A particle physicist's perspective on Topological Insulators.

Andreas Karch University of Washington

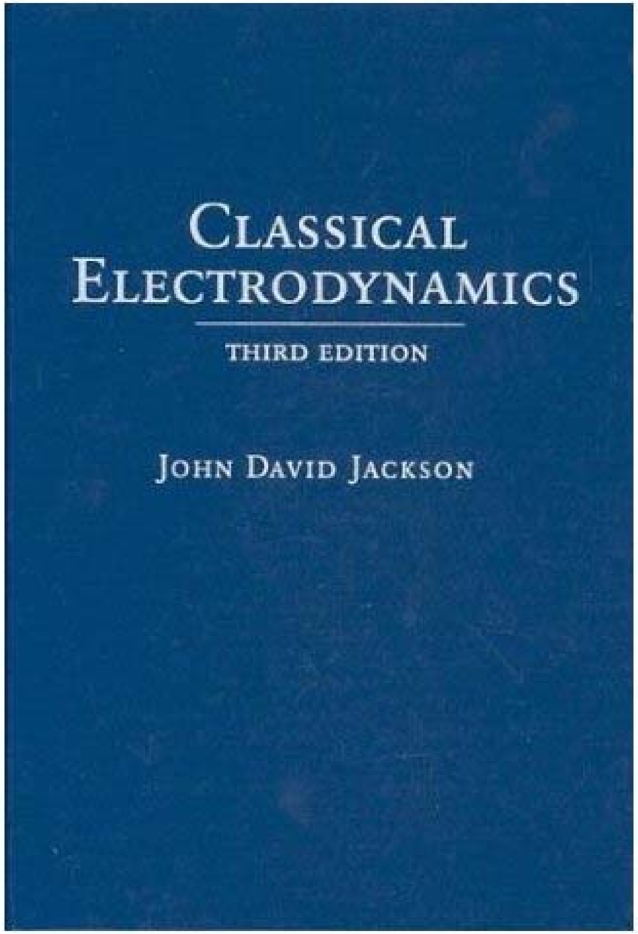
"Electric-Magnetic Duality and Topological Insulators",
by AK, Phys.Rev.Lett. 103:171601

"Fractional topological insulators in three dimensions",
with J.Maciejko, X.-L. Qi, S. Zhang, Phys.Rev.Lett. 105:246809

and more recent work with Hoyos, Jensen, Maciejko and Takayanagi

A particle physicist's perspective on Topological Insulators.

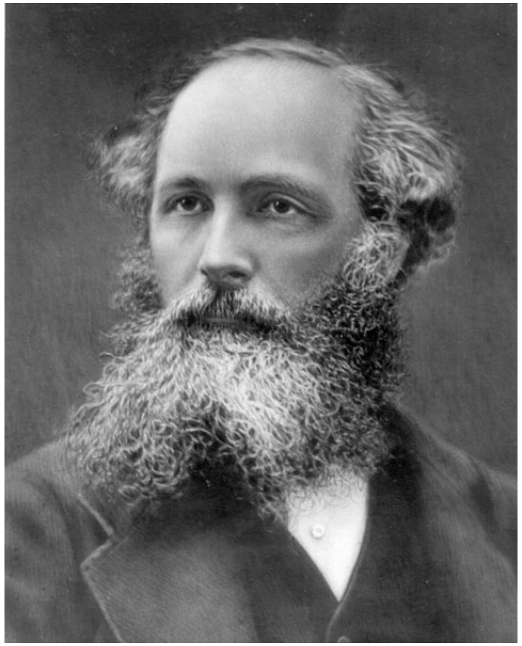
Description of Insulators:



$$
\begin{aligned}
\vec{\nabla} \cdot \vec{D} &= \rho_e \\
\vec{\nabla} \cdot \vec{B} &= \rho_m = 0 \\
\vec{\nabla} \times \vec{H} &= \vec{j}_e \\
\vec{\nabla} \times \vec{E} &= \vec{j}_m = 0
\end{aligned}
$$

$$
\vec{D} = \epsilon \vec{E} \qquad \vec{H} = \frac{\vec{B}}{\mu}
$$

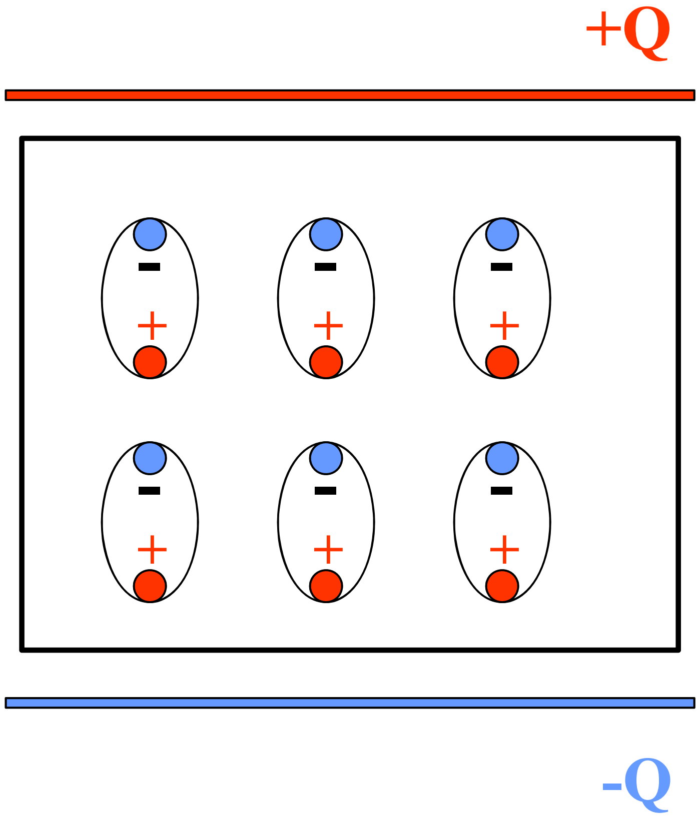
Where does this come from?



$$\vec{D} = \epsilon_0 \vec{E} \qquad \vec{H} = \frac{\vec{B}}{\mu_0}$$

Why replace ε_0 with ε?

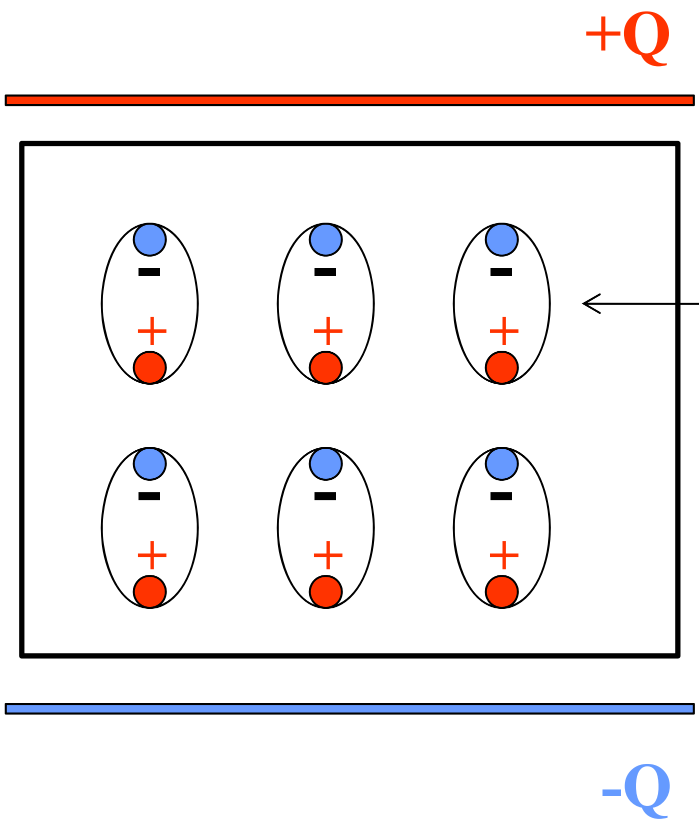
ε from ε_0: microscopic picture



free (macroscopic) charge

(appears explicitly as source in Maxwell's equation)

ε from ε_0



"bound" charges

implies polarization:

$$\vec{\nabla} \cdot \vec{P} = \rho_{bound}(\vec{E})$$

ε from ε_0

For small electric field expand:

$$\vec{P} = \vec{P}_0 + \chi\epsilon_0\vec{E} + \mathcal{O}(E^2)$$

ε from ε_0

For small electric field expand:

$$\vec{P} = \cancel{\vec{P}_0} + \chi\epsilon_0\vec{E} + \mathcal{O}(E^2)$$

typically vanishes. no built in polarization
non-zero P_o: ferroelectric
[[P_o=0 for unbroken rotation invariance]]

ε from ε_0

For small electric field expand:

$$\vec{P} = \chi \epsilon_0 \vec{E} + \mathcal{O}(E^2)$$

negligible for "small" electric fields
(small compared to intrinsic scales)

ε from ε_0

For small electric field expand:

$$\vec{P} = \chi \epsilon_0 \vec{E} + \mathcal{O}(E^2)$$

negligible for “small” electric fields (small compared to intrinsic scales)

“Low energy effective field theory”

ε from ε_0

For small electric field expand:

$$\vec{P} = \chi \epsilon_0 \vec{E} + \mathcal{O}(E^2)$$

negligible for “small” electric fields (small compared to intrinsic scales)

“To a physicist, everything is a harmonic oscillator”

ε from ε_0

For small electric field expand:

$$\vec{P} = \chi \epsilon_0 \vec{E} + \mathcal{O}(E^2)$$

electric susceptibility
parametrize complete response by a single number;
in principle computable

Reorganize:

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}(\vec{E}) = \epsilon \vec{E}$$

with $$\epsilon = \epsilon_0 (1 + \chi)$$

Similar for the magnetic field (non-ferromagnetic insulator):

$$\vec{H} = \frac{1}{\mu_0} \vec{B} - \vec{M}(\vec{B}) = \frac{1}{\mu} \vec{B}$$

Reorganize:

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Similar for the magnetic field (non-ferromagnetic insulator):

$$\vec{H} = \frac{1}{\mu_0} \vec{B} - \vec{M}(\vec{B}) = \frac{1}{\mu} \vec{B}$$

Question: **Why not include in D a term linear in B?**
Why not include in H a term linear in E?

In particle physics language:

2-derivative effective action:

$$S_{eff} = \int d^3x\, dt \left(\frac{\epsilon \vec{E}^2}{2} - \frac{\vec{B}^2}{2\mu} \right)$$

$$D = \frac{\partial \mathcal{L}}{\partial E}, \qquad H = -\frac{\partial \mathcal{L}}{\partial B}$$

Question:

Why not include in D a term linear in B?
Why not include in H a term linear in E?
That is, why not include a θ-term?

$$\theta\, F_{\mu\nu} F_{\sigma\tau} \epsilon^{\mu\nu\sigma\tau} \sim \theta\, \vec{E} \cdot \vec{B}$$

Time reversal symmetry.

Time Reversal:

$$t \longrightarrow -t$$

charge density: ρ **even**

current density:

$$\vec{j} = \rho \vec{v}$$ **odd**

Time reversal.

Time Reversal:

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\frac{\vec{x}_1 - \vec{x}_2}{|\vec{x}_1 - \vec{x}_2|^3}$$

even

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times (\vec{x}_1 - \vec{x}_2)}{|\vec{x}_1 - \vec{x}_2|^3}$$

odd

Time Reversal Invariant Insulators

Generalized Constitutive Relation:

even **even** **odd**

$$\vec{D} = \epsilon\vec{E} - \frac{\alpha\theta}{\pi}\vec{B}$$

$$\vec{H} = \frac{\vec{B}}{\mu} + \frac{\alpha\theta}{\pi}\vec{E}$$

only time reversal invariant for

$$\theta = 0$$

A particle physicist's perspective on Topological Insulators.

Time Reversal Invariant Insulators

Generalized Constitutive Relation:

$$\vec{D} = \epsilon\vec{E} - \frac{\epsilon_0\alpha\theta}{\pi}c\vec{B}$$

$$\vec{H} = \frac{\vec{B}}{\mu} + \frac{\alpha\theta}{\pi}\frac{\vec{E}}{c\mu_0}$$

Quantization of Magnetic Flux ensures T invariance at

$$\theta = 0$$

or

$$\theta = \pi$$

(Qi, Hughes, Zhang)

Topological Insulators

Need 3 parameters to describe media: ε, μ, and:

$\theta = 0$	Topologically trivial insulators
$\theta = \pi$	Topologically non-trivial insulators

Topological Insulators

Need 3 parameters to describe media: ε, μ, and:

$\theta = 0$	Topologically trivial insulators	**vacuum, air, NaCl,MgF_2 …..**
$\theta = \pi$	Topologically non-trivial insulators	

Topological Insulators

Need 3 parameters to describe media: ε, μ, and:

$\theta = 0$	Topologically trivial insulators	**vacuum, air, NaCl, MgF_2 …..**
$\theta = \pi$	Topologically non-trivial insulators	**$Bi_{1-x}Sb_x$, Bi_2Se_3, Bi_2Te_3, Sb_2Te_3, TlBi(Sb)Te(Se, S)$_2$, LaPtBi, ...**

θ uniquely determined by bandstructure.

Flux Quantization:

Modified Constitutive Relation from: $$S_\theta = \frac{\theta}{2\pi}\frac{e^2}{2\pi}\int d^3x\, dt\, \vec{E}\cdot\vec{B}$$

Study theory on Euclidean 4-manifold. iS_θ/θ integer quantized (due to magnetic flux quantization).

In Path Integrals only $\exp\left(iS_\theta[\vec{E},\vec{B}]\right)$ matters

θ is 2 π periodic!

Consequence of Flux Quantization:

This is the abelian version of "θ-vacuaa" of QCD.

(Callan, Dashen, Gross 1976, Jackiw&Rebbi, 1976))

$\theta = -\pi$ equivalent to $\theta = \pi$

$\theta = \pi$ is time reversal invariant.

Relation to Band Structure? Topology?

Topology

= **classifying different geometries without introducing an explicit notion of distance.**

(movie from wikipedia)

Topology

= **classifying different geometries without introducing an explicit notion of distance.**

{A,R} {B} {C,G,I,J,L,M,N,S,U,V,W,Z}
{D,O} {E,F,T,Y} {H} {P,Q} {K,X}

Equivalence classes of the English alphabet
in uppercase sans-serif font (Calibri)

Topology

= classifying different geometries without introducing an explicit notion of distance.

2,1 **0,2** **2,0**

{A,R} {B} {C,G,I,J,L,M,N,S,U,V,W,Z}

{D,O} {E,F,T,Y} {H} {P,Q} {K,X}

0,1 **3,0** **4,0** **1,1** **4,0**

Different topologies typically classified by **integers** (= Topological Quantum Numbers)

For example: number of boundaries.
number of loops

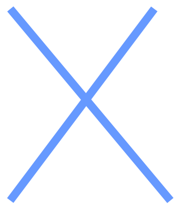
Topology

= **classifying different geometries without introducing an explicit notion of distance.**

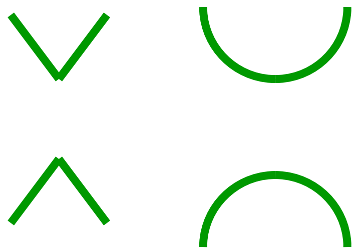
{A,R} {B} {C,G,I,J,L,M,N,S,U,V,W,Z}

{D,O} {E,F,T,Y} {H} {P,Q} {K,X}

topologically H



topologically X (singular limit)



topologically 2 copies of V

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Bandstructure and Topology

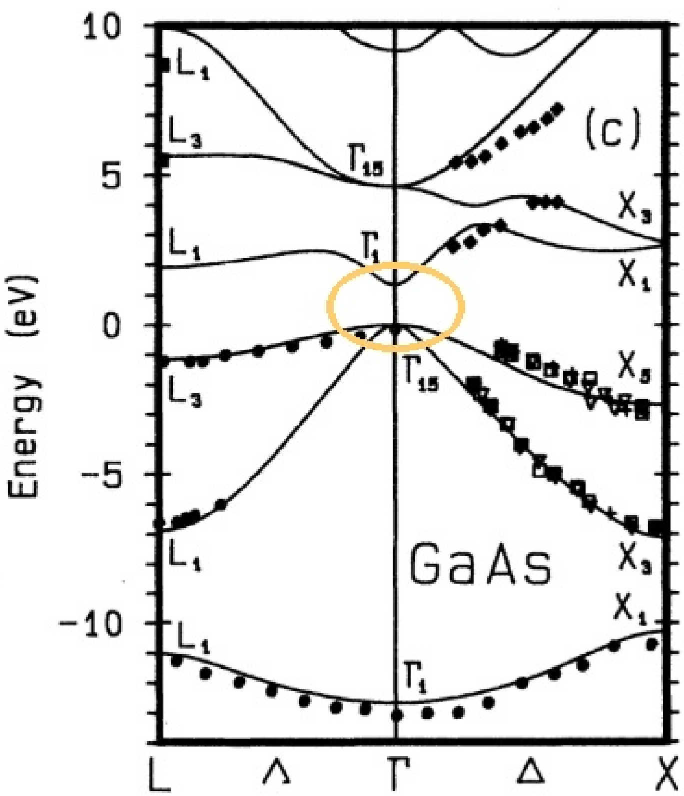
Beautiful Math. But why physics?

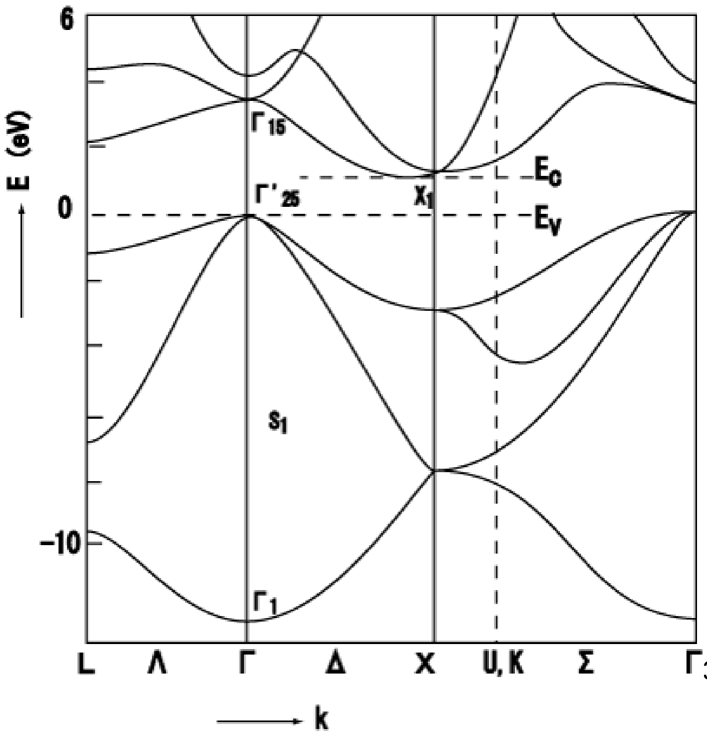
Topology of what? The sample?

Bandstructure and Topology

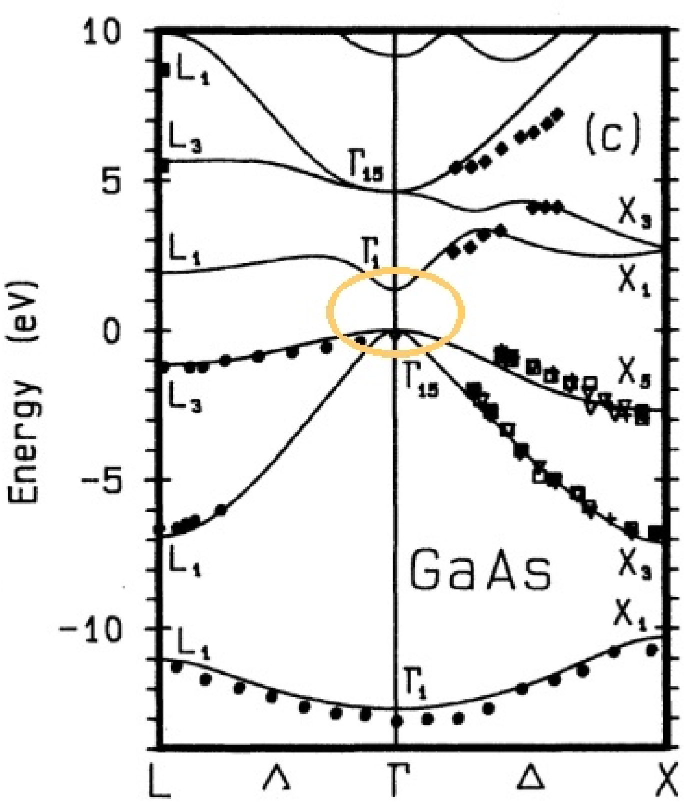
Non-interacting electrons in periodic potential:

BANDSTRUCTURE





Bandstructure and Topology

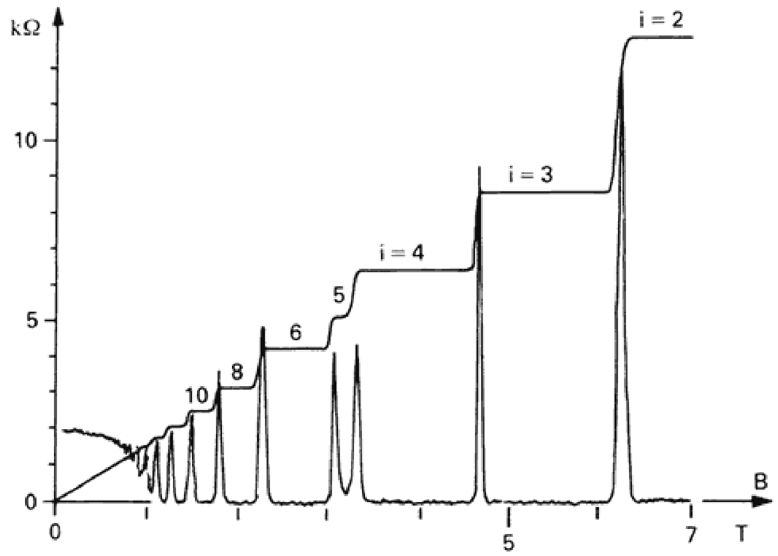


Band structure can have non-trivial topology.

Topological Quantum Numbers are strictly integer.

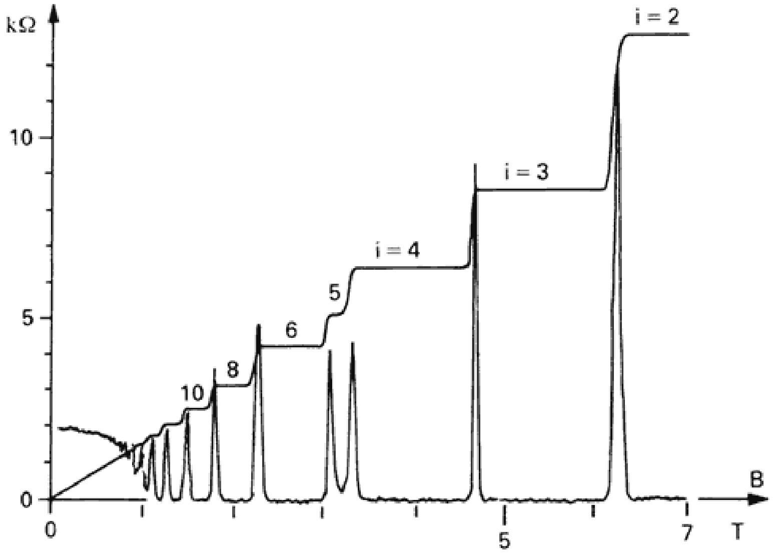
Protected under deformations that **do not close the gap!**

TKNN theory of Quantum Hall.



$$\sigma_{xy} = n\frac{e^2}{h}$$

TKNN theory of Quantum Hall.



$$\sigma_{xy} = n\frac{e^2}{h}$$

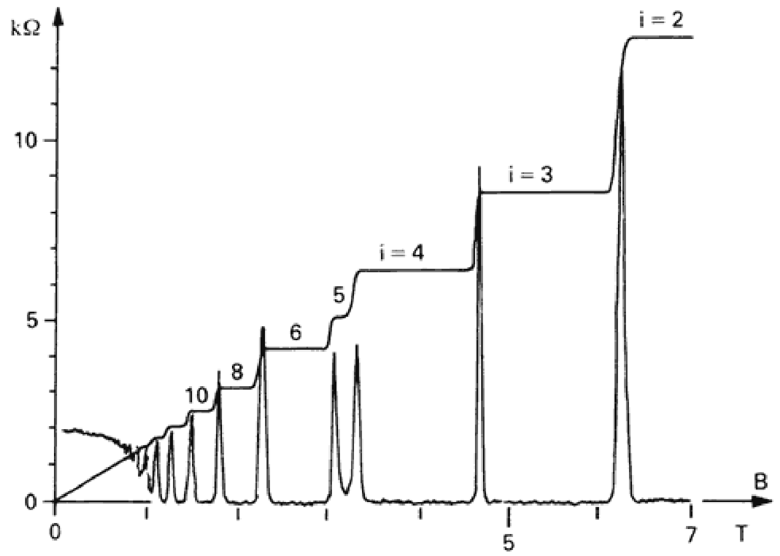
TKNN theory of Quantum Hall.

(Thouless, Kohmoto, Nightingale, den Nijs)

$$\sigma_{xy} = \frac{e^2}{h}\frac{1}{2\pi}\int dk_x \int dk_y f_{xy}(\mathbf{k})$$

$$f_{xy}(\mathbf{k}) = \frac{\partial a_y(\mathbf{k})}{\partial k_x} - \frac{\partial a_x(\mathbf{k})}{\partial k_y}$$

$$a_i(\mathbf{k}) = -i \sum_{\alpha \in \text{ occ}} \langle \alpha \mathbf{k}| \frac{\partial}{\partial k_i} |\alpha \mathbf{k}\rangle\,, \; i = x, y.$$



“Berry Connection” has quantized flux.

$$\sigma_{xy} = n\frac{e^2}{h}$$

TKNN for topological insulator.

Multi-Band-Berry-Connection.

(Qi, Hughes, Zhang)

(see also Fu, Kane, Mele)

$$\theta \equiv 2\pi P_3(\theta) = \frac{1}{16\pi^2}\int d^3\mathbf{k}\,\epsilon^{ijk}\mathrm{Tr}\{[f_{ij}(\mathbf{k}) - \frac{2}{3} i a_i(\mathbf{k})\cdot a_j(\mathbf{k})]\cdot a_k(\mathbf{k})\}$$

$$\begin{aligned} f_{ij}^{\alpha\beta} &= \partial_i a_j^{\alpha\beta} - \partial_j a_i^{\alpha\beta} + i\,[a_i, a_j]^{\alpha\beta}, \\ a_i^{\alpha\beta}(\mathbf{k}) &= -i\,\langle \alpha, \mathbf{k}| \frac{\partial}{\partial k_i} |\beta, \mathbf{k}\rangle \end{aligned}$$

Note:

- QHZ invariant is Z_2 (not Z) valued.
- Non-zero θ indicates strong L · S coupling

Next: Implications of generalized constitutive relation?

Implications of Generalized Constitutive Relation.

Physics 514 Homework Set #3 Winter 2010

Due in class 1/26/10

300 pts

3. `(100 pts)` A topological insulator is a material (e.g. $\mathrm{Bi_2Te_3}$) with constitutive relations

$$\vec{D} = \epsilon\vec{E} - \alpha\vec{B}, \quad \vec{H} = \frac{\vec{B}}{\mu} + \alpha\vec{E}$$

where α is the fine structure constant. For simplicity let us assume that $\epsilon \approx \epsilon_0$ and $\mu \approx \mu_0$ and only investigate the effect of the non-trivial "topological magneto-electric" effect, that is the apperance of $\vec{B}$ in $\vec{D}$ and the appearance of $\vec{E}$ in $\vec{H}$. The equations of electro- and magneto-statics are unmodified

$$\vec{\nabla} \cdot \vec{D} = \rho_e, \quad \vec{\nabla} \cdot \vec{B} = \rho_m, \quad \vec{\nabla} \times \vec{H} = \vec{j}_e, \quad \vec{\nabla} \times \vec{E} = \vec{j}_m$$

where ρ and $\vec{j}$ denote the free charge and current densities and subscripts e and m denote electric and magnetic charges respectively. We know that for physical charges $\rho_m = \vec{j}_m = 0$. Consider a planar interface between such a topological insulator at $z < 0$ and vacuum at $z > 0$.

a. Derive the boundary conditions obeyed by magnetic and electric fields at the interface (assuming no free surface charges or currents).

Implications of Generalized Constitutive Relation.

Physics 514 Homework Set #3 Winter 2010

Due in class 1/26/10

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Implications of Generalized Constitutive Relation.

Physics 514 — Homework Set #3 — Winter 2010

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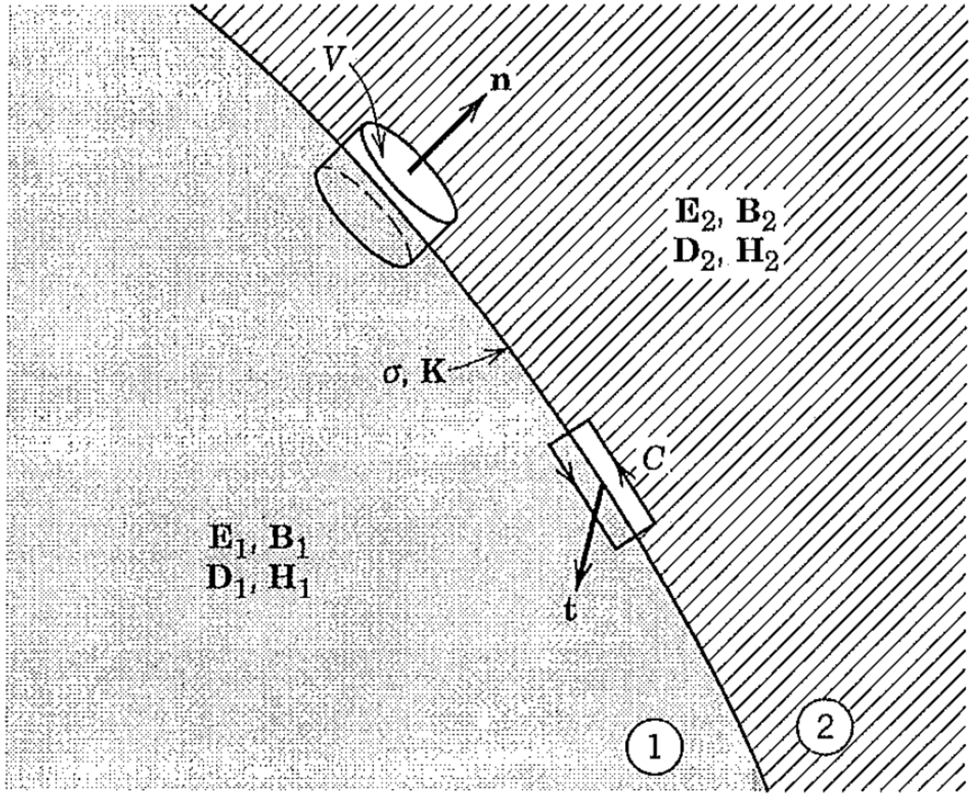
Boundary Conditions

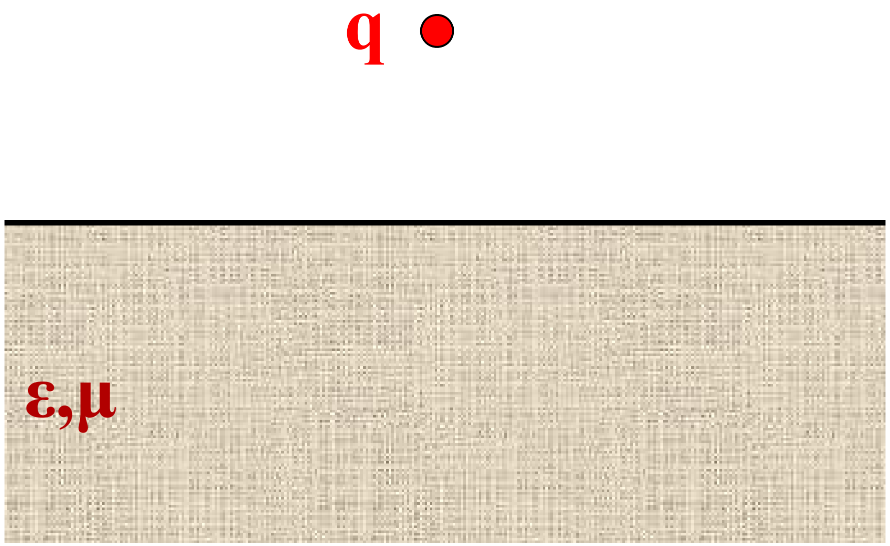
Maxwell unmodified $\rightarrow$ BC unmodified



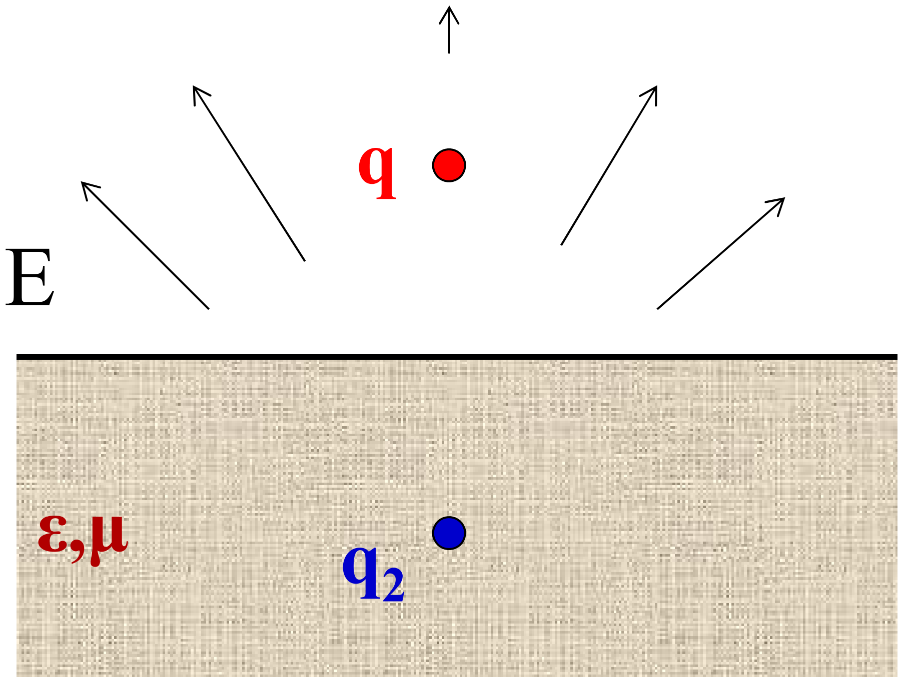
$E_{||}$, $H_{||}$ continuous
$B_{\perp}$, $D_{\perp}$ continuous

(in the absence of macroscopic surface charge or current densities)

$E=?$



Method of Images:



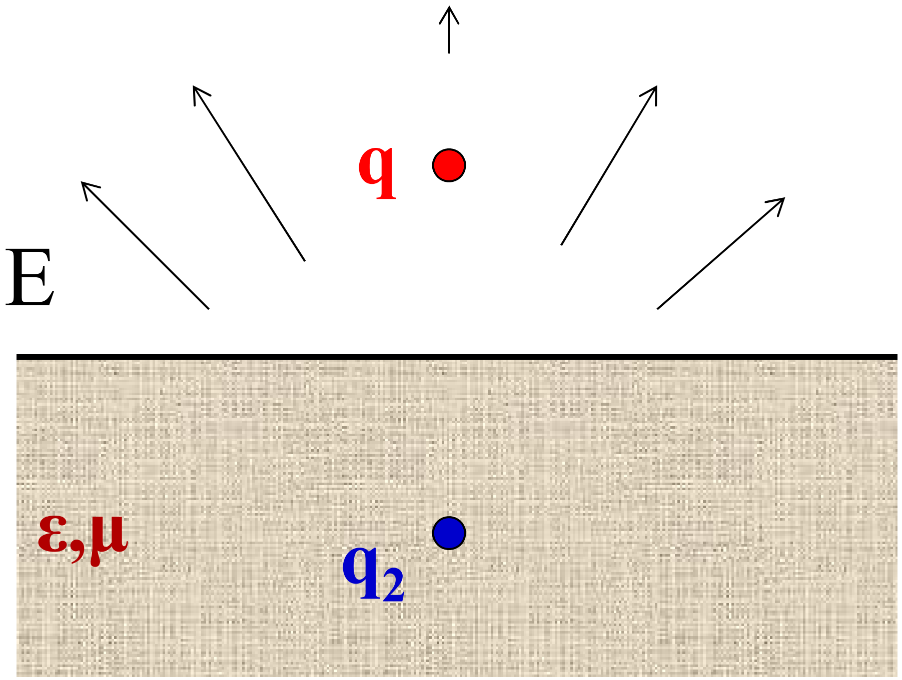
$$E_{||},\ D_{\perp}\ \text{continuous}$$

$$\downarrow$$

$$q_2 = -\left(\frac{\epsilon - \epsilon_0}{\epsilon + \epsilon_0}\right) q$$

Discontinuity in $E_{\perp}$:
microscopic surface charge density

Method of Images:



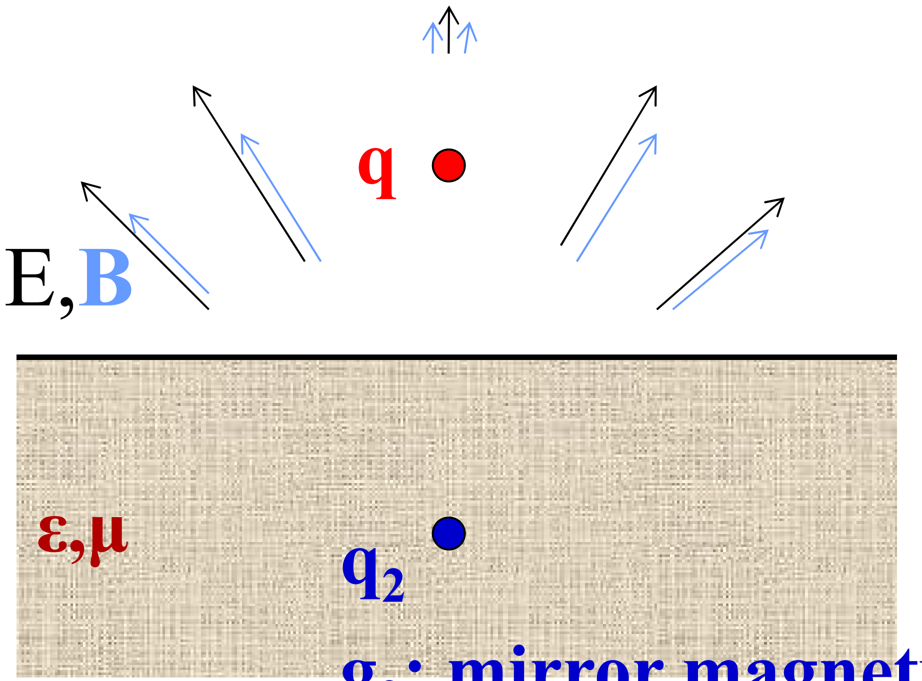
Now: $\boxed{\vec{H} = \frac{\vec{B}}{\mu_0}}$

$E_{\|},\ H_{\|}$ continuous
$B_{\perp},\ D_{\perp}$ continuous

$$\boxed{\vec{H} = \frac{\vec{B}}{\mu} + \frac{\alpha\theta}{\pi}\vec{E}}$$

non-zero

Method of Images:



g_2: **mirror magnetic monopole charge**

Now: $\boxed{\vec{H} = \frac{\vec{B}}{\mu_0}}$

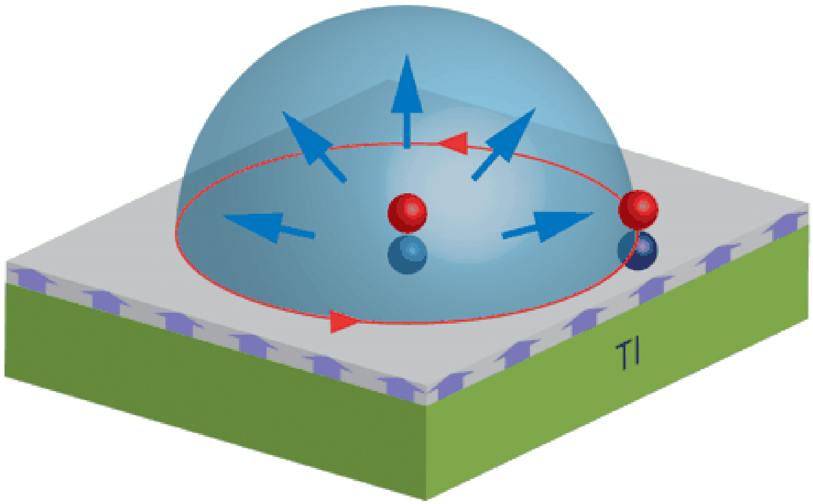
$E_\parallel,\ H_\parallel$ continuous
$B_\perp,\ D_\perp$ continuous

$$\boxed{\vec{H} = \frac{\vec{B}}{\mu} + \frac{\alpha\theta}{\pi}\vec{E}}$$

non-zero

Discontinuity in $B_\parallel$:
microscopic surface **current** density

Magnetic Monopoles in TI



Mirror charge of an electron
is a **magnetic monopole**
(Qi, Li, Zang, Zhang *Science*)

first pointed out by Lee and Sikivie for
"axion domain walls"

$$q_1 = q_2 = \frac{1}{\epsilon_1} \frac{(\epsilon_1 - \epsilon_2)(1/\mu_1 + 1/\mu_2) - 4\alpha^2 P_3^2}{(\epsilon_1 + \epsilon_2)(1/\mu_1 + 1/\mu_2) + 4\alpha^2 P_3^2} q$$

$$g_1 = -g_2 = -\frac{4\alpha P_3}{(\epsilon_1 + \epsilon_2)(1/\mu_1 + 1/\mu_2) + 4\alpha^2 P_3^2} q$$

$$P_3 = \frac{\theta}{2\pi}$$

Maxwell has E/q ↔ B/g symmetry. So why so complicated?

Duality Covariant Mirror Charges (AK)

Duality:

$SL(2,\mathbb{Z})$

$$\begin{pmatrix} \vec{D} \\ 2\alpha\vec{B} \end{pmatrix} = \Lambda \begin{pmatrix} \vec{D}' \\ 2\alpha\vec{B}' \end{pmatrix}, \quad \begin{pmatrix} 2\alpha\vec{E} \\ \vec{H} \end{pmatrix} = (\Lambda^T)^{-1} \begin{pmatrix} 2\alpha\vec{E}' \\ \vec{H}' \end{pmatrix}$$

$$\begin{pmatrix} \rho_e \\ 2\alpha\rho_m \end{pmatrix} = \Lambda \begin{pmatrix} \rho'_e \\ 2\alpha\rho'_m \end{pmatrix}, \quad \begin{pmatrix} \vec{j}_e \\ 2\alpha\vec{j}_m \end{pmatrix} = \Lambda \begin{pmatrix} \vec{j}'_e \\ 2\alpha\vec{j}'_m \end{pmatrix}$$

Constitutive Relation:

$$\begin{pmatrix} \vec{D} \\ 2\alpha\vec{B} \end{pmatrix} = \mathcal{M} \begin{pmatrix} 2\alpha\vec{E} \\ \vec{H} \end{pmatrix}$$

Mirror Charges: $\vec{q}^{(2)} = -\vec{q}^{(1)} = (\mathcal{T}+1)^{-1}(\mathcal{T}-1)\,\vec{q}.$

$$\mathcal{T} = \mathcal{M}_1\mathcal{M}_2^{-1} \qquad \mathcal{M} = \Lambda\mathcal{M}'\Lambda^T$$

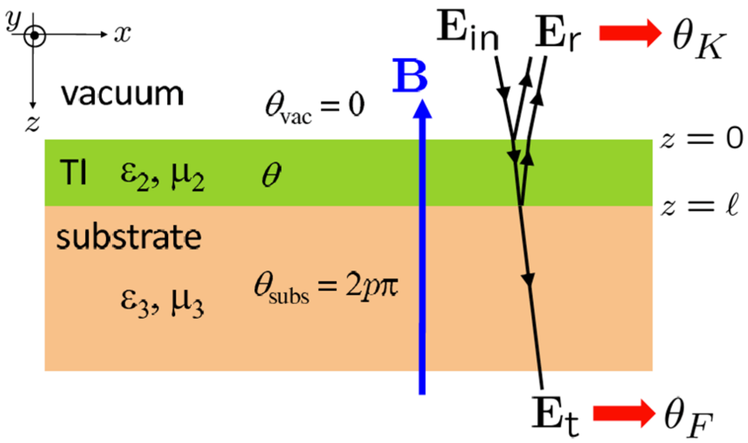
$$\mathcal{T} = \Lambda\mathcal{T}'\Lambda^{-1}$$

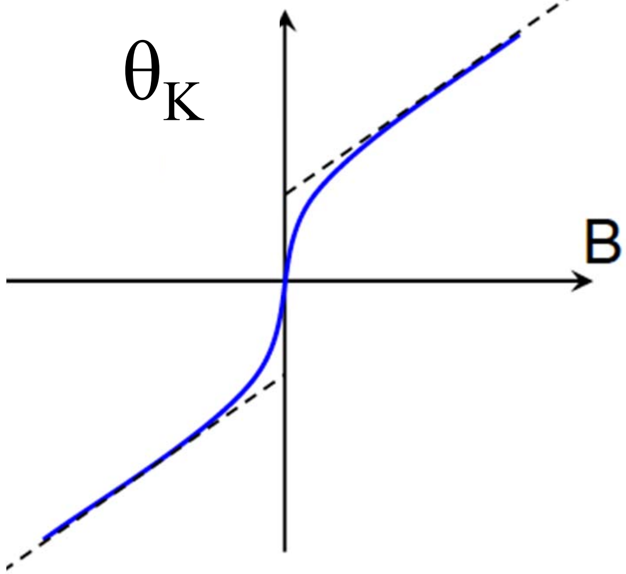
Faraday Rotation

Plane waves reflected off/transmitted through the interface experience rotation in their polarization proportional to $\alpha\theta$ = Kerr/Faraday effect.

Typically happens in the presence of magnetic fields but here it survives in the limit of zero (time reversal breaking) external field.

Faraday/Kerr effect:





Faraday/Kerr effect:

Physics 514 **Homework Set #7** **Winter 2010**

Due in class 3/2/10

300 pts

1. `(100 pts)` Reflection of the surface of a topological insulator: Consider a monochromatic electromagnetic wave of frequency ω incident from vacuum on a topological insulator. For simplicity consider the case of normally incident light. Recall that for a topological insulator

$$\vec{D} = \epsilon \vec{E} - \alpha \vec{B}, \quad \vec{H} = \frac{\vec{B}}{\mu} + \alpha \vec{E}$$

where we work in units where, in vacuum, $\epsilon_0 = \mu_0 = c = 1$, so ϵ and μ for the insulator are pure numbers.

a. What are the boundary conditions that the electric and magnetic fields have to obey at the surface?

b. Determine the amplitude and polarization of the transmitted and the reflected wave. In particular, establish that both transmitted and reflected wave experience a "Faraday rotation", that is the direction of polarization gets rotated by an angle α_F. Determine α_F both for the reflected and the transmitted wave (for the reflected wave the Faraday effect is often referred to as the Kerr effect). While the Faraday effect is commont in magnetic (time reversal breaking) materials, its appearance in time reversal invariant materials is another peculiarity of topological insulators.

A Microscopic model?

Low Energy Effective Theory:

$\boxed{\text{Flux Quantization}} \longrightarrow \boxed{\theta = \text{Integer} \cdot \pi}$

IR

Is there something
in between?

Full Band Structure:

$\boxed{\text{Band Topology}} \longrightarrow \boxed{\theta = \text{TKNN-like-invariant}}$

UV

A Microscopic model?

Low Energy Effective Theory

IR

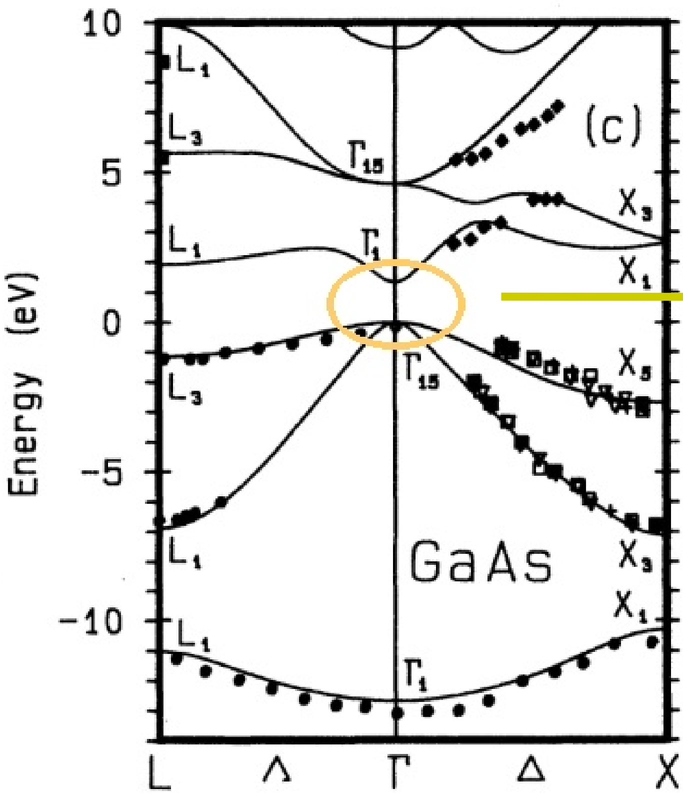
Free Dirac Equation $\longrightarrow$

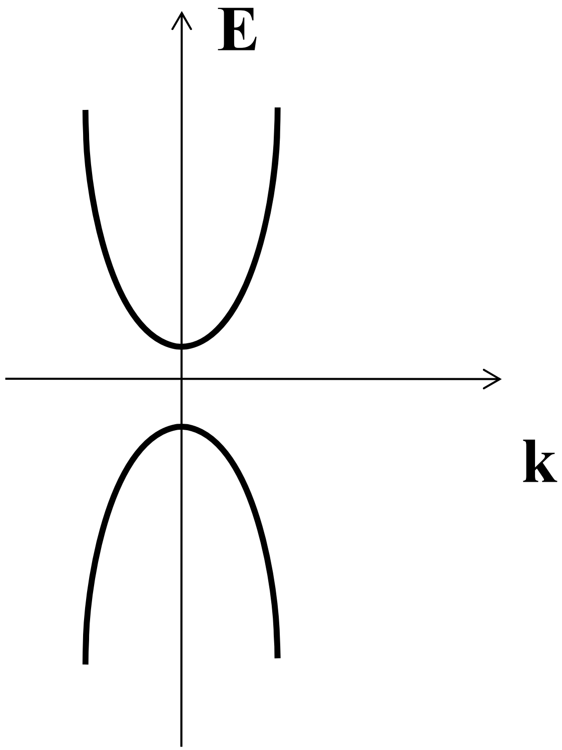
- θ from chiral anomaly
- experimental signatures
- generalization to **fractional** TI

Full Band Structure

UV

Microscopic Model:





Spectrum of free massive Dirac fermion.

A Microscopic Model

A microscopic model: Massive Dirac Fermion.

$$\mathcal{L} = \bar{\psi}(i\partial_\mu \gamma^\mu - M)\psi$$

Time Reversal: $M \longrightarrow M^*$

Time reversal system has real mass.

positive or negative.

for energies << M $\downarrow$

$\theta=0$ or $\theta=\pi$

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chiral anomaly – robust against interactions.

$\theta=0$ or $\theta=\pi$

Chiral rotation and ABJ anomaly.

Massless theory invariant under chiral rotations:

$$\psi \to e^{-i\phi\gamma_5/2}\psi$$

Symmetry of massive theory if mass transforms:

$$M \to e^{i\phi}M$$

Phase can be rotated away! Chose M positive.

Chiral rotation and ABJ anomaly.

But in the quantum theory chiral rotation is anomalous. Measure transforms.

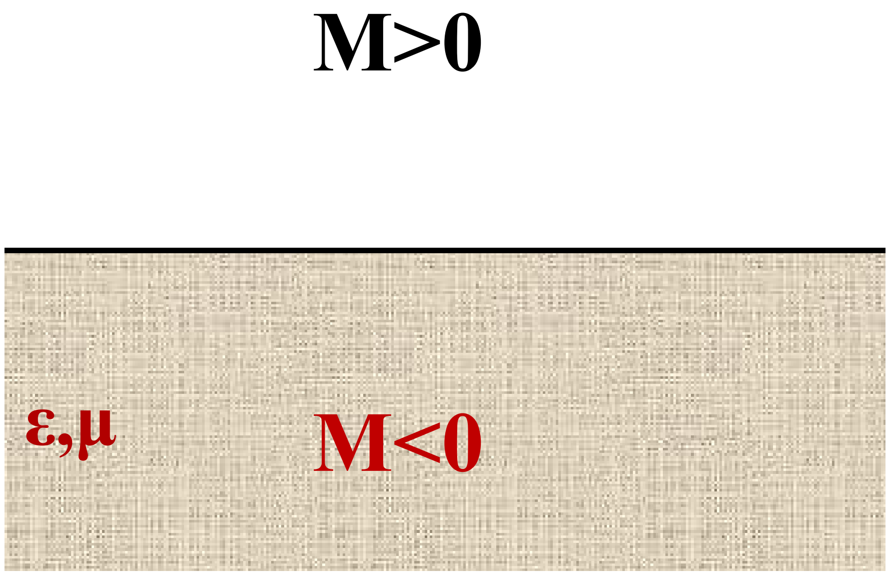
$$\Delta\mathcal{L} = C\alpha\frac{\phi}{32\pi^2}\,\mathrm{tr}\;\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$

$$C = \sum_{fields} q^2 = 1\cdot 1^2 = 1$$

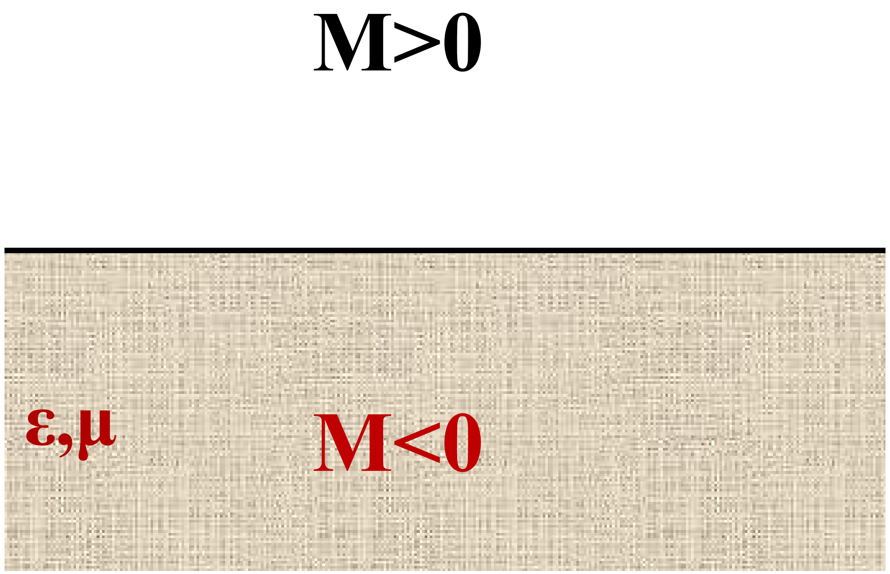
Single field with unit charge.

$$\theta \to \theta - C\phi$$

Microscopic Model of Interface.



Microscopic Model of Interface.



Mass has to cross zero.

Microscopic Model of Interface.

M>0

Mass has to cross zero.

↓

Localized Zero Mode.

M<0

Domain Wall Fermion.

Microscopic Model of Interface.

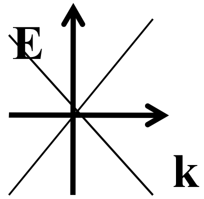
M>0

Mass has to cross zero.

Localized 2+1 d Zero Mode.

E ~ p

Single Dirac Cone!

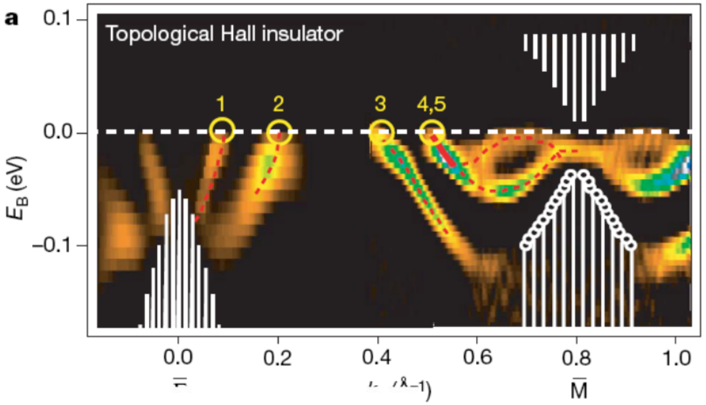


$$\sigma_{xy} = \frac{1}{2} n \frac{e^2}{h}$$

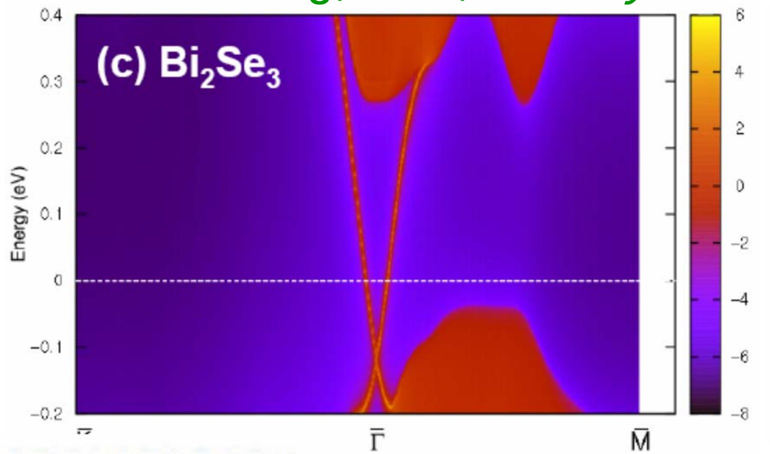
Impossible in pure 2+1.
E.g. graphene: 4 Dirac Cones.

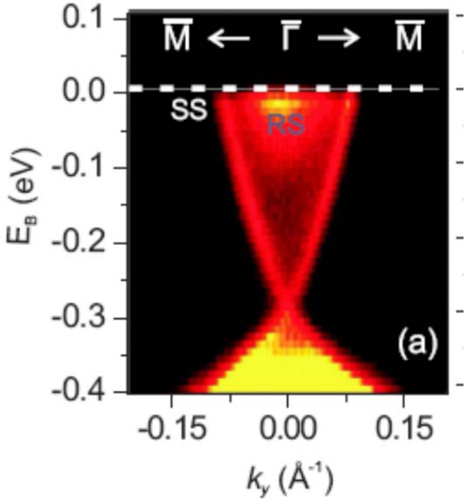
Excellent Signature!

Hasan group, *Nature* 2008

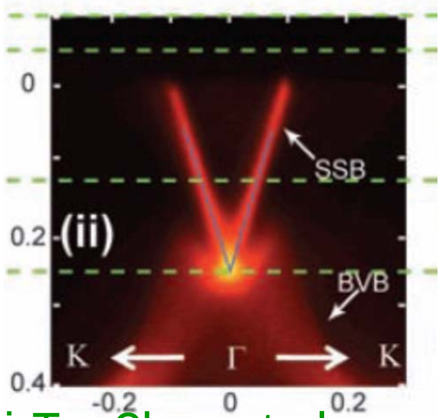


H. J. Zhang, et al, *Nat Phys* 2009

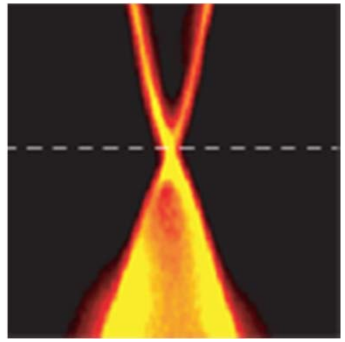




Hasan group, Bi_2Se_3 *Nat Phys* 2009



Bi_2Te_3 Chen et al *Science* 2009



$TlBiTe_2$ Kuroda et al *Phys.Rev.Lett* 2010

Fractional Topological Insulators?

Recall from Quantum Hall physics:

electron

fractionalizes into m partons

$$\sigma_{xy} = n\frac{e^2}{h} \xrightarrow{e^- \text{ interactions}} \sigma_{xy} = \frac{n}{m}\frac{e^2}{h}$$

(m odd for fermions)

Quantum Hall

Fractional Quantum Hall

Fractional Topological Insulators?

TI = half of an integer quantum hall state on the surface

expect: fractional TI = half a fractional QHS

Hall quantum = half of 1/odd integer.

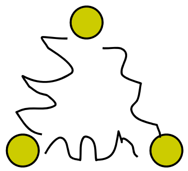
Can we get this from charge fractionalization?

Partons.

Microscopic Model:

chiral anomaly $\longrightarrow$ $\theta/\pi = \sum (\text{charge})^2$

e⁻ =



electron breaks up into m partons.

(m odd so e^- is fermion)

$$\theta / \pi = \sum (\text{charge})^2 = m \cdot \left(\frac{1}{m}\right)^2 = \frac{1}{m}$$

(if partons form a TI = have negative mass)

How to make a fractional TI?

Need: Strong electron/electron interactions

(so electrons can potentially fractionalize)

Strong spin/orbit coupling

(so partons can form topological insulator)

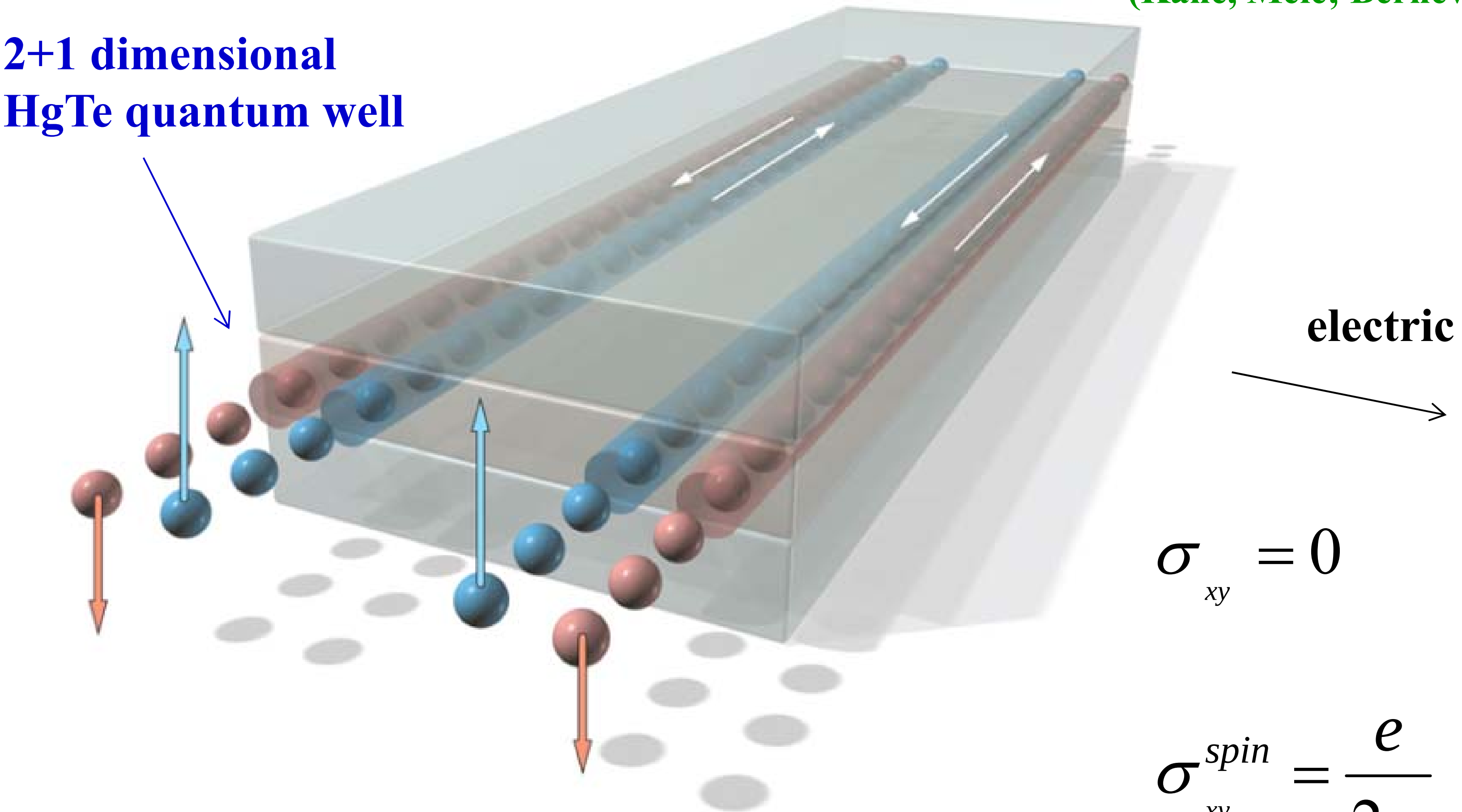
How can one tell if a given material is a fractional TI (in theory/in practice)?

Transport! Fractional Hall + Kerr.

Generalizations.

Quantum Spin Hall Effect in HgTe

(Kane, Mele; Bernevig Zhang)



$$\sigma_{xy} = 0$$

$$\sigma_{xy}^{spin} = \frac{e}{2\pi}$$

Fractional Quantum Spin Hall.

- Quantum Spin Hall is also well described by time reversal invariant free Dirac equation

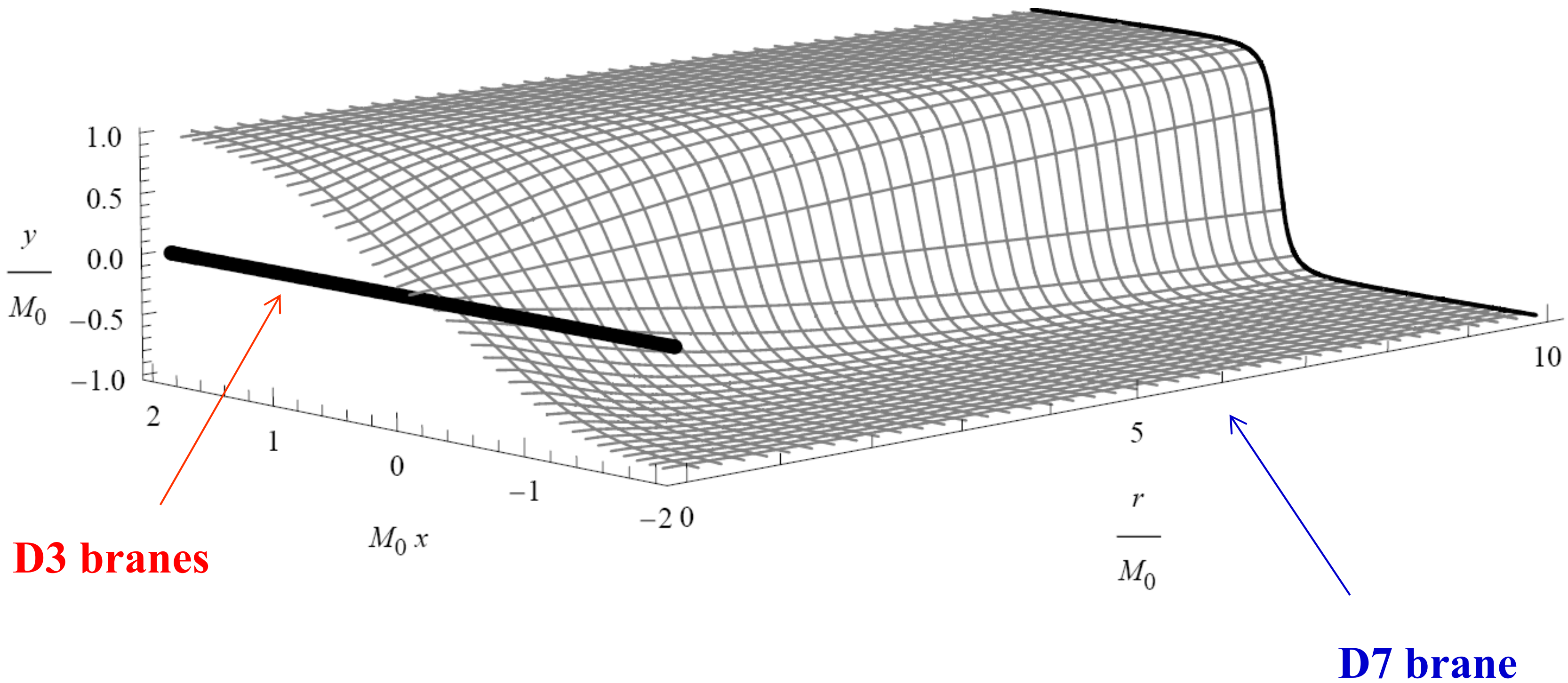
- Transport again determined by anomaly; robust against interactions. “Quantum R-Hall-Effect”.

- Fractionalization of charge gives fractional transport coefficients.

(AK, Maciejko, Takayanagi)

holographic fTI

(Jensen, Hoyos, AK)



Basic phenomenon indeed robust against strong interactions (in the effective theory).

Summary.

Particle
Theory

TOPOLOGICAL
INSULATORS

Condensed
Matter Physics