Modified Gravity Explains Dark Matter?

Taishi Katsuragawa

Refs:
Works in progress
Brief Introduction to Modified Gravity

Background
• General Relativity
• Dark Energy and Dark Matter

Why Modified Gravity?
General Relativity

General Relativity (GR) is simple but successful.

Einstein-Hilbert (EH) action

\[ S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} R, \quad \kappa^2 = 8\pi G \]

Einstein equation

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa^2 T_{\mu\nu} \]

\( T_{\mu\nu} \): Energy-momentum tensor

Cosmic History

[Planck (2013)]

Gravitational Waves

[LIGO (2016)]
GR万歳！

congratulations! GR!!
There are still mysteries in our Universe: Dark Energy (DE) and Dark Matter (DM)

**Dark Energy**

Energy to accelerate the expansion of the current Universe.

cf.) Type Ia supernova, CMB, BAO

**Dark Matter**

Invisible matter besides ordinary matters

cf.) Galaxy rotation curve etc.

[Amanullah et. al (2010)]

[Begeman, Broeils, and Sanders (1991)]
\[ S = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g} \left[ R - 2\Lambda \right] + S_{SM+DM} \]

**Cosmological constant (Λ)**  **Cold Dark Matter (CDM)**

Two questions remain...
- What is the cosmological constant?
- What is the origin of CDM?

**Cosmological Constant (CC) problems**
- Fine tuning (why so small?)
- Coincidence (why observed value?) etc.

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Constant vs. Dynamical Field

Cosmological constant?
- Simple and consistent with observation
- DE “=” Cosmological constant?

Equation of state of DE: $p = w \rho$ (p: pressure $\rho$: energy density)
- If $w < -1/3$, we can explain late-time acceleration.
- DE is not necessarily cosmological constant ($w = -1$).
- Dynamical Dark Energy

<table>
<thead>
<tr>
<th>Value of $w$</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w = 1/3$</td>
<td>Radiation (relativistic matter)</td>
</tr>
<tr>
<td>$w = 0$</td>
<td>Dust (non-relativistic matter)</td>
</tr>
<tr>
<td>$-1 &lt; w &lt; -1/3$</td>
<td>Quintessence</td>
</tr>
<tr>
<td>$w = -1$</td>
<td>Cosmological Constant</td>
</tr>
<tr>
<td>$w &lt; -1$</td>
<td>Phantom</td>
</tr>
</tbody>
</table>
How to introduce new dynamical field DOF?

→ New Matter or **Modified Gravity**

**Dynamical Dark Energy**

\[
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa^2 T_{\mu\nu}
\]

- Modification of gravity sector
- Modification of matter sector

= Modified gravity

- The modification leads to the emergence of new DOF.
- New DOF causes deviations from GR.
  - to explain the Dark Energy \(\(_{\smile}(\forall \smile)\_\)
  - to bring undesirable deviations \(\_\cdot\omega\cdot\_\)
  - Modifications are constrained by observations.
F(R) Gravity and Scalaron

F(R) Gravity
- Weyl Transformation
- Equivalence to Scalar-Tensor Theory

Scalaron
- Matter coupling to SM Particles
F(R) Gravity

Basics on F(R) gravity

Action of F(R) gravity

\[ S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} F(R) \]

[Buchdahl (1970)]

\[ \int d^4x \sqrt{-g} R \]

Replace: \( R \to F(R) \)

- EoM with matter field

\[ F_R(R) R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} F(R) + (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu) F_R(R) = \kappa^2 T_{\mu\nu} \]

- Trace of the EOM

\[ \Box F_R(R) = \frac{1}{3} \kappa^2 T + \frac{1}{3} [2F(R) - F_R(R)R] \]

The Ricci scalar is dynamical although \( R = -\kappa^2 T \) in GR.
From F(R) to Scalar-Tensor Theory

(1) Rewrite the action with an auxiliary field

\[
S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[ F_A(A) R - \left\{ F_A(A) A - F(A) \right\} \right]
\]

where A is auxiliary scalar field, and \( F_R(R) = \partial_R F(R) \)

- EoM of auxiliary field A

\[
F_{AA}(A) (R - A) = 0 \quad \rightarrow \quad A = R \quad \text{if} \quad F_{RR}(R) \neq 0
\]

(2) Transform the metric

Weyl Transformation

\[ g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = \Omega^2(x) g_{\mu\nu} \]

Jordan frame : \( g_{\mu\nu} \rightarrow \) Einstein frame : \( \tilde{g}_{\mu\nu} \)
From F(R) to Scalar-Tensor Theory

• Choose the Weyl trans. as

$$\Omega^2(x) = F_R(R) \equiv e^{2\sqrt{1/6\kappa}\varphi(x)}, \quad \varphi(x) = \frac{\sqrt{6}}{2\kappa} \ln F_R(R)$$

F(R) gravity in Einstein frame

$$S = \int d^4x \sqrt{-\tilde{g}} \left[ \frac{1}{2\kappa^2} \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right]$$

where

$$V(\varphi) = \frac{1}{2\kappa^2} \frac{F_R(R)R - F(R)}{F_R^2(R)}$$

After the Weyl trans., F(R) gravity can be expressed in terms of GR with scalar field $\varphi(x)$

– Mathematical equivalence to Scalar-Tensor theory

We call the scalar field as Scalaron
Scalaron Couplings with Matters

Short Summary

\[ S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} F(R) \]

\[ \overset{(1)}{S} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[ F_A(A)R - \{F_A(A)A - F(A)\} \right] \]

\[ \overset{(2)}{S} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-\tilde{g}} \tilde{R} + \int d^4x \sqrt{-\tilde{g}} \left[ -\frac{1}{2} \tilde{g}^{\mu\nu} (\partial_\mu \varphi) (\partial_\nu \varphi) - V(\varphi) \right] \]

Consider the matter sector

\[ S_{\text{Matter}} = \int d^4x \sqrt{-g} \mathcal{L} (g^{\mu\nu}, \Psi) \]

\[ = \int d^4x \sqrt{-\tilde{g}} e^{-4\sqrt{1/6\kappa\varphi(x)}} \mathcal{L} \left( e^{2\sqrt{1/6\kappa\varphi(x)}} \tilde{g}^{\mu\nu}, \Psi \right) \]

Dilatonic coupling btw. Scalaron and matter field

- Weak interaction because of gravitational origin
- Suppressed by Planck mass \( \kappa = 1/M_{\text{pl}}, M_{\text{pl}} = 10^{19}\text{[GeV]} \)
Chameleon Mechanism

Screening Mechanism
  • Solar-System Constraint

Chameleon Mechanism
  • Environment Dependence
Screening Mechanism

 Modifications to GR introduce additional DOF. However, the Solar-System constraints often exclude modifications.

- The fifth force $\varphi$ should act only on large scale, and it should be screened on small scale.

$\varphi$ is responsible for modification of gravity

Solar system

$\varphi$ is screened on small scale

Inter-stellar (or -galactic) space
Test of Gravity and Screening Mechanism

Modified Gravity

Screening Mechanism

Newton Potential

Psaltis (2008)
Chameleon Mechanism

Viable F(R) gravity possesses **Chameleon mechanism**

- Restrictive constraints from obs. in Solar System
- Scalaron effective potential couples to trace of $T_{\mu\nu}$

\[
\tilde{\Box} \varphi = \partial \varphi V_{\text{eff}}(\varphi), \quad V_{\text{eff}}(\varphi) = V(\varphi) - \frac{1}{4} e^{-4\sqrt{1/6} \kappa \varphi} T_{\mu}^{\mu}
\]

**Chameleon Mechanism**

\[
T_{\mu}^{\mu} = -\rho
\]

(for dust)

Large $\rho_+$

$\rho_+^2$

Small $\rho_-$

$\rho_-^2$

\[
V_{\text{eff}}
\]

\[
\kappa \varphi
\]

**Scalaron mass**

\[
m_\varphi = V''_{\text{eff}}(\varphi_{\text{min}})
\]

In high-density region, scalar field is heavy and suppressed.

In low-density region, scalar field is light and acts as DE.
Scalaron as Dark Matter Candidate

Objectives

Stability of Scalaron
• Coupling with SM Particles
• Decay width and Lifetime
Applications of Modified Gravity

How can we use the modifications for unanswered questions?

= Application of modified gravity
  – Cosmology (DE etc.)
  – Astrophysics (massive NS, BH, GW etc.)
  – Particle Physics?

Objective. 1

Quantization of new DOF = New particle?
  – Beyond Standard Model (SM) particle is introduced from the “beyond GR” sector.
  – New constraints from the viewpoint of particle physics.
Can the new particle be a DM candidate?
- The origin is gravitational sector
- New particle has very weak interactions with matter
- New particle can be massive

Objective. 2

DM candidate in modified gravity?
• New constraints on modified gravity by converting the existing constraints on DM.
• Unified treatment of DM and DE in one theory
Can Scalaron be a DM?

Properties of Scalaron

- Heavy in the Solar-System (or around the Earth) by the Chameleon Mechanism
- Interaction to SM particle is suppressed by the Planck mass ($e^{\kappa \varphi} \sim 1 + \kappa \varphi$)

They suggest the Scalaron could be a CDM.
- Can F(R) gravity explain DM problem?
  
  [Nojiri and Odintsov (2008), Choudhury et al. (2015)]

To study the Scalaron as DM candidate
- Stability = Decay process and Lifetime [TK and S. Matsuzaki (2017)]
- Relic abundance
- Direct detection experiment
  
  In progress
Coupling to Matter: Massless vector

Massless vector field $A_\mu(x)$

$$\mathcal{L}_V (g^{\mu\nu}, A_\mu) = -\frac{1}{4} g^{\alpha\mu} g^{\beta\nu} F_{\alpha\beta} F_{\mu\nu}$$
$$= -\frac{1}{4} e^{4\sqrt{1/6} \kappa \varphi} \tilde{g}^{\alpha\mu} \tilde{g}^{\beta\nu} F_{\alpha\beta} F_{\mu\nu}$$

Field strength is invariant under the Weyl trans.

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu$$

No Coupling to Scalaron through field strength

$$S = \int d^4 x \sqrt{-\tilde{g}} \ e^{-4\sqrt{1/6} \kappa \varphi} \mathcal{L}_V (g^{\mu\nu}, A_\mu)$$
$$= \int d^4 x \sqrt{-\tilde{g}} \mathcal{L}_V (\tilde{g}^{\mu\nu}, A_\mu)$$
Coupling to Matter: Massless fermion

Massless fermion field $\psi(x)$

$$\mathcal{L}_F (\gamma^\mu, \psi) = i \bar{\psi}(x) \gamma^\mu \nabla_\mu \psi(x)$$

where

$$\gamma^\mu(x) = e_a^\mu(x) \gamma^a , \{ \gamma^\mu, \gamma^\nu \} = 2g^{\mu\nu}$$

$$\nabla_\mu \psi(x) = \partial_\mu \psi(x) + \frac{1}{8} \omega_{\mu ab}(x) [\gamma^a, \gamma^b] \psi(x)$$

$$w_{\mu ab}(x) = e_{a\nu} (\partial_\mu e^\nu_b + \Gamma^\nu_{\mu\rho} e^\rho_b)$$

Action in the Einstein frame

$$S = \int d^4 x \sqrt{-g} \mathcal{L}_F (\gamma^\mu, \psi)$$

$$= \int d^4 x \sqrt{-\tilde{g}} \left[ e^{-3\sqrt{1/6\kappa_-}} i \bar{\psi} \tilde{\gamma}^\mu \tilde{\nabla}_\mu \psi - \frac{3i}{2} \sqrt{\frac{1}{6\kappa_-} e^{-3\sqrt{1/6\kappa_-}} (\partial_\mu \varphi) \bar{\psi} \tilde{\gamma}^\mu \psi} \right]$$

No coupling after field redefinition

$$\psi \rightarrow \psi' = e^{-3/2\sqrt{1/6\kappa_-}} \psi$$

$$S = \int d^4 x \sqrt{-\tilde{g}} i \bar{\psi}' \tilde{\gamma}^\mu \tilde{\nabla}_\mu \psi'$$
The scalaron would affect the quantum dynamics of fermion field although the scalaron coupling can be eliminated by field redefinition in classical dynamics.

- Path integral measure induces the anomaly

\[
\psi(x) = \sum_n a_n \psi_n(x), \quad \bar{\psi}(x) = \sum_n \hat{a}_n \bar{\psi}_n
\]

\[
\psi'(x) = (1 + \phi(x))\psi(x), \quad \phi(x) \equiv \frac{3}{2} \sqrt{\frac{1}{6}} \kappa \varphi(x)
\]

\[
\Pi_n da_n d\hat{a}_n \rightarrow \Pi_n da'_n d\hat{a}'_n \cdot \mathcal{J}^{-2}
\]

\[
\mathcal{J} = \exp \left[ i \int d^4 x \phi(x) \cdot \frac{g^2}{4(4\pi)^2} \text{tr}[F_{\mu\nu}^2] \right]
\]

The couplings with massless vector fields show up.

\[
\mathcal{L}_{\text{anomaly}} = -\frac{g^2}{2(4\pi)^2} \phi \text{tr}[F_{\mu\nu}^2]
\]
Coupling to Matter: Massive vector field $A_\mu(x)$

$$\mathcal{L}_{V-\text{mass}}(g^{\mu\nu}, A_\mu) = -\frac{1}{2} m_V^2 e^{2\sqrt{1/6}\kappa\varphi} \tilde{g}^{\mu\nu} A_\mu A_\nu$$

Action in the Einstein frame

$$S = \int d^4 x \sqrt{-g} \mathcal{L}_{V-\text{mass}}(g^{\mu\nu}, A_\mu)$$

$$= \int d^4 x \sqrt{-\tilde{g}} [\mathcal{L}_{V-\text{mass}}(\tilde{g}^{\mu\nu}, A_\mu) + \mathcal{L}_{V-\varphi}(\tilde{g}^{\mu\nu}, A_\mu, \varphi)]$$

$$\mathcal{L}_{V-\varphi}(\tilde{g}^{\mu\nu}, A_\mu, \varphi) = -\frac{1}{2} m_V^2 \left( e^{-2\sqrt{1/6}\kappa\varphi} - 1 \right) \tilde{g}^{\mu\nu} A_\mu A_\nu$$

Expand the interacting Lagrangian w.r.t. $|\kappa\varphi| \ll 1$

Coupling to Scalaron through the mass term.

$$\mathcal{L}_{V-\varphi}(\tilde{g}^{\mu\nu}, A_\mu, \varphi) = \frac{2\kappa\varphi}{\sqrt{6}} \cdot \frac{1}{2} m_V^2 \tilde{g}^{\mu\nu} A_\mu A_\nu + \mathcal{O}(\kappa^2 \varphi^2)$$
After field redefinition, massive fermion field $\psi'(x)$

$$\mathcal{L}_{F-\text{mass}}(\psi) = -m_F e^{\sqrt{1/6} \kappa \varphi} \bar{\psi}' \psi'$$

Action in the Einstein frame

$$S = \int d^4x \sqrt{-g} \mathcal{L}_{\text{mass}} (g^{\mu \nu}, \psi)$$

$$= \int d^4x \sqrt{-\tilde{g}} [\mathcal{L}_{F-\text{mass}}(\psi') + \mathcal{L}_{F-\varphi} (\psi', \varphi)]$$

$$\mathcal{L}_{F-\varphi} (\psi', \varphi) = -m_F \left( e^{-\sqrt{1/6} \kappa \varphi} - 1 \right) \bar{\psi}' \psi'$$

Coupling toScalars through the mass term.

$$\mathcal{L}_{F-\varphi} (\psi', \varphi) = \frac{\kappa \varphi}{\sqrt{6}} \cdot m_F \bar{\psi}' \psi' + \mathcal{O}(\kappa^2 \varphi^2)$$
Coupling to SM Particles

For massless vector field (Photon, Gluon)

\[ \mathcal{L} = -\frac{3g^2}{4(4\pi)^2} \left( \frac{3}{2} \sqrt{\frac{\kappa}{6} \varphi} \right) \text{tr} \left[ F_{\mu\nu}^2 \right] + \mathcal{O}(\kappa^2 \varphi^2) \]

For massive vector field (Weak bosons)

\[ \mathcal{L} = \frac{2\kappa \varphi}{\sqrt{6}} \cdot \frac{1}{2} m^2 \tilde{g}^{\mu\nu} A_\mu A_\nu + \mathcal{O}(\kappa^2 \varphi^2) \]

For massive fermion field (Quarks, Leptons)

\[ \mathcal{L} = \frac{\kappa \varphi}{\sqrt{6}} \cdot m_F \bar{\psi}' \psi' + \mathcal{O}(\kappa^2 \varphi^2) \quad \psi \rightarrow \psi' = e^{-3/2} \sqrt{1/6 \kappa \varphi} \psi \]
As to the couplings to diphoton and digluon, the scalaron couplings are generated at one-loop level of the SM perturbation.

\[ \phi \rightarrow \gamma \gamma \]

\[ \phi \rightarrow g g \]

corresponding to leading order contribution in original fermion field \( \psi(x) \)
Consider the early Universe after EW phase transition but before QCD phase transition.

Lifetime $\Gamma_{\phi}^{-1} \geq 10^{17} \text{[s]}$ (age of Universe)

$\rightarrow$ Lifetime changes in the cosmic history
Scalaron Mass in Cosmic History

Scalaron mass depends on the environment in the Universe.

- We need to construct the time evolution of $T^\mu_\mu$
- For perfect fluid, $T^\mu_\mu = - (\rho - 3p)$

$$V_{\text{eff}}(\varphi) = V(\varphi) + \frac{1}{4}e^{-4\sqrt{1/6}\kappa\varphi}(\rho - 3p)$$

We find

$$\rho - 3p = \frac{gT^4}{2\pi^2} x^2 \int_0^\infty d\xi \frac{\xi^2}{\sqrt{\xi^2 + x^2}} \frac{1}{e^{\sqrt{\xi^2 + x^2}} \pm 1} \quad x = \frac{m}{T}, \quad \xi = \frac{p}{T}$$

• At high temp. (relativistic)

$$\rho - 3p \approx \frac{g}{12} m^2 T^2$$

• At low temp. (non-relativistic)

$$\rho - 3p \approx \rho \approx mg \left( \frac{mT}{2\pi} \right)^{3/2} e^{-m/T}$$
Time evolution of $T_{\mu}^{\mu}$

To compare $\rho$ and $T_{\mu}^{\mu} = \rho - 3\rho$
Time evolution of $T^\mu_\mu$

To compare $\rho$ and $T^\mu_\mu = \rho - 3\rho$
Starobinsky model

Particular model of F(R) gravity

Starobinsky model for late-time acceleration

\[ F(R) = R - \beta R_c \left[ 1 - \left( 1 + \frac{R^2}{R_c^2} \right)^{-n} \right] \]

[Starobinsky (2007)]

where \( R_c \sim \Lambda \) is constant curvature, and \( \beta, n > 0 \)

Starobinsky model in large-curvature limit \( R \gg R_c \)

- Chameleon mechanism works in dense regime

\[ F(R) \approx R - \beta R_c \left[ 1 - \left( \frac{R_c}{R} \right)^{2n} \right] \]

where \( \beta R_c \approx 2\Lambda \)

- Scalaron mass

\[ m_\phi^2 = \frac{2\Lambda}{6n(2n + 1)\beta} \left( \frac{\kappa^2 (\rho - 3p)}{2\Lambda} \right)^{2(n+1)} \]

increasing function of \( \rho - 3p \)
R^2 correction

- Singularity problem in F(R) Gravity

\[ V_{\text{eff}} \]

\[ \varphi = 0 \leftrightarrow R = \infty \]

\[ m_\varphi \gg M_{\text{pl}} \]

\[ T^\mu_\mu \text{ increases} \]

Potential minimum

- In order to prevent the scalaron mass from reaching the Planck mass, we add the R^2 term

\[ F(R) = R - \beta R_c \left[ 1 - \left( 1 + \frac{R^2}{R_c^2} \right)^{-n} \right] + \alpha R^2 \]

[Dev, Jain, Jhingan, Nojiri, Sami, Thongkool (2008)]
History of Scalaron Mass

Large curvature limit,

\[ m_\varphi \approx \frac{1}{6\alpha(1 + 2\kappa^2\alpha(\rho - 3\rho))} \]

Starobinsky model

\[ \alpha = \frac{1}{6M^2}, \quad M = 10^{13}[\text{GeV}] \]

As inflaton

\[ \alpha = 10^{22}[\text{GeV}^{-2}] \]

Upper bound from Eot-Wash experiments
Conclusion
Summary and Conclusion

• Modified gravity has been investigated so far to explain the dark energy.

• We are studying if the modified gravity can explain the dark matter.

• F(R) gravity predicts the new scalar field, and we study it as a new dark matter candidate.

• We studied...
  – Interactions btw. scalaron and SM particles
  – Time-evolution of the scalaron mass

• Future works
  – Relic density and direct detection etc.
Thank you for your attention.
(and sorry for my bad talk)