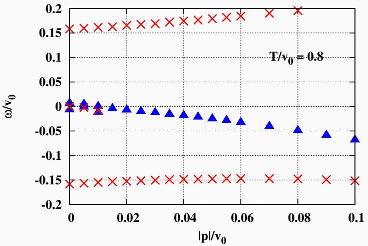


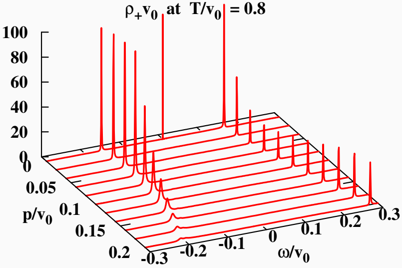


$$\omega - |\mathbf{p}| - \mathrm{Re}\ \Sigma_+(\omega, |\mathbf{p}|, T) = 0$$

High Temperature Region

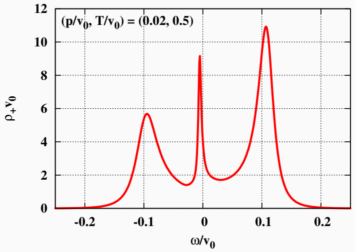
$$T/v_0 = 0.8\ , \quad T/M_W(T) \simeq 4.9\ , \tag{11}$$

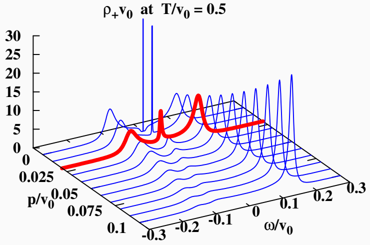




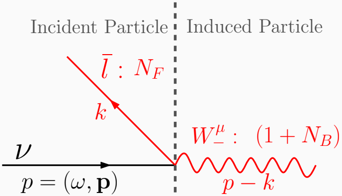
$$\omega - |\mathbf{p}| - \mathrm{Re}\ \Sigma_+(\omega, |\mathbf{p}|, T) = 0$$

Spectral Density at $(|\mathbf{p}|/v_0, T/v_0) = (0.02, 0.5)$





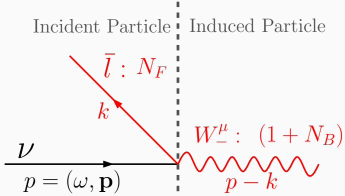
Landau Damping



Landau Damping (Fig.) makes the imaginary part being finite in the spacelike region:

$$\mathrm{Im}\Sigma_+^{(\nu)} \ni \int_k \delta\left[\omega+|\mathbf{k}|-\sqrt{|\mathbf{p}-\mathbf{k}|^2+M_{\mathrm{W}}^2}\right] \times\left[N_{\mathrm{F}}(1+N_{\mathrm{B}})+N_{\mathrm{B}}(1-N_{\mathrm{F}})\right]\cdot\left[\cdots\right]. \quad (12)$$

Landau Damping Suppression



- For a small external momentum $(\omega, \mathbf{p})$ and a not small $M_{\rm W}$, the Landau Damping becomes kinematically limited in the loop integral $\int_k$:

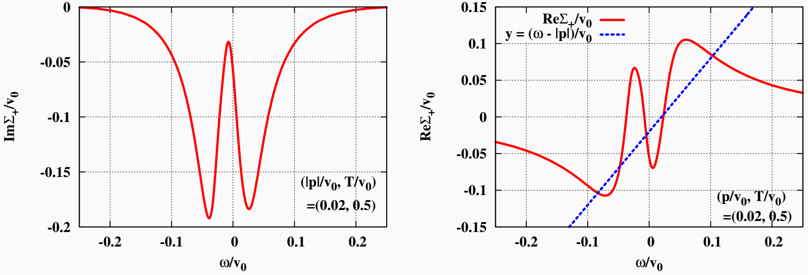
$$G(x) = \int_x^{\infty} dx\; x N_{\rm B}(x) = \sum_{n=1}^{\infty} \frac{e^{-nx}}{n^2}\left[1+nx\right] \;, x = \frac{\omega^2 - |\mathbf{p}|^2 - M_{\rm W}^2}{2T(\omega - |\mathbf{p}|)} > 0 \;. \quad (13)$$

  Here, the HTL limit $T \gg M_{\rm W}, \omega, |\mathbf{p}|$ leads to $x \to 0$ and $G(x) \to \zeta(2)$.

- The condition $T \sim M_{\rm W}$ and the resultan finite x leads to $G(x) < \zeta(2)$ and thereby the suppression of the Landau damping.

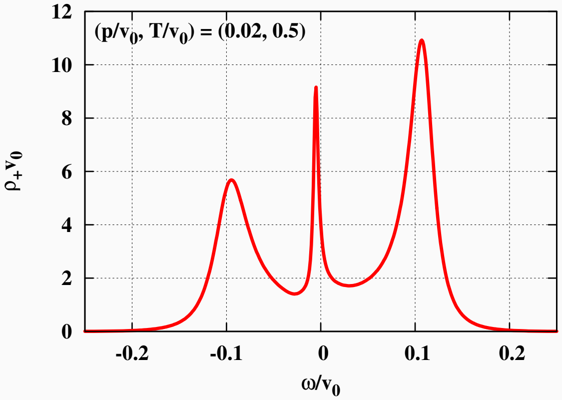
Self-Energy at Three-Peak Region

$$\frac{T}{v_0} = 0.5 \ , \quad \frac{|\mathbf{p}|}{v_0} = 0.02 \ . \tag{14}$$



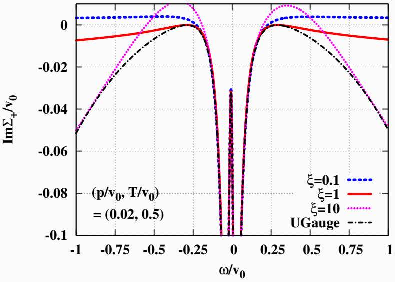
In the right panel, the crossing points corresponds to the solutions of
$\omega - |\mathbf{p}| - \mathrm{Re}\ \Sigma_+(\omega, |\mathbf{p}|, T) = 0.$

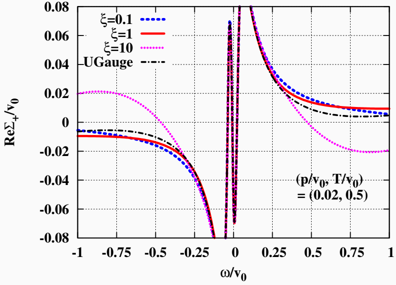
Spectral Density at $(|\mathbf{p}|/v_0, T/v_0) = (0.02, 0.5)$



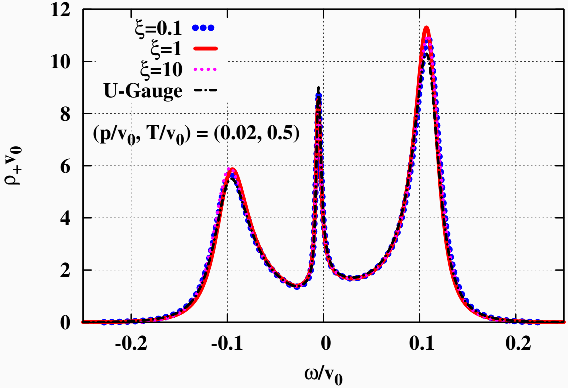
ξ Depenence of Self-Energy in Three-Peak Region

$$\frac{T}{v_0} = 0.5 \ , \quad \frac{|\mathbf{p}|}{v_0} = 0.02 \ . \tag{15}$$





ξ Depenence of Three-Peak Spectral Density



Sphaleron Freeze-out Temperature

The net baryon number $N_{\rm b}$ is produced in the sphaleron process when the changing rate of $N_{\rm b}$ is larger than the expanding rate of the universe,

$$\left| \frac{1}{N_{\rm b}} \frac{dN_{\rm b}}{dt} \right| \geq H(T) \ , \tag{16}$$

where,

$$H(T) = 1.66 \sqrt{N_{\rm dof}} \frac{T^2}{M_{\rm PL}} \simeq T^2 \times 1.41 \times 10^{-18} \ ({\rm GeV}) \ , \tag{17}$$

$$\frac{1}{N_{\rm b}} \frac{dN_{\rm b}}{dt} = -1023 \cdot g^7 v(T) \exp\left[-1.89 \frac{4\pi v(T)}{gT} \right] \ , \tag{18}$$

and we obtain

$$T \geq T_* \simeq 160 \ {\rm GeV} \ , \quad T_*/v_0 \simeq 0.65 \ . \tag{19}$$

Neutrino Spectral Density around $T = T_*$

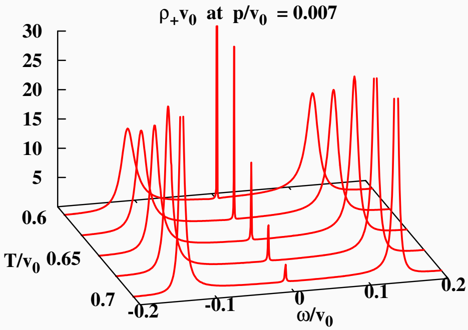


Table of Contents

Summary

- We have investigated the spectral properties of standard-model left-handed neutrinos at finite T around the electroweak scale in a way where the gauge invariance is manifestly checked (R_ξ gauge).
- The spectral density of SM neutrino has the three-peak structure with the ultrasoft mode with a physical significance when $T/M_{\mathrm{W,Z}} \gtrsim 1$.
- The collective excitation which involves the ultrasoft mode can appear at temperature comparable to T_*. Thus, the three-peak collective modes could affect the leptogenesis at $T \gtrsim T_*$, and therefore, the baryogenesis.

Future Plan

- It is desirable to estimate how large the effects of two-loop or higher-order diagrams are on the neutrino spectral density.
- The present formulation should be extended to include the bare fermion mass effects.
- It is interesting to evaluate the lepton number creation with respect to the non-trivial spectral properties of the neutrinos as well as the charged leptons by adopting the explicit model of the leptogenesis.
- The spectral density of the right-handed neutrinos?

Thanks for Your Attention!