Lattice studies of the conformal window

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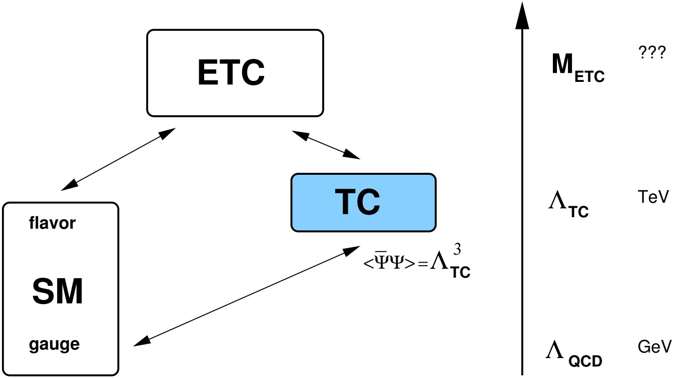
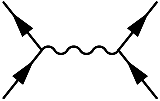
Outline


ETC
TC
flavor
SM
gauge
$<\bar{\Psi}\Psi> = \Lambda_{TC}^3$
M_{ETC}
???
Λ_{TC}
TeV
Λ_{QCD}
GeV

- ETC: mass term for the SM fermions, FCNC



$$\longrightarrow \qquad \Delta\mathcal{L} \propto \frac{1}{M_{\rm ETC}^2}\langle\bar\Psi\Psi\rangle_{\rm ETC}\;\bar\psi\psi, \;\; \frac{1}{M_{\rm ETC}^2}\bar\psi\psi\;\bar\psi\psi$$

- running of $\langle\bar\Psi\Psi\rangle$: **(near) conformal IR behaviour**

$$\boxed{\langle\bar\Psi\Psi\rangle_{\rm ETC} = \langle\bar\Psi\Psi\rangle_{\rm TC}\;\exp\left(\int_{\Lambda_{\rm TC}}^{M_{\rm ETC}}\frac{d\mu}{\mu}\,\gamma(\mu)\right)}$$

[holdom, lane, appelquist, luty, sannino, chivukula,...]

[Banks & Zaks 82]

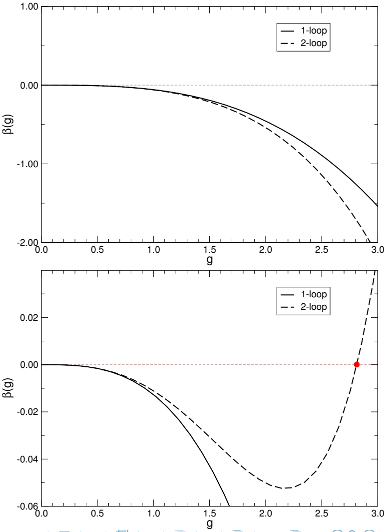
$$\beta(\bar{g}) = \mu \frac{d}{d\mu}\bar{g}(\mu) = -\beta_0 \bar{g}^3 - \beta_1 \bar{g}^5 + O(\bar{g}^7)$$

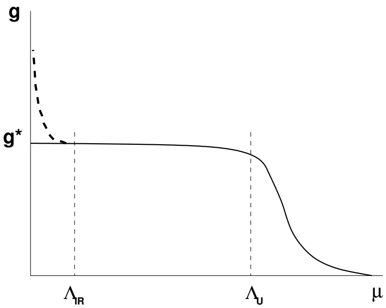
$$\begin{aligned} \beta_0 > 0 &\implies \text{asymptotic freedom, } n_f < n_f^{\mathrm{af}} \\ \beta(\bar{g}) < 0 &\implies \text{confining theories} \\ \beta(\bar{g}^*) = 0 &\implies \textbf{IR fixed point} \end{aligned}$$

conformal window: IR zero of the β function

$$\boxed{n_f^c < n_f < n_f^{\mathrm{af}}}$$

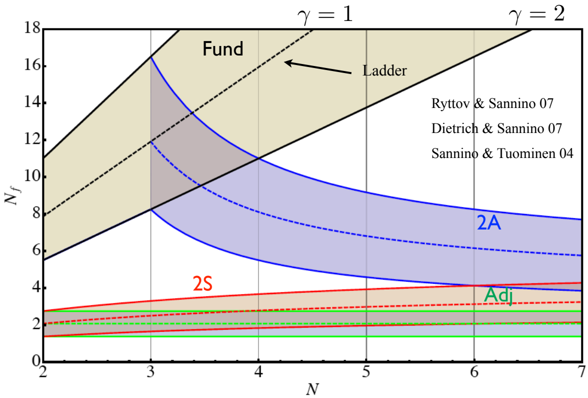
	$(4\pi)^2\beta_0$	$(4\pi)^4\beta_1$
SU(3), $n_f = 3$ fund	9	64
SU(3), $n_f = 12$ fund	3	-50
SU(2), $n_f = 2$ adj	2	-40





- asymptotic freedom at high-energy scale Λ_{UV}
- scale invariance at large distances $\Longrightarrow$ no massive spectrum
- large-distance dynamics: anomalous dimensions at the IRFP
- nonperturbative IRFP: strong coupling, large anomalous dimensions
- scale invariance explicitly broken by a mass term/finite volume

SU(N) Phase Diagram



Action with a cutoff μ: $S(g_i) = \sum_i g_i \mu^{4-d_i} O_i$

RG transformation: $\mu \mapsto \mu' = \mu/b$ – **at constant physics**

$$g_i' = R_i(g)$$

$\hookrightarrow$ flow in the (infinite-dimensional) parameter space.

Fixed point:

$$\boxed{g_i^* = R_i(g^*)}$$

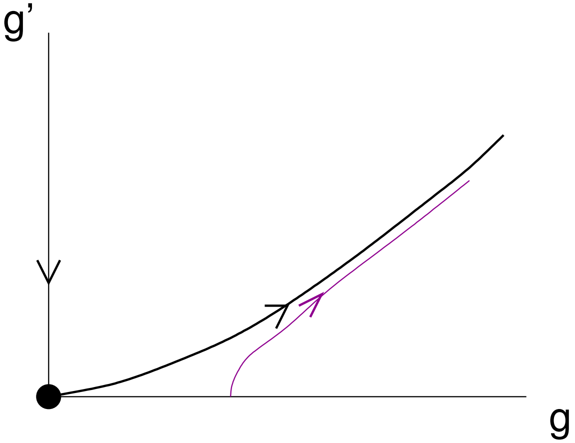
In the vicinity of the fixed point:

$$\begin{aligned} \delta g_i' &= \left. \frac{\partial R_i}{\partial g_k} \right|_* \delta g_k \\ v_i' &= b^{y_i} v_i \end{aligned}$$

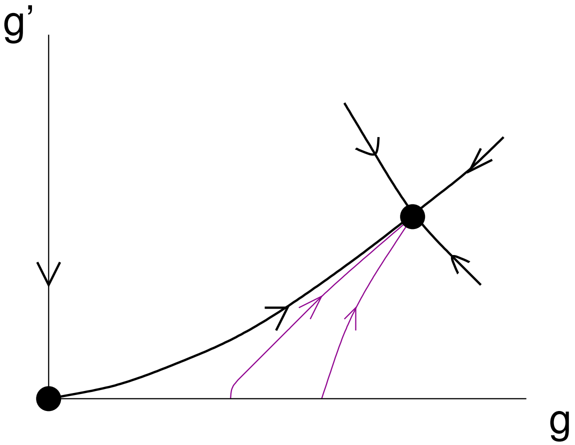
y_i **characterize the fixed point**

$y_i > 0$ relevant parameters

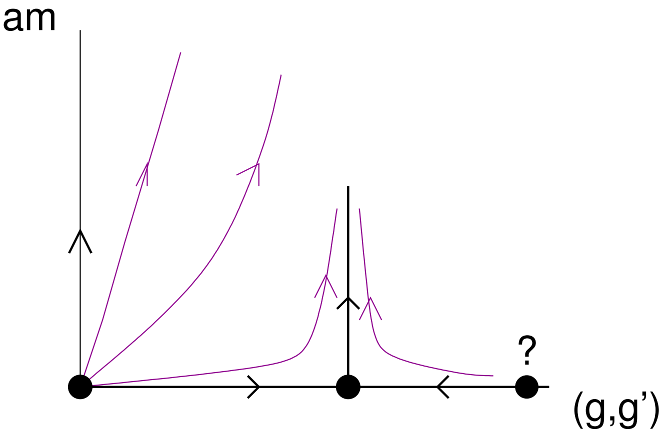
YM theory - g lattice bare coupling, g' space of irrelevant couplings



flow in the $m = 0$ plane - assuming only the mass is relevant in the IR



flow in the presence of a mass deformation



[luty 09, degrand09, **ldd&zwicky 10**]

$$\begin{aligned} C_H(t;g,m,\mu) &= \int d^3x \, \langle H(x,t) H(0)^\dagger \rangle \\ &\sim F_H^2 \, e^{-M_H t} \end{aligned}$$

RG transformation:

$$\begin{aligned} Z_H(\mu)^2 C_H(t;g,m,\mu) &= Z_H(\mu')^2 C_H(t;g',m',\mu') \\ C_H(t;g,m,\mu) &= b^{-2\gamma_H} C_H(t; b^{y_g} g, b^{y_m} m, \mu') \end{aligned}$$

naive dimensional analysis:

$$C_H(t; b^{y_g} g, b^{y_m} m, \mu') = b^{-2d_H} C_H(tb^{-1}; b^{y_g} g, b^{y_m} m, \mu)$$

choosing b such that $b^{y_m} m = 1$:

$$C_H(t;g,m,\mu) = m^{2\Delta_H/y_m} \mathcal{F}(tm^{1/y_m}) \, , \qquad \Delta_H = d_H + \gamma_H$$

$$\boxed{M_H \sim m^{1/(1+\gamma_*)}}$$

[ldd&zwicky 10]

$$T_{\phi_1 O \phi_2}(g, m, \mu) = \langle \phi_1 | O | \phi_2 \rangle |_{g,m,\mu}$$

RG transformation:

$$\begin{aligned} T_{\phi_1 O \phi_2}(g, m, \mu) &= b^{-\gamma_O} T_{\phi_1 O \phi_2}(g', m', \mu') \\ &= b^{-\gamma_O} T_{\phi_1 O \phi_2}(b^{y_g} g, b^{y_m} m, \mu/b) \end{aligned}$$

naive dimensional analysis:

$$T_{\phi_1 O \phi_2}(b^{y_g} g, b^{y_m} m, \mu/b) = b^{-d_O + d_1 + d_2} T_{\phi_1 O \phi_2}(b^{y_m} m, \mu)$$

choosing b such that $b^{y_m} m = 1$

$$T_{\phi_1 O \phi_2}(g, m, \mu) \sim m^{(\Delta_O + d_1 + d_2)/y_m}$$

trace anomaly

$$\theta_\alpha^\alpha = \frac{\beta}{2} G^2 + n_f(1+\gamma_m) m\bar{q}q$$

matrix elements of the energy-momentum tensor

$$\langle H(p)|\theta_{\alpha\beta}|H(p)\rangle = 2p_\alpha p_\beta$$

hence we can write the trace at the fixed point

$$2M_H^2 = n_f(1+\gamma_m)\langle H(p)|m\bar{q}q|H(p)\rangle$$

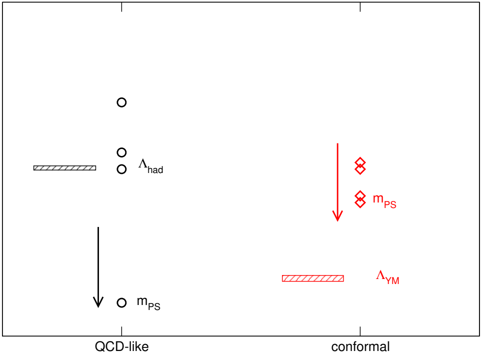
using the scaling law for the matrix element

$$\boxed{M_H \sim m^{1/y_m}}$$

In the presence of a mass deformation (and infinite volume)

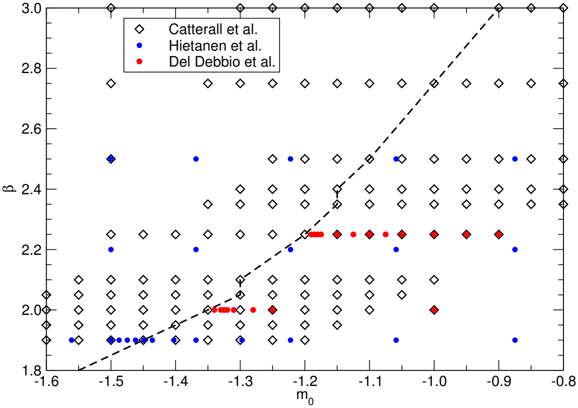
- no parametric separation between the PS and the other channels
- all masses/decay constants scale like m^{1/y_m} (but $M \sim F(0)/L$!!)

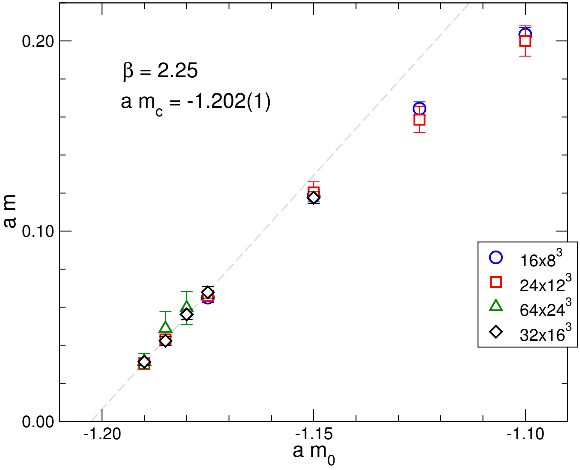
$\hookrightarrow$ very different spectrum from a QCD-like theory [Miransky 94]



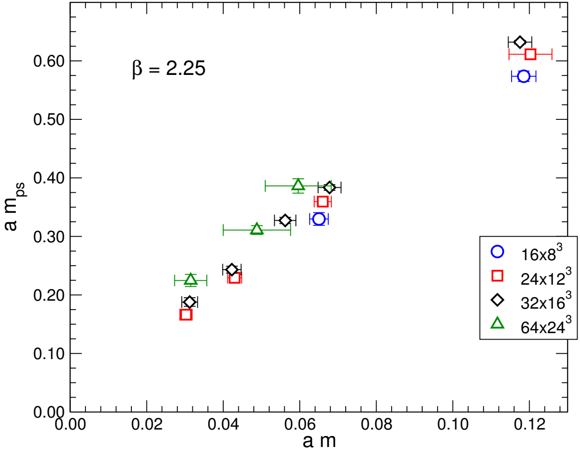
[catterall et al 07, **ldd et al 08/10**, rummukainen et al 08]

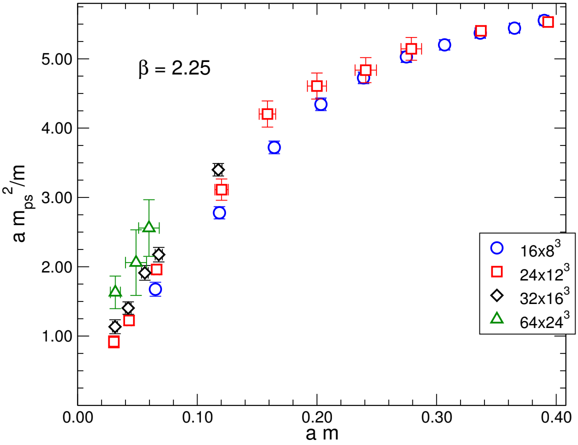
MC simulations with $n_f = 2$ Wilson fermions

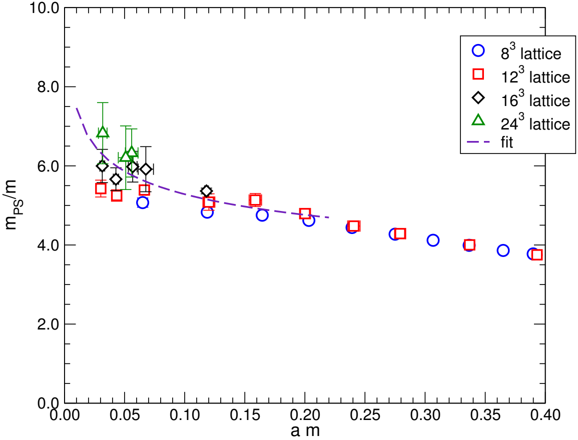




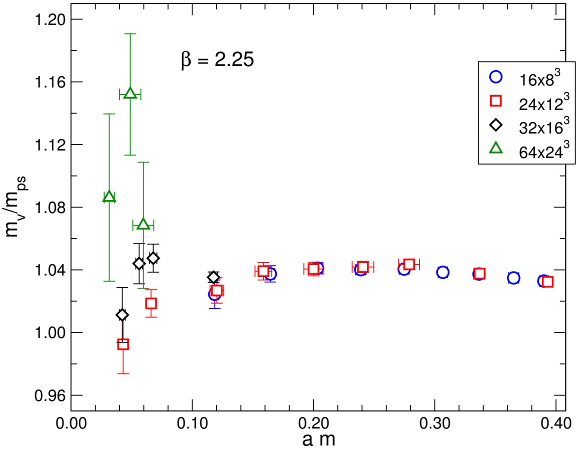
β = 2.25
a m_c = -1.202(1)
a m
a m_0
0.00
0.10
0.20
-1.20
-1.15
-1.10
16x8^3
24x12^3
64x24^3
32x16^3


β = 2.25
0.60
0.50
0.40
0.30
0.20
0.10
0.00
a m_ps
0.00
0.02
0.04
0.06
0.08
0.10
0.12
a m
16x8^3
24x12^3
32x16^3
64x24^3


β = 2.25
$a\, m_{ps}^2/m$
$a\, m$
0.00
0.10
0.20
0.30
0.40
1.00
2.00
3.00
4.00
5.00
16x8³
24x12³
32x16³
64x24³

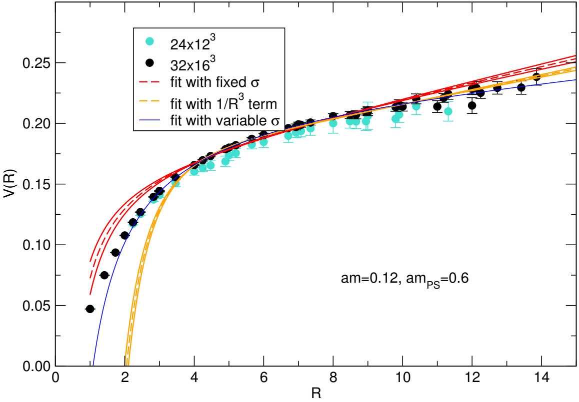


$$\log m_{\mathrm{PS}} = \frac{1}{1+\gamma} \log m + C = \mathbf{0.85} \log m + C$$

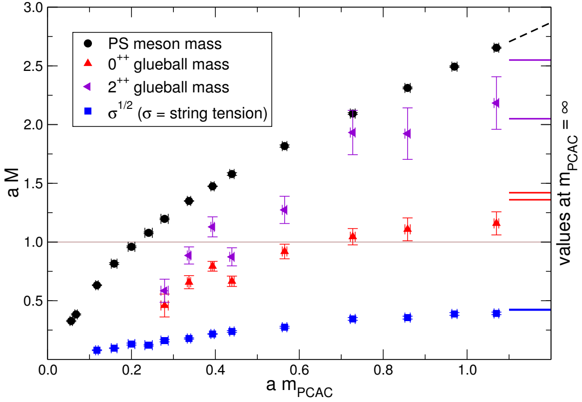

1.20
1.16
1.12
1.08
1.04
1.00
0.96
m_V/m_{ps}
0.00
0.10
0.20
0.30
0.40
a m
β = 2.25
16x8³
24x12³
32x16³
64x24³

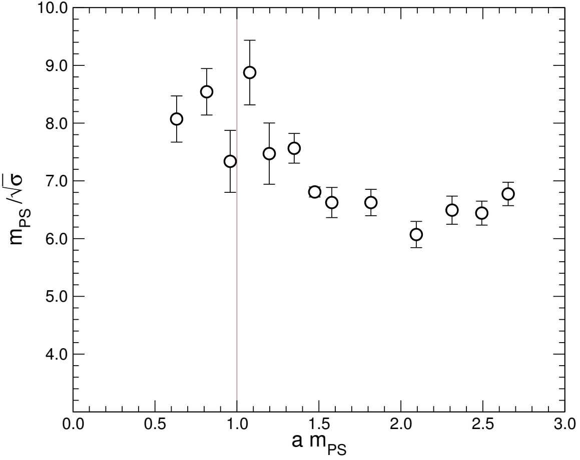
SU(2) adjoint

Gluonic sector - 1



SU(2) adjoint
Gluonic sector - 2




10.0
9.0
8.0
7.0
6.0
5.0
4.0
$m_{PS}/\sqrt{\sigma}$
0.0
0.5
1.0
1.5
2.0
2.5
3.0
a m_{PS}

RG transformation

$$C_H(t;\hat{m},L,\mu) = b^{-2\gamma_H} C_H(t;\hat{m}',L,\mu')\,,$$

naive dimensional analysis

$$C_H(t;\hat{m},L,\mu) = b^{-2(d_H+\gamma_H)} C_H(b^{-1}t; b^{y_m}\hat{m}, b^{-1}L,\mu)\,.$$

choosing b such that $b^{-1}L = L_0$

$$C_H(t;\hat{m},L,\mu) = \left(\frac{L}{L_0}\right)^{-2\Delta_H} C_H\left(\frac{t}{L/L_0}; x\frac{1}{\mu L_0^{y_m}}, L_0, \mu\right)\,,$$

where the scaling variable is $x = L^{y_m} m$.

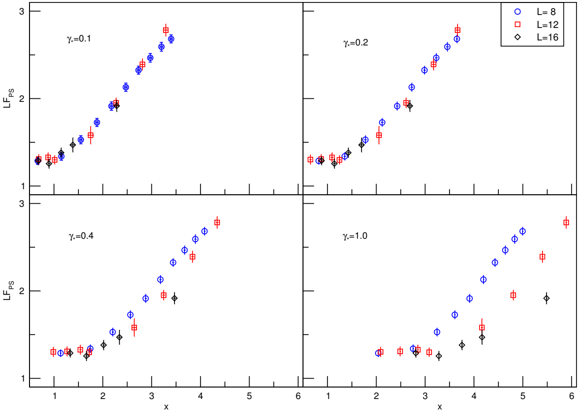
$$M_H = L^{-1} f(x)\,,$$

$f(x) \to 0$ when $x \to 0$
m scaling in the thermodynamic limit

$$f(x) \sim x^{1/y_m}\,, \quad \text{as } x \to \infty\,.$$

SU(2) adjoint

Finite-size scaling



RG analysis

$$\langle \bar{q}q \rangle \sim m^{\eta_{\bar{q}q}}$$

where

$$\eta_{\bar{q}q} = \frac{\Delta_{\bar{q}q}}{y_m} = \frac{3-\gamma_*}{1+\gamma_*}$$

relation to the eigenvalue density

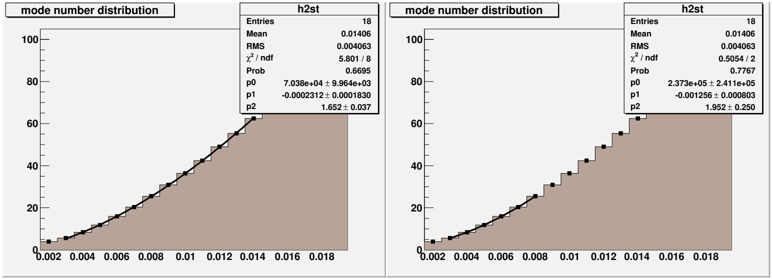
$$\langle \bar{q}q \rangle = -2 \int_0^{\mu/m} dx \, \frac{\rho(mx)}{1+x^2} + \mathcal{A}(m)\,,$$

$$\langle \bar{q}q \rangle \overset{m\to 0}{\sim} m^{\eta_{\bar{q}q}} \quad \Leftrightarrow \quad \rho(\lambda) \overset{\lambda\to 0}{\sim} \lambda^{\eta_{\bar{q}q}}\,.$$

This in turn implies:

$$\eta_{\bar{q}q}\big|_{\mathrm{QCD-like}} = 0\,, \qquad \eta_{\bar{q}q}\big|_{\mathrm{mCGT}} > 0\,,$$

$$\nu(M,m) = \int_{-\Lambda}^{+\Lambda} d\lambda\, \rho(\lambda)\,, \qquad \Lambda = \sqrt{M^2 - m^2}$$
$$\nu(\Lambda,0) = C(\Lambda - g)^{\eta_{\bar{q}q}+1}$$



Schrödinger functional [ALPHA collaboration]

$$Z[\eta] = e^{-\Gamma[\eta]} = \int_{L\times L^3} \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\, \exp(-S[U,\psi,\bar{\psi}])$$

Dirichlet boundary conditions at $t=0, L$, dependent on η

$$\boxed{\frac{k}{\bar{g}^2(L)} = \left.\frac{\partial\Gamma}{\partial\eta}\right|_{\eta=0}, \qquad \bar{g}^2 = g_0^2 + O(g_0^4)}$$

Lattice step scaling function

$$\Sigma(u, a/L) = \left.\bar{g}^2(bL)\right|_{\bar{g}^2(L)=u, m=0}$$

Step scaling function

$$\sigma(u) = \lim_{a\to 0} \Sigma(u, a/L)$$

fixed point: $$\boxed{\sigma(u^*) = u^*}$$

Definition of the renormalized mass

$$\boxed{\partial_\mu (A_R)_\mu = 2\bar{m} P_R}$$

$$\begin{aligned} (A_R)_\mu(x) &= Z_A \bar{\psi}(x)\gamma_\mu\gamma_5\psi(x) \\ (P_R)(x) &= Z_P \bar{\psi}(x)\gamma_5\psi(x) \end{aligned}$$

Lattice step scaling function

$$\begin{aligned} Z_P(g_0, L/a) &= c\,\frac{\sqrt{3f_1}}{f_P(L/2)} \\ \Sigma_P(u, a/L) &= \frac{Z_P(g_0, bL/a)}{Z_P(g_0, L/a)}, \quad \bar{g}^2(L) = u \end{aligned}$$

Step scaling function

$$\sigma_P(u) = \lim_{a\to 0} \Sigma_P(u, a/L)$$

- existence of the IRFP is scheme-independent

$$\beta(\bar{g}) = \mu \frac{d}{d\mu} \bar{g}(\mu)$$

change of scheme: $\bar{g}' = \Phi(\bar{g})$, $\Phi'(\bar{g}) > 0$

$$\boxed{\beta'(\bar{g}') = \frac{\partial}{\partial \bar{g}} \Phi(\bar{g}) \beta(\bar{g})}$$

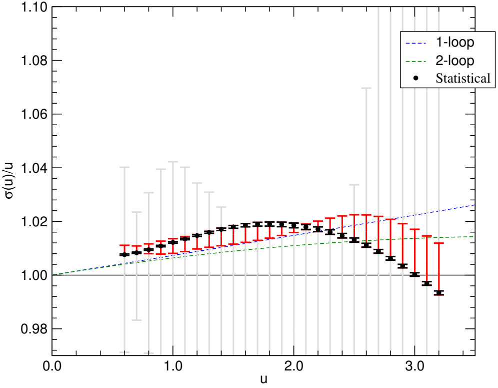
universality of the first two coefficients

- β-function away from IRFP is not universal!

- $\bar{m}' = \bar{m} F(\bar{g})$

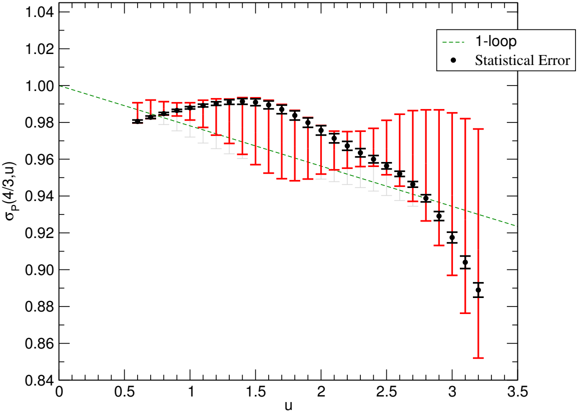
$$\boxed{\gamma' = \gamma + \beta \frac{\partial}{\partial \bar{g}} \log F}$$

$n_f = 2$ Wilson, Schrödinger functional coupling [Hietanen et al 08, Bursa et al 09]



$$\Delta(L_1, L_2) = 1/g^2(L_2) - 1/g^2(L_1)$$

$n_f = 2$ Wilson, Schrödinger functional mass [Bursa et al 09]



$$\log|\sigma_P(u,b)| = -\gamma \log b \quad \Longrightarrow \quad 0.05 < \gamma < 0.56$$

- tools have been developed and tested - keep looking for better tools
- preliminary results need to be put on solid ground
  - benchmark codes/analysis
  - control systematics: small masses & large volumes
  - improved actions
- all methods have systematics
  more confident when different approaches yield consistent results
- anomalous dimensions are small
- SU(2) adjoint evidence for conformality
- deformations away from conformality
  - $n_f < n_f^c$
  - mass deformation
  - 4fermi interactions
- lattice results to be taken into account for model building
- talk to phenomenologists!