#### A Stringy Mechanism for A Small Cosmological Constant

- X. Chen, Shiu, Sumitomo, Tye, arxiv:1112.3338, JHEP 1204 (2012) 026
- Sumitomo, Tye, arXiv:1204.5177
- Sumitomo, Tye, in preparation

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#### Motivation

### Dark Energy

#### Late time expansion



#### Awarded Nobel Prize in 2011!







Acceleration  

$$\frac{3\ddot{a}}{a} = -4\pi G(3p + \rho)$$
The universe is accelerating if  $\rho < -3p$   
or pressure-density ratio:  $w \equiv \frac{p}{\rho} < -\frac{1}{3}$   
Cosmological scale  
EOM (Friedmann eq.)  
 $H = \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G\rho}{3}}$  for flat background  
Observationally  $\Omega_{\Lambda} \sim 0.7$   $\longrightarrow$  DE domination  
 $\rho_{0} = \frac{3H_{0}^{2}}{8\pi G}\Omega_{\Lambda} \sim 10^{-122}M_{P}^{4}$ 

#### Two possibilities

• For **cosmological constant** WMAP+BAO+SN suggests  $w = -1.10 \pm 0.14$  (64% CL) for a flat universe  $\Omega_k = -\frac{k}{a_0^2 H_0^2} = 0$ 



#### For time-varying DE

WMAP+BAO+H0+D∆t+SN suggests

$$w = w_0 + w_a(1 - a(t))$$

$$w_0 = -0.93 \pm 0.13$$
  
 $w_a = -0.41^{+0.72}_{-0.71}$  (68% CL)

e.g. Stringy Quintessence models





### Stringy Landscape

There are many types of vacua in string theory, as a result of a variety of (Calabi-Yau) compactification.

 $ds_{10}^2 = ds_4^2 + ds_6^2$ 

A class of Calabi-Yau gives Swiss-cheese type of volume.

$$\mathcal{V}_6 = \gamma_1 (T_1 + \bar{T}_1) - \sum_{i=2} \gamma_i (T_i + \bar{T}_i),$$

E.g. workable models: [Denef, Douglas, Florea, 04]

•  $\mathbb{P}^4_{[1,1,1,6,9]}$ :  $h^{1,1} = 2, h^{2,1} = 272$ 

• 
$$\mathcal{F}_{11}$$
:  $h^{1,1} = 3$ ,  $h^{2,1} = 111$ 

• 
$$\mathcal{F}_{18}$$
:  $h^{1,1} = 5$ ,  $h^{2,1} = 89$ 

All can be stabilized (a la KKLT), but in various way.

Any implication of multiple vacua?

#### Keys in this talk

#### Product distribution



We apply this mechanism for cosmological constant (CC)

Before proceeding...

I have to say

we **don't** solve cosmological constant problem completely.

But here,

we introduce a tool to make cosmological constant smaller, maybe up to a certain value.

"A Stringy Mechanism for A Small Cosmological Constant"

# Moduli stabilization ~random approach~

#### Gaussian suppression on stability

Various vacua in string landscape

Aass matrix given randomly at extrema

How likely stable minima exist?

Positivity of mass matrix  $\longrightarrow$  Positivity of Hessian  $\partial_{\phi_i} \partial_{\phi_j} V \Big|_{\min}$ 

Real/complex symmetric matrix

• Gaussian Orthogonal Emsemble  
[Aazami, Easther, 05], [Dean, Majumdar, 08], [Borot, Eynard, Majumdar, Nadal, 10]  

$$Z = \int dM_{ij} e^{-\frac{1}{2}\text{tr }M^2}, M = M^T$$

$$\mathcal{P} = \exp\left[-\frac{\ln 3}{4}N^2 + \frac{\ln(2\sqrt{3}-3)}{2}N - \frac{1}{24}\ln N - 0.0172\right]_{10^{-5}}^{10^{-7}}$$

$$\frac{\ln 3}{4} \sim 0.275, \frac{\ln(2\sqrt{3}-3)}{2} \sim -0.384$$

#### Hierarchical setup

• Assuming hierarchy between diag. and off-diag. comp.

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Actual models are likely to have minima at AdS.

+ uplifting term toward dS vacua.

Hessian = A + B where A: diagonal positive definite with  $\sigma_A$ B: GOE with  $\sigma_B$ 

Still Gaussianly suppressed, but a chance for dS

$$\mathcal{P} = a \ e^{-bN^2 - cN}$$

#### [X. Chen, Shiu, YS, Tye, 11]

When applying a model in type IIA, quite tiny chance remains.



• Assuming more randomness in SUGRA at SUSY AdS

 $\mathcal{P} = e^{-bN^2}$ 

[Bachlechner, Marsh, McAllister, Wrase, 12]

### Moduli stabilization ~concrete models~

### Type IIB

Sources:  $H_3$ ,  $F_1$ ,  $F_3$ ,  $\tilde{F}_5$ , dilaton, localized sources Metric:  $ds_{10}^2 = e^{2A}ds_4^2 + e^{-2A}d\tilde{s}_6^2$ 

Calabi-Yau

Then EOM becomes [Giddings, Kachru, Polchinski, 02]

$$\tilde{\nabla}^{2}(e^{4A} - \alpha) = \frac{e^{2A}}{6 \operatorname{Im} \tau} |iG_{3} - *_{6}G_{3}|^{2} + e^{-6A} |\partial(e^{4A} - \alpha)|^{2} + (\text{local sources})$$
LHS=0 when integrating out positive contributions

 $e^{4A} = \alpha$ ,  $iG_3 = *_6 G_3$ : imaginary self-dual condition

where  $\alpha$  is a function in  $\tilde{F}_5$ ,  $G_3 = F_3 - \tau H_3$ ,  $\tau = C_0 + i e^{-\phi}$ 

#### No-scale structure

Take a scaling:  $\tilde{g}_{mn} \rightarrow \lambda \ \tilde{g}_{mn}$ 

 $e^{4A} = \alpha$ ,  $iG_3 = *_6 G_3$ : invariant

The other equations are also unchanged.

No-scale structure

superpotential  $W_0 = \int G_3 \wedge \Omega$  is independent of Kahler

4D effective potential with  $K = -3 \ln(T + \overline{T})$ ,  $W_0 = \text{const}$ 

$$V = e^{K/M_P^2} \left( K^{IJ} D_I W_0 \ \overline{D_J W_0} - \frac{3}{M_P^2} |W|^2 \right) = 0$$

Kahler directions remain flat.

### A bonus in type IIB

Hierarchical structure of mass matrix/potential helps to stabilize moduli at positive cosmological constant.

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[X. Chen, Shiu, YS, Tye, 12]

Moduli stabilization with positive cosmological constant

- Fluxes Complex structure & dilaton
- Non-perturbative effect,  $\alpha'$ -correction, localized branes

🛑 Kahler

[KKLT, 03], [Balasubramanian, Berglund, Conlon, Quevedo, 05], [Balasubramanian, Berglund, 04]...



### KKLT

Non-trivial potential for Kahler is generated by NP-corrections.

E.g. Gluino condensation on D7-branes

D7-branes wrapping the four cycle:  $W_{NP} = A e^{-\tilde{a} 8\pi^2/g_{D7}} = A e^{-aT}$ 

Together with the superpotential from fluxes:  $W = W_0 + W_{NP}$ 



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#### Large Volume Scenario

[Balasubramanian, Beglund, Conlon, Quevedo, 05]

 $\alpha'$ -corrections can break no-scale structure too.

 $\mathcal{O}(\alpha'^3)$ -correction in type II action [Becker, Becker, Haack, Louis, 02]

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$$K = -2\ln\left(\mathcal{V} + \frac{\xi}{2}\left(-i(\tau + \bar{\tau})\right)^{3/2}\right) - \ln(-i(\tau + \bar{\tau})) + \cdots$$
  
scales differently

E.g.  $\mathbb{P}^{4}_{[1,1,1,6,9]}$  model (assuming complex sector is stabilized)

$$\mathcal{V} = \frac{1}{9\sqrt{2}} \left( t_1^{3/2} - t_2^{3/2} \right), \qquad W = W_0 + A_1 e^{-a_1 T_1} + A_2 e^{-a_2 T_2}$$

Solution:  $W_0 \sim -20$ ,  $A_1 \sim 1$ ,  $t_1 \sim 10^6$ ,  $t_2 \sim 3^{10^{-24}}$  $V_{\min} \sim -10^{-25}$ : AdS vacua

 $\Rightarrow |W_0| \gg |W_{NP}|, \ \mathcal{V} \gg \xi$ : naturally realized

### Kahler uplifting

[Balasubramanian, Berglund, 04], [Westphal, 06], [Rummel, Westphal, 11], [de Alwis, Givens, 11]

Same setup as that of LVS

$$K = -2\ln\left(\mathcal{V} + \frac{\xi}{2}\right) + \cdots, \qquad \mathcal{V} = \gamma_1(T_1 + \overline{T}_1) - \sum_{i=2} \gamma_i(T_i + \overline{T}_i),$$
$$W = W_0 + A_1 e^{-a_1 T_1} + \sum_{i=2} A_i e^{-a_i T_i}$$
Interested in a region where this term plays a roll.

less large volume than LVS, but still  $|W_0| \gg |W_{NP}|$ ,  $\mathcal{V} \gg \xi$ 

E.g. single modulus [Rummel, Westphal, 11]

$$V \sim -\frac{W_0 a_1^3 A_1}{2 \gamma_1^2} \left( \frac{2C}{9 x_1^{9/2}} - \frac{e^{-x_1}}{x_1^2} \right), \qquad C = \frac{-27 W_0 \xi a_1^{3/2}}{64 \sqrt{2} \gamma_1 A_1}, x_1 = a_1 t_1$$

When  $W_0A_1 < 0$ , the  $C \propto \xi$  term contributes the uplifting.

### KKLT vs Kahler uplifting



Backreaction of  $\overline{D3}$ ? A singularity exists, but finite action Safe or not? [DeWolfe, Kachru, Mulligan, 08], [McGuirk, Shiu, YS, 09], [Bena, Giecold, Grana, Halmagyi, Massai, 09-12], [Dymarsky, 11],...



### Statistical approach

Further approximation  

$$\frac{V}{M_P^4} = -\frac{W_0 a_1^3 A_1}{2 \gamma_1} \left( \frac{C}{9 x_1^{9/2}} - \frac{e^{-x_1}}{x_1^2} \right), \qquad C = \frac{-27 W_0 \xi a_1^{\frac{3}{2}}}{64 \sqrt{2} \gamma_1^2 A_1}, \qquad x_1 = a_1 t_1$$
[Rummel, Westphal, 11]

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The stability constraint with positive CC at stationery points:



Neglecting the parameters  $a_1, \gamma_1, \xi$ , the model is simplified to be

$$\Lambda = w_1 w_2 (c - c_0), \qquad c_0 \le c = \frac{w_1}{w_2} < c_1 \qquad (w_1 = -W_0, w_2 = A_1, c \propto C)$$

#### Stringy Random Landscape

Starting with the simplified potential:

[YS, Tye, 12]

$$\Lambda = w_1 w_2 (c - c_0), \qquad c_0 \le c = \frac{w_1}{w_2} < c_1$$

Since  $W_0$ ,  $A_1$  are given model by model (various ways of stabilizing complex moduli), here we impose reasonable randomness on parameters.

 $w_1, w_2 \in [0, 1]$ , uniform distribution (for simplicity)

Probability distribution function

$$P(\Lambda) = N_0 \int dc \int dw_1 dw_2 \,\,\delta(w_1 w_2 (c - c_0) - \Lambda) \,\,\delta\left(\frac{w_1}{w_2} - c\right)$$

 $N_0$ : normalization constant

Divergence in product distribution

When 
$$z = w_1 w_2$$
,  

$$P(z) = \int dw_1 dw_2 \, \delta(w_1 w_2 - z) = \frac{1}{2} \ln \frac{1}{z} \qquad \text{log divergence at } z = 0$$
With constraint?  $\Lambda = w_1 w_2 (c - c_0), \qquad \underbrace{c_0 \leq c}_{\text{positivity}} = \frac{w_1}{w_2} \underbrace{< c_1}_{\text{stability}}$ 

$$\longrightarrow P(\Lambda) = \frac{c_1}{c_1 - c_0} \ln \frac{c_1 - c_0}{c_1 \Lambda} \qquad \text{still diverging!!}$$

Comparison to the full-potential (randomizing  $W_0$ ,  $A_1$  without approx.)



#### Zero-ness of parameters

We assumed the parameters  $W_0$ ,  $A_1$  passing through zero value, but is it true?

- E.g.  $T^6$  model:  $W_0 = \left(c_1 + \sum d_i U_i\right) \left(c_2 + \sum e_i U_i\right)S$ SUSY condition  $W_0 = 2 (c_1 + c_2 s) \frac{\prod_k (d_k - e_k s)}{\sum_i (d_i + e_i s) \prod_{j \neq i} (d_j - e_j s)}$   $s = \operatorname{Re}(S)$ easy to be zero
- Brane position dependence of  $A_1$  [Baumann, Dymarsky, Klebanov, Maldacena, McAllister, Murugan, 06]  $A_1 = \hat{A}_1(U_i) (f(X_i))^{1/n}, \quad f(X_i) = \prod X_i^{p_i} - \mu^q$

 $f(X_i) = 0$  when D3-brane hits D7-brane (divisor, at  $\mu$ )

known as Ganor zero

#### Comments on sum distribution

Sum distribution smooths out the divergence and moves the peak.

E.g. 
$$z = x_1^{n_1} + x_2^{n_2} + \dots + x_p^{n_p}$$

- Each has divergent peak:  $P(w_i = x_i^{n_i}) \propto w_i^{-1 + \frac{1}{n_i}}$
- Independent of each other, no correlations.

 $\Rightarrow$  But uncorrelated summation gives  $P(z) \propto z^{-1+\sum_{n_i}^1}$ .

When all  $n_i = 2$ , and  $x_i \in \text{normal distribution}$ ,

$$P(z) = \frac{e^{-p/2}z^{-1+p/2}}{2^{p/2}\Gamma(p/2)}$$

known as Chi-squared distribution



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### Bousso-Polchinski

4-form quantization

$$S = \int d^4x \sqrt{-g} \left( \frac{1}{M_P^2} R - \Lambda_{\text{bare}} - \frac{Z}{2 \times 4!} F_4^2 \right)$$
$$\longrightarrow \Lambda = \Lambda_{\text{bare}} + \frac{1}{2} \sum_{i=1}^{J} n_i^2 q_i^2$$

Assume randomness in Bousso-Polchinski;

 $n_i$ : random integer,  $0 \le q_i \le 1$ : uniform,

 $-100 \le \Lambda_{\text{bare}} \le 0$ : uniform

But... Moduli fields couple each term  

$$\Lambda \sim -W_0 A_1 \left( \frac{C}{9x_1^{9/2}} - \frac{e^{-x_1}}{x_1^2} \right)$$
correlation generated via stabilization





Multi-moduli analyses

## Multi-moduli stabilization

[Sumitomo, Tye, in preparation]

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Again, we work in the region:  $|W_0| \gg |W_{NP}|$ ,  $\mathcal{V} \gg \xi$ .

Assuming stabilization of complex structure moduli and dilaton at higher energy scale,

$$\begin{split} \frac{V}{M_P^4} &= -\frac{A_1 W_0 a_1^3}{2 \gamma_1} \left( \frac{2C}{9\tilde{\mathcal{V}}^3} - \frac{x_1 e^{-x_1}}{\tilde{\mathcal{V}}^2} - \sum_{i=2} \frac{B_i x_i e^{-x_i}}{\tilde{\mathcal{V}}^2} \right),\\ \tilde{\mathcal{V}} &= x_1^{3/2} - \sum_{i=2} \delta_i x_i^{3/2}, \ x_i = a_i t_i, \ C = \frac{-27 W_0 \xi a_1^{3/2}}{64\sqrt{2}\gamma_1 A_1}, \ B_i = \frac{A_i}{A_1}, \ \delta_i = \frac{\gamma_i a_i^{3/2}}{\gamma_1 a_1^{3/2}} \end{split}$$

• Now we have  $N_K \times N_K$  mass matrix.

All upper-left sub-determinants are positive (Sylvester's criteria).  $N_K$  extremal equations +  $N_K$  stability constraints

• Stability at positive CC requires  $B_i > 0$ .

Uplifting is controlled by the first term.



#### Cosmological moduli problem

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Reheating for BBN:  $T_r \ge \mathcal{O}(10) \text{ MeV}$   $T_r \sim \sqrt{M_P \Gamma_{\phi}}, \ \Gamma_{\phi} \sim \frac{m_{\phi}^3}{M_P}$ 

 $m_{\phi} \geq \mathcal{O}(10) \text{ TeV} \sim 10^{-15} M_P$ 

What happens in lightest (physical) moduli mass?



#### More peaked parameters

So far we assumed uniform distribution for  $W_0$ ,  $A_i$ . But realistic models have a number of complex moduli and others.

Different distributions for  $W_0, A_i$ 

Consider the effect of multiple independent parameters.

$$W_0 = -w_1 w_2 \cdots w_n, \qquad A_i = y_1^{(i)} y_2^{(i)} \cdots y_n^{(i)}$$
$$0 \le w_i \le 15^{\frac{1}{n}}, \ 0 \le y_j^{(i)} \le 1, \text{ all obey uniform distribution.}$$

Now,  

$$P(W_0) = \frac{1}{15(n-1)!} \left( \ln \frac{15}{|W_0|} \right)^{n-1}, \quad \prod_{i=1}^{n-1} P(A_i) = \frac{1}{(n-1)!} \left( \ln \frac{1}{A_i} \right)^{n-1}$$
See how CC is affected by "n"

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### Cosmological constant

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We cannot simply consider effect of the coefficient.

 $\frac{V}{M_{P}^{4}} = -\frac{A_{1}W_{0}a_{1}^{3}}{2\gamma_{1}} \left(\frac{2C}{9\tilde{\mathcal{V}}^{3}} - \frac{x_{1}e^{-x_{1}}}{\tilde{\mathcal{V}}^{2}} - \sum_{i=2}\frac{B_{i}x_{i}e^{-x_{i}}}{\tilde{\mathcal{V}}^{2}}\right)$ Dynamics also affects. The result:  $\langle \Lambda \rangle_{N_{\kappa}=1} = 4.7 \times 10^{-3} n^{0.080} e^{-1.40 n}$  $\langle \hat{\Lambda} \rangle$ Red:  $N_{K} = 1$  $\langle \Lambda \rangle_{N_K=2} = 3.7 \times 10^{-3} n^{0.97} e^{-1.49 n}$ 0.001 Blue:  $N_K = 2$ 10-5  $\langle \Lambda \rangle_{N_K=3} = 3.4 \times 10^{-3} n^{1.5} e^{-1.55 n}$ Green:  $N_{K} = 3$ 10-7 More than the effect of 10-9 the coefficient!  $\langle A_1 W_0 \rangle \sim 15 \ e^{-1.39 \ n}$ 5 10

#### Moduli mass

We worry about the cosmological moduli problem.

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 $\langle m_{\min}^2 \rangle_{N_K=1} = 0.18 \, n^{0.14} e^{-1.40 \, n}$  $\langle m_{\min}^2 \rangle_{N_K=2} = 0.061 \, n^{0.73} e^{-1.56 \, n}$  $\langle m_{\min}^2 \rangle_{N_K=3} = 0.039 \, n^{1.2} e^{-1.66 \, n}$ Compare with CC

 $\langle \Lambda \rangle \propto e^{-1.40 n}, e^{-1.49 n}, e^{-1.55 n}$ 

Suppression in mass is getting larger as increasing  $N_K$ .

also suggests

### Estimation

Using the estimated functions, we get

$N_K(=h^{1,1})$	1	2	3
$\langle \Lambda \rangle \sim 10^{-122} M_P^4$	<i>n</i> ~ 197	<i>n</i> ~ 188	<i>n</i> ~ 182
$\langle m^2 \rangle \sim 10^{-30} M_P^2$	$n \sim 48$	$n \sim 44$	$n \sim 42$

*n*: number of product in  $W_0$ ,  $A_i$ 

Rather considerable number, e.g.

•  $\mathbb{P}^4_{[1,1,1,6,9]}$ :  $h^{1,1} = 2$ ,  $h^{2,1} = 272$  •  $\mathcal{F}_{11}$ :  $h^{1,1} = 3$ ,  $h^{2,1} = 111$ 

and the other moduli (e.g. brane position, open string) come in a complicated way, like

•  $A_1 = \hat{A}_1(U_i) (f(X_i))^{1/n}, \ f(X_i) = \prod X_i^{p_i} - \mu^q$ 

While, without help of product distribution in  $W_0$ ,  $A_i$ 

 $N_K \sim 10100$  for  $\langle \Lambda \rangle \sim 10^{-122} M_P^4$ ,  $N_K \sim 1350$  for  $\langle m^2 \rangle \sim 10^{-30} M_P^2$ 

#### Mass matrix

Physical mass matrix is a linear combination of  $\partial_{x_i} \partial_{x_j} V|_{\min}$ .

Assuming uniformly distributed  $-15 \le W_0 \le 0$ ,  $0 \le A_i \le 1$ ,

$$\left< \left| \partial_{x_i} \partial_{x_j} V \right|_{\min} \right| \right> \sim 10^{-3} \times \begin{pmatrix} 7 & 4 & \cdots & \cdots & 4 \\ 4 & 60 & 1 & \cdots & 1 \\ \vdots & 1 & \ddots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 1 \\ 4 & 1 & \cdots & 1 & 60 \end{pmatrix}$$
 some hierarchical structures

Though off-diagonal comp. are relatively suppressed, eigenvalue repulsion gets more serious when increasing  $N_K$ .

e.g. 2 × 2 matrix: 
$$\begin{pmatrix} a & b \\ b & c \end{pmatrix} \implies \lambda_{\pm} = \frac{1}{2} \left( a + c \pm \sqrt{(a - c)^2 + 4b^2} \right)$$

The lowest mass eigenvalue is generically suppressed more than CC.

Summary & Discussion

### Summary & Discussion

• Stringy Random Landscape

We may expect that stringy motivated models have the following properties:

- Product of parameters
- Correlation of each term by dynamics

 $\Rightarrow$  Both works for smaller CC.

- A number of Kahler moduli Correlation makes CC smaller. But the effect is modest.
- A number of complex moduli and other moduli
   Those are likely to produce more peakiness in parameters
   Interesting to see detailed effect in concrete models

#### Summary & Discussion

A potential problem

Lightest moduli mass is suppressed simultaneously.

Cosmological moduli problem before reaching  $\Lambda \sim 10^{-122} M_P^4$ .

Other than "product" and "correlation" effect, "eigenvalue repulsion" also makes the value smaller.

This is presumably a generic problem when taking statistical approach without fine-tuning.

Once finding a way out, the stringy mechanism naturally explain why CC is so small.

Thermal inflation, coupling suppression to SM, or some other corrections may help?