

# Scalar and other spectrum in $N_f=12$ QCD

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## Outline

- **Introduction**
- **Scalar from fermionic operator (flavor singlet)**
- **Scalar from Gluonic operator ( $0^{++}$  Glueball )**
- **Discussion**
- **summary**

# Introduction

# LatKMI project (2011-)



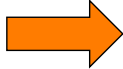
Y.Aoki, T.Aoyama, M. Kurachi, T. Maskawa, K.-i. Nagai,  
HO, E. Rinaldi, A. Shibata, K.Yamawaki, T.Yamazaki

## Systematic study of flavor dependence in Large $N_f$ QCD using single setup of the lattice simulation

Our goals:

- Understand the flavor dependence of the theory
- Find the conformal window
- Find the walking regime and investigate the anomalous dimension

Status (lattice):

- $N_f=16$ : likely conformal
- $N_f=12$ : consistent with conformal  this talk
- $N_f=8$ : studies suggests chiral broken phase and walking behavior.
- $N_f=4$ : chiral broken and enhancement of chiral condensate

Lattice gauge theory + numerical simulation : powerful tool

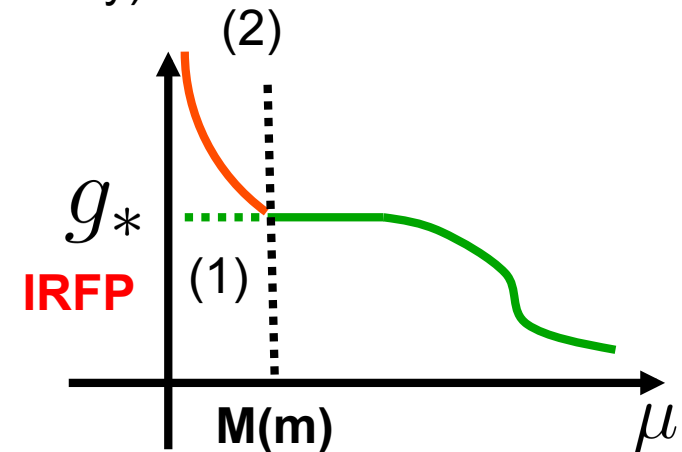
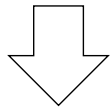
## ■ Possible signals of the conformal behavior on lattice

Introducing the bare mass (explicit breaking of conformality)

$$\delta\mathcal{L} = -m\bar{q}q$$

If the theory is in the conformal window,

1.  $m \rightarrow 0$ , box size  $L \rightarrow \infty$ , CFT is realized. (no mass gap)
2.  $m \neq 0$ , this theory has mass gap at low energy ( $M$ ) and give rise to asymptotic state (bound state).



- All hadron mass spectrums & decay constant  $\rightarrow$  [Hyper scaling](#) [Miransky '96]

$$m_X \propto m^{1/(1+\gamma^*)} \quad \gamma^* : \text{mass anomalous dimension}$$

- In the conformal phase, glueball could be lighter than hadron.  
[Miransky, **Phys.Rev. D59 (1999) 105003**]
- In the walking regime, light scalar may be identified with a dilaton as a NG-boson of the breaking of scale invariance.

Note: Lattice setup of conformal gauge theory with small bare mass  
 $\rightarrow$  a simple realization of the [conformal](#) technicolor model.

# “Higgs boson”

- Higgs like particle (125 GeV) is found at LHC.
- Consistent with the Standard Model Higgs.  
But true nature is so far unknown.
- Many candidates for beyond the SM.  
one possibility
  - (walking) technicolor
    - “Higgs” = dilaton (pNGB) due to breaking of the approximate scale invariance

# Our work

Observables: [Glueball \(O++\)](#), [flavor singlet scalar](#)

Is this lighter than pion? If so, Good candidate of “Higgs” (techni-dilaton).

## Lattice setup

- SU(3), Nf=12 flavor
- **HISQ** (staggered) fermion

Volume (= L<sup>3</sup> x T)

- L =18, T=24
- L =24, T=32
- L =30, T=40

Bare coupling constant ( $\beta = \frac{6}{g^2}$ )

- beta=4

bare quark mass

- mf=(0.05), 0.06, 0.08, 0.1, 0.12, 0.16  
(0.05 ... fermion only)

L	T	mf	#config
18	24	0.06	2800
		0.08	5000
		0.10	5000
		0.12	5000
		0.16	5000
24	32	0.05	2700
		0.06	14000
		0.08	15000
		0.10	9000
30	40	0.05	1200
		0.06	2000
		0.08	6700
		0.10	4000

# Lattice results of Scalar in $N_f=12$

## Fermionic observables

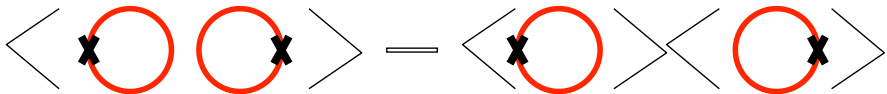
- All results are preliminary.
- Calculation method <- previous talk



# Scalar from fermionic observables

Observables: [Flavor singlet scalar](#) ( $\sigma$ )

$$C_\sigma(t) = \langle \sum_i^{N_f} \bar{\psi}_i \psi_i(t) \sum_j^{N_f} \bar{\psi}_j \psi_j(0) \rangle = N_f(-C(t) + N_f D(t))$$

$$\mathcal{O}_F(t) \equiv \bar{\psi}_i \psi_i(t), \quad D(t) = \langle \mathcal{O}_F(t) \mathcal{O}_F(0) \rangle - \langle \mathcal{O}_F(t) \rangle \langle \mathcal{O}_F(0) \rangle$$


## Staggered fermion case

- Scalar interpolating operator  $C(t)$  can couple to two states of

$$(\mathbf{1} \otimes \mathbf{1}) \ \& \ (\gamma_4 \gamma_5 \otimes \xi_4 \xi_5)$$

- 0+(non-singlet scalar) :  $C(2t)_+ \rightarrow a_0$  (continuum limit)
- 0-(scPion) :  $C(2t)_- \rightarrow \text{scPion}$  (continuum limit)

$$C_\pm(2t) \equiv 2C(2t) \pm C(2t+1) \pm C(2t-1)$$

- [Flavor singlet](#) scalar can be evaluated with disconnected diagram.

$$C_\sigma(2t) = -C_+(2t) + 3D_+(2t)$$

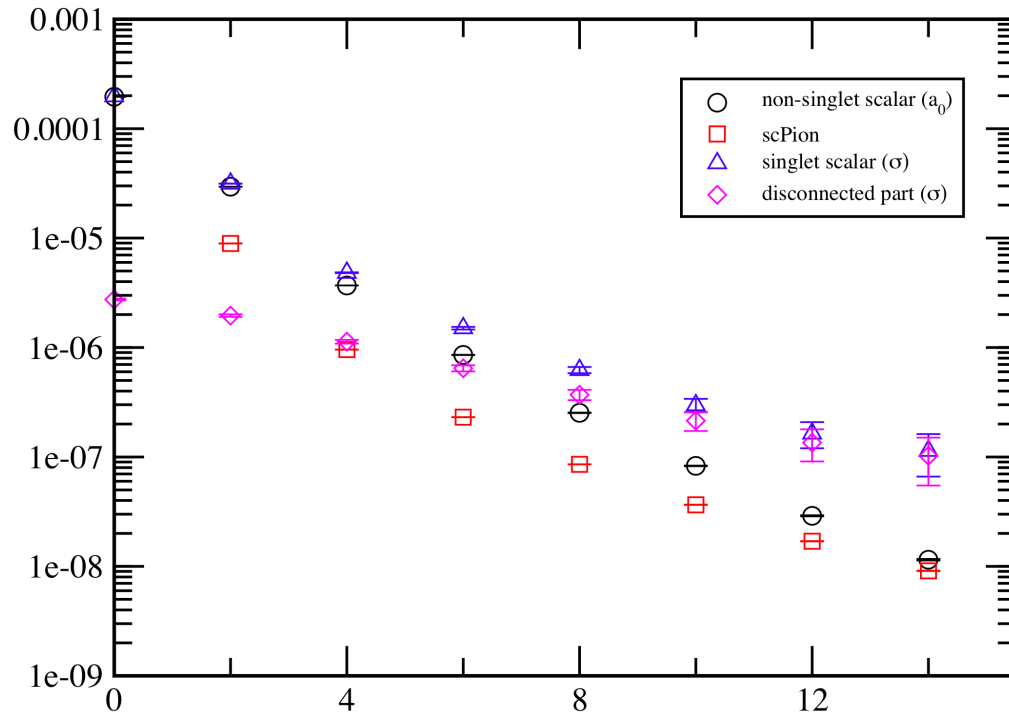
meson correlator, L=24 T=32  $\beta=4.0$  mf=0.06

$$0^+(a_0) : C_+(2t) = 2C(2t) + C(2t + 1) + C(2t - 1)$$

$$0^-(\text{scPion}) : C_-(2t) = 2C(2t) - C(2t + 1) - C(2t - 1)$$

$$0^+(\sigma) : C_\sigma(2t) = -C_+(2t) + 3D_+(2t)$$

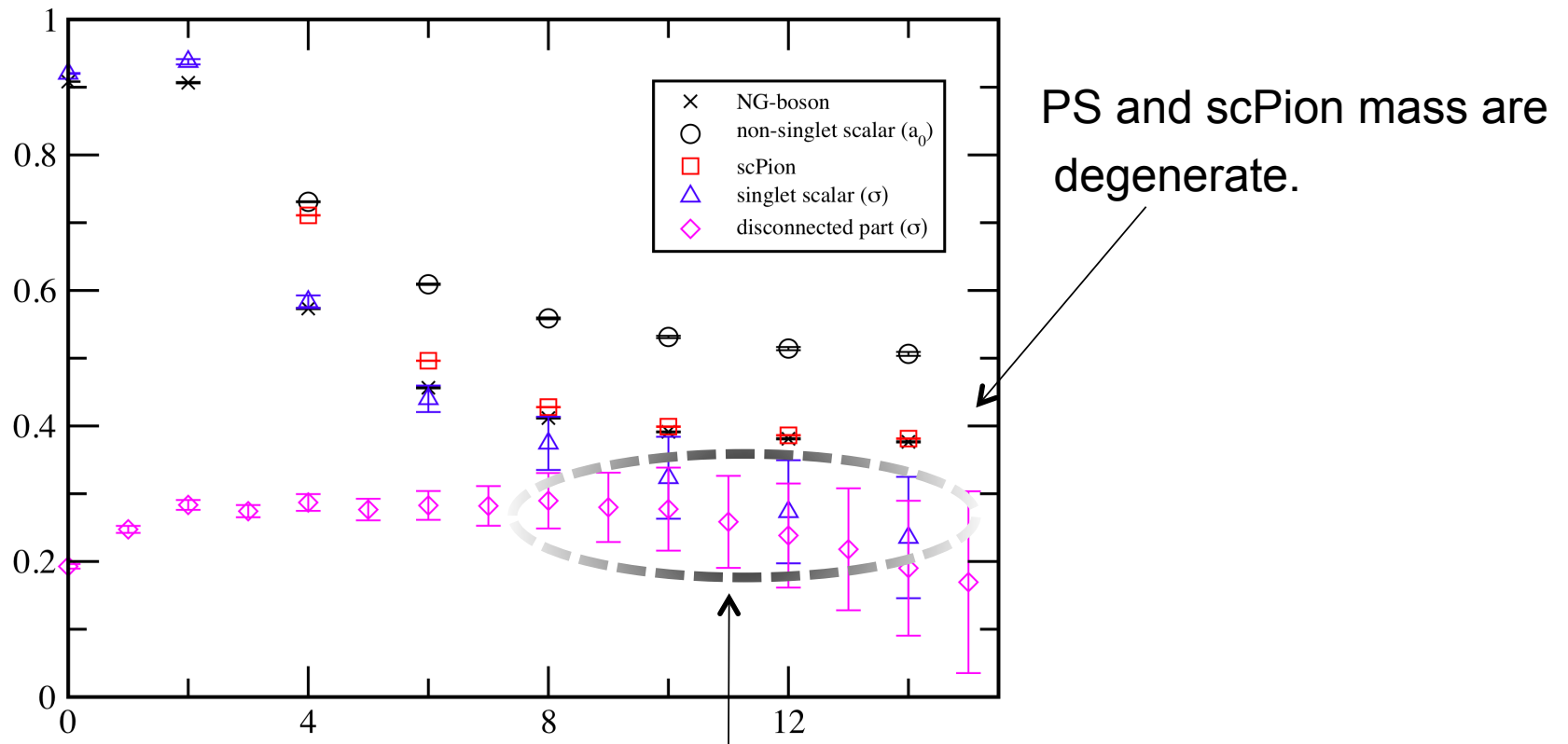
$$0^+ : 3D_+(2t)$$



Scalar meson effective mass L=24 T=32  $\beta=4.0$  mf=0.06  
 (Yamazaki san's slide with NG-boson effective mass)

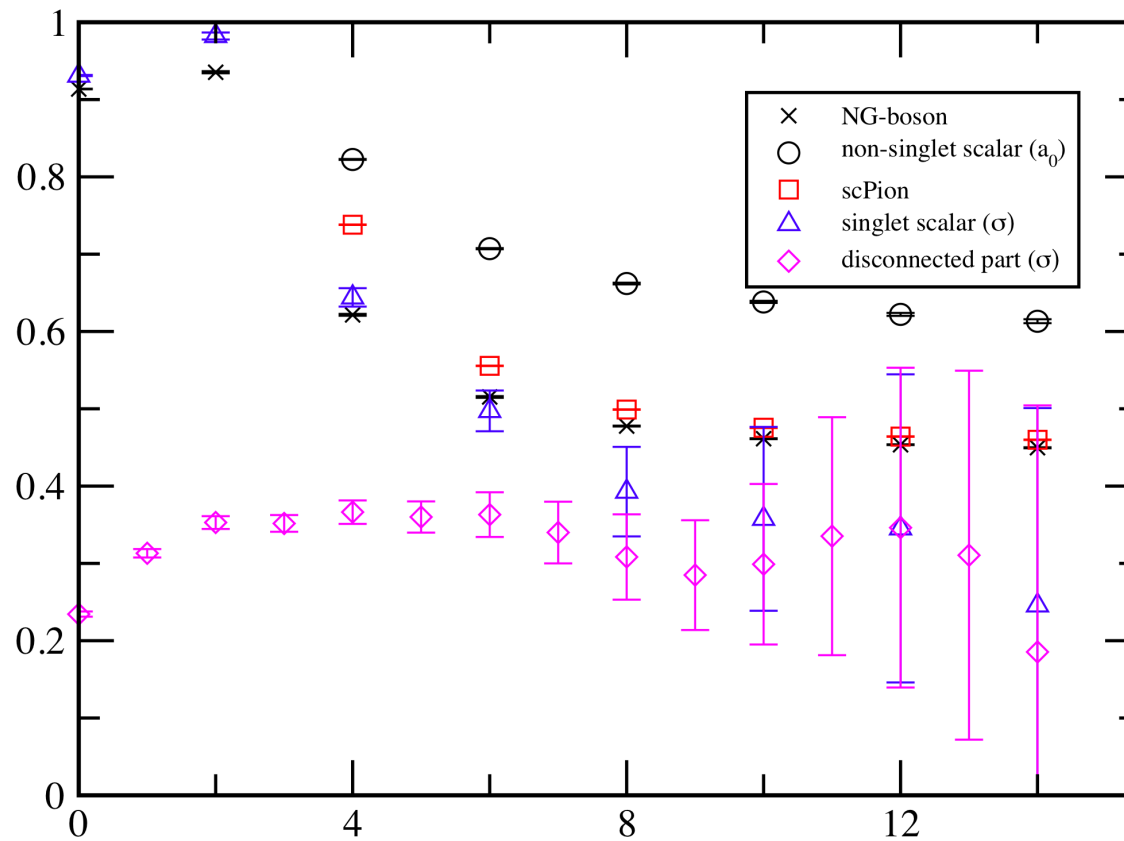
$$C_\sigma(2t) = -C_+(2t) + 3D_+(2t) \rightarrow A_\sigma e^{-m_\sigma t}$$

$$D(t) = A_\sigma e^{-m_\sigma t} + A_{a_0} e^{-m_{a_0} t} \rightarrow A'_\sigma e^{-m_\sigma t} \quad (\text{if } m_\sigma < m_{a_0})$$



- Disconnected correlator gives same effective mass as full correlator.
- Flavor non-singlet scalar is heavier than singlet scalar.

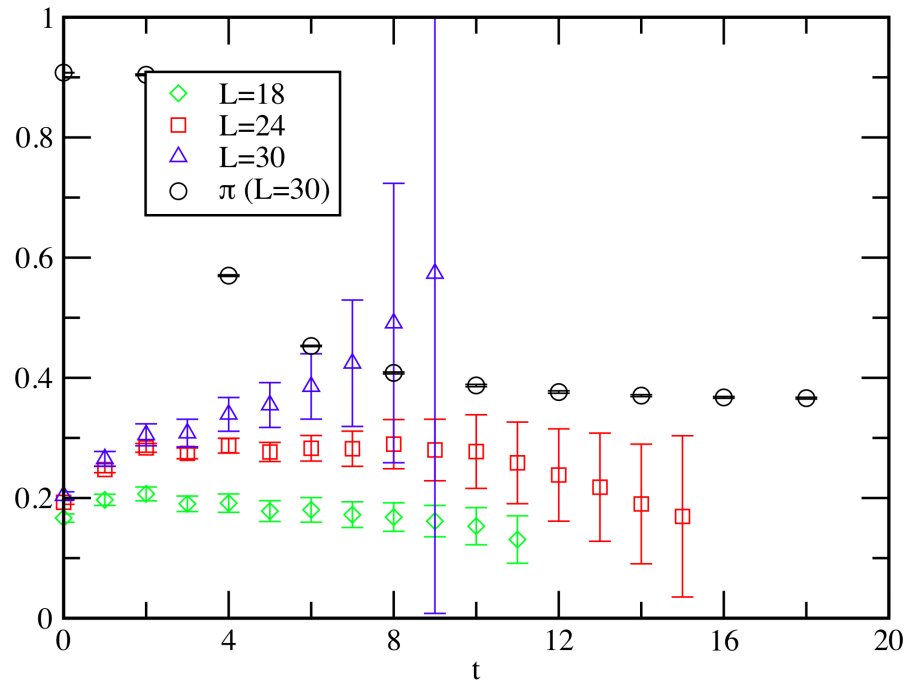
# Meson effective mass L=24 T=32 $\beta=4.0$ mf=0.08



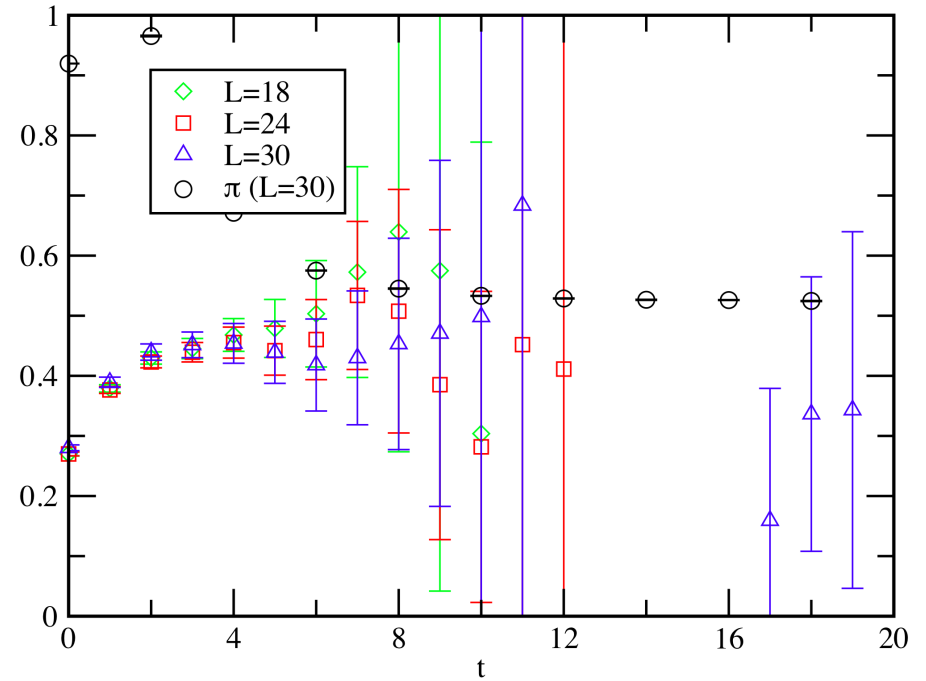
- Good plateau for disconnected correlator for small  $t$ .
- Flavor non-singlet scalar is heavier than singlet scalar.

Volume dependence of effective mass from  $D(t)$   
 $L=18, 24, 32$   $\beta=4.0$   $mf=0.06$  &  $0.10$

$m=0.06$



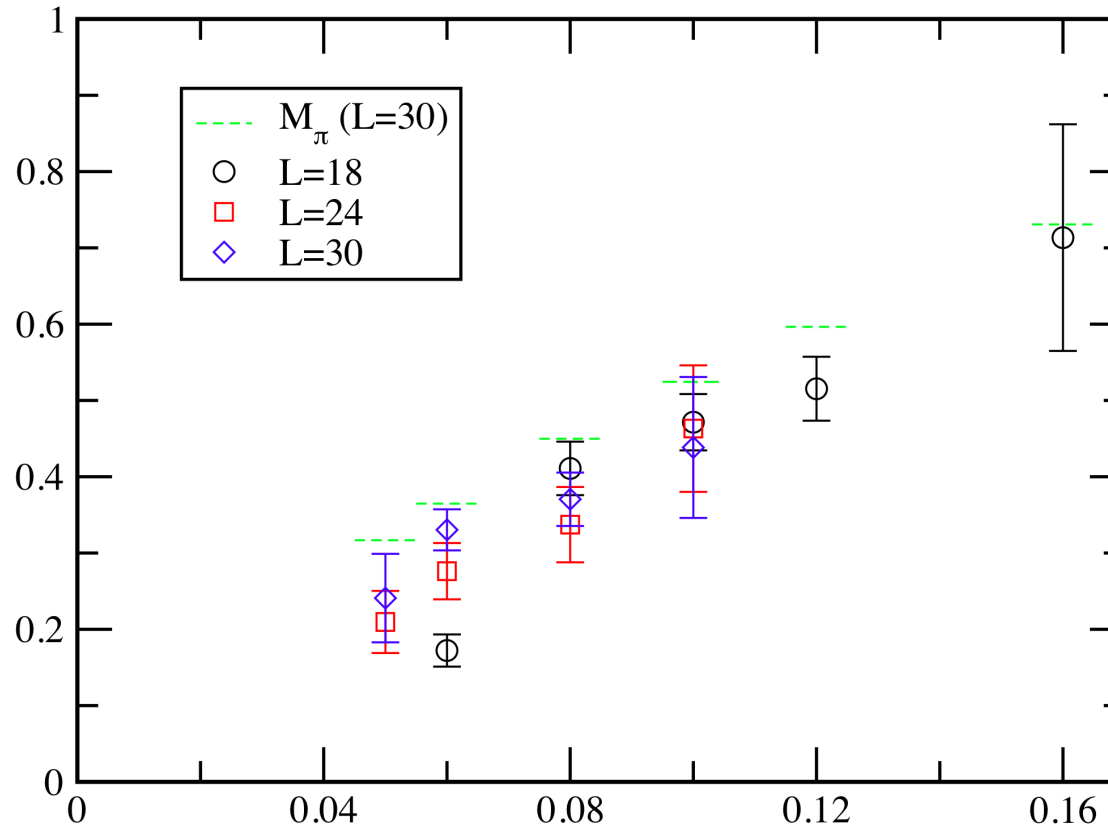
$m=0.10$



- At  $m=0.10$ , all the results for  $L=18, 24, 30$  are consistent.
- At  $m=0.06$ ,  $m\sigma(L=18) < m\sigma(L=24)$  and large statistical fluctuation in  $L=30$ .

## Fit result of scalar meson mass from $D(t)$

*preliminary*



- Fermion mass dependence is observed.
- Finite volume effect can be controlled. (L=24 and 30 are consistent).  
For lighter fermion mass, L=18 data has large a finite volume effect.
- **Scalar(0+) (at L=24) is lighter than pion.**

# Lattice results of Scalar in $N_f=12$

## Gluonic observables

- All results are preliminary.

# Glueball spectrum

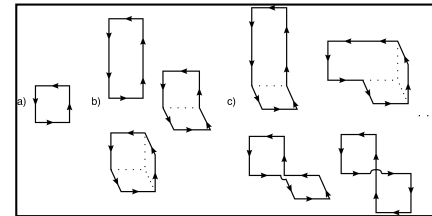
- Not measured yet in large Nf flavor QCD
- Measurement of 0++ state -> scalar state
- Possible candidate of light dilaton (scalar)

$J$	$A_1$	$A_2$	$E$	$T_1$	$T_2$
0	1	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	1	0	1	1
4	1	0	1	1	1

## Basic Method

- Mass eigenstates are classified by the cubic group.
- Gauge invariant operator (Wilson loop) with suitable representation must to be constructed.

$$\mathcal{O}_G(t) = \frac{1}{L^3} \sum_{x \in L^3} \text{Tr} \left( \prod_{l \in \mathcal{W}(x)} U_l \right)$$



- Correlation function in scalar channel is defined with vacuum subtraction;

$$\mathcal{O}^{(A_1)}(t) - \langle 0 | \mathcal{O}^{(A_1)} | 0 \rangle$$

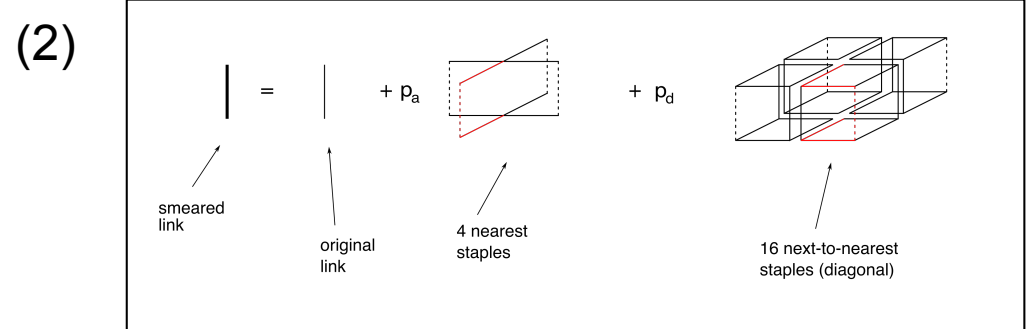
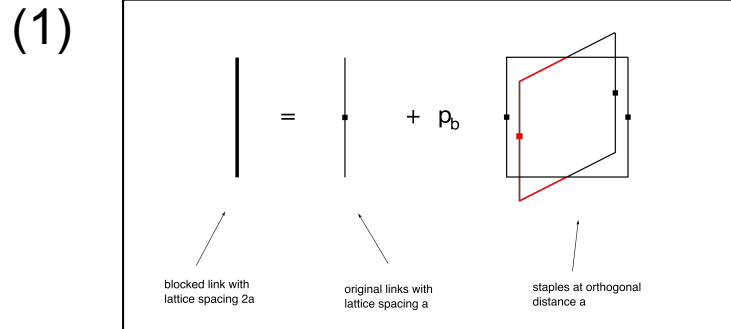


# Improvement for Glueball operators

Ref. [E. Gregory et. al.1208.1858] in QCD study

Very noisy (disconnected, typically heavy in QCD)

- Blocking(1) and smearing(2) technique to reduce the fluctuation
- Variational method (3) (many operators) -> plateau from small



(3) • Matrix correlator by operators: 
$$C_{ij}(t) = \sum_{\tau} \langle 0 | \mathcal{O}_i^{\dagger}(\tau + t) \mathcal{O}_j(\tau) | 0 \rangle$$

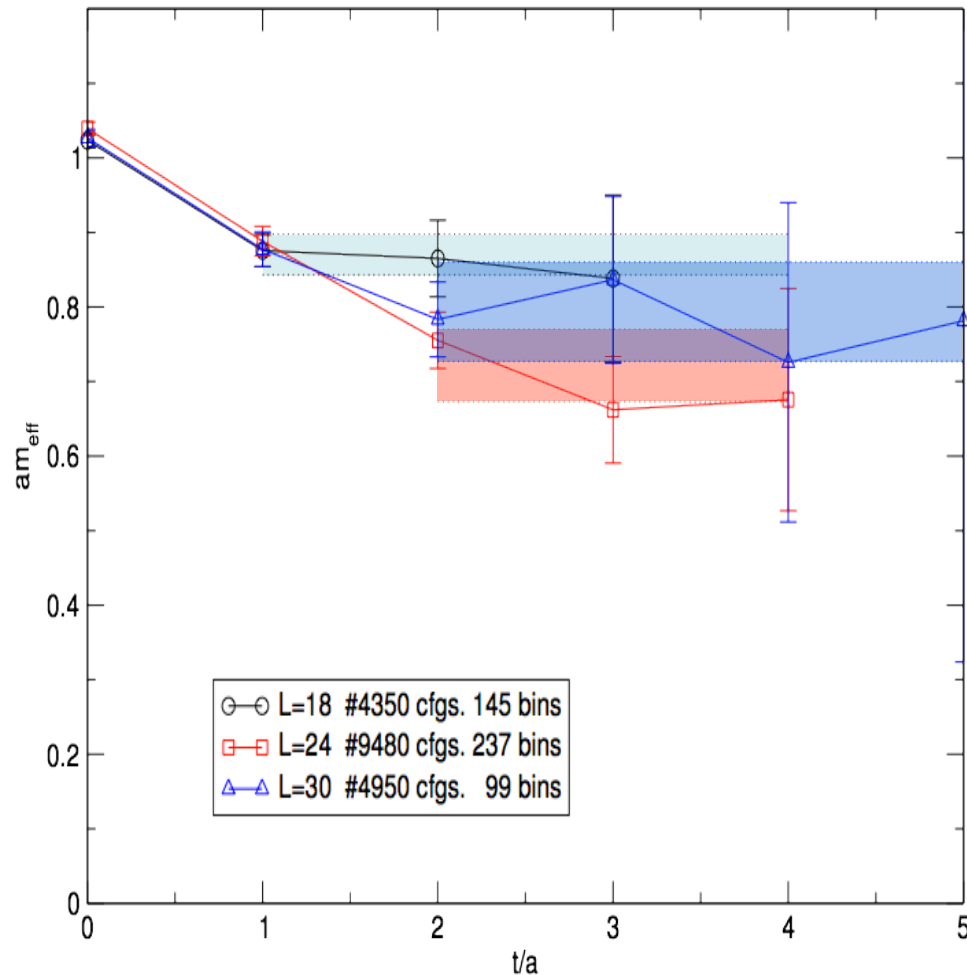
• Generalized eigenvalue problem: 
$$C_{ij}(t)v_j^{\alpha} = \lambda^{\alpha}v_i^{\alpha} \quad \Phi_{\alpha}(t) = \sum_{i=1}^n v_i^{\alpha} \mathcal{O}_i(t)$$

- The largest eigenstate (ground state) correlator fit

$$\langle \Phi_{\alpha}^{\dagger}(t) \Phi_{\alpha}(0) \rangle = |c_{\alpha}|^2 \left( e^{-m_{\alpha}t} + e^{-m_{\alpha}(T-t)} \right)$$

# Results: scalar glueball

Scalar glueball effective mass  $\beta=4.0$   $am_f=0.10$



$$m_{0^{++}} > m_{\pi}$$

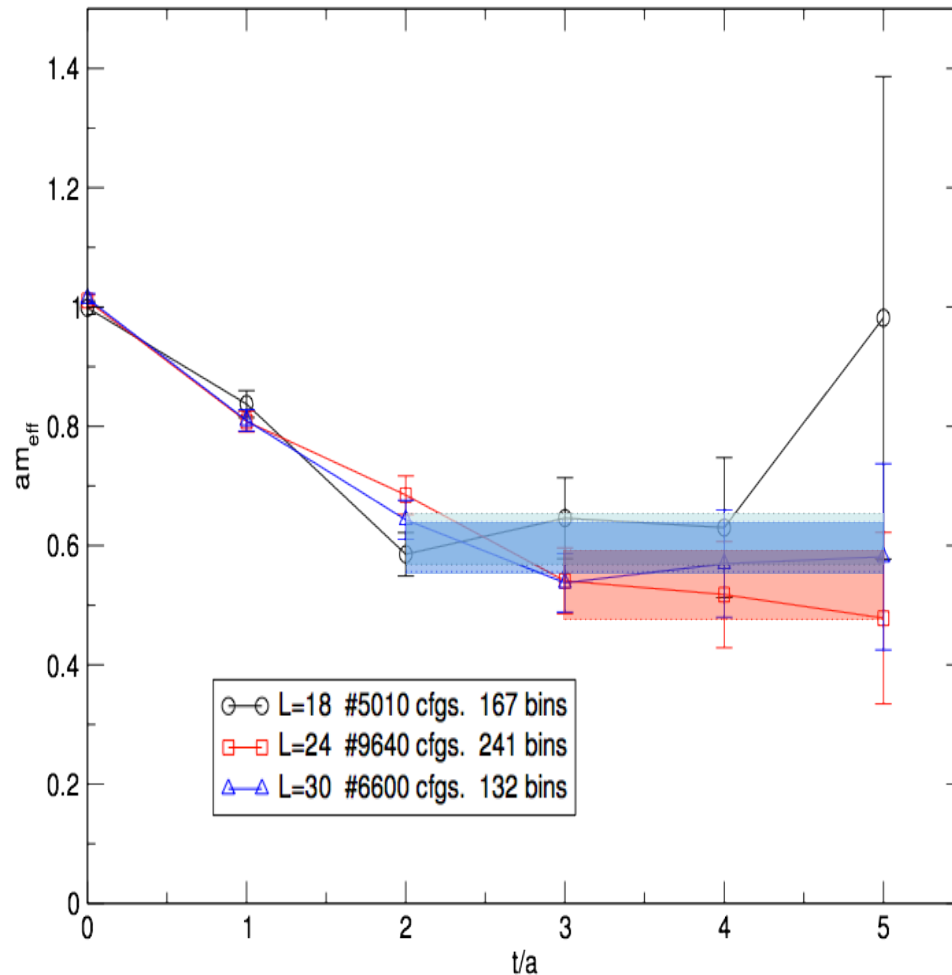
$$L=18 \rightarrow am=0.847(26)$$

$$L=24 \rightarrow am=0.722(49)$$

$$L=30 \rightarrow am=0.787(61)$$

# Results: scalar glueball

Scalar glueball effective mass  $\beta=4.0$   $am_f=0.08$



$$m_{0^{++}} \geq m_{\pi}$$

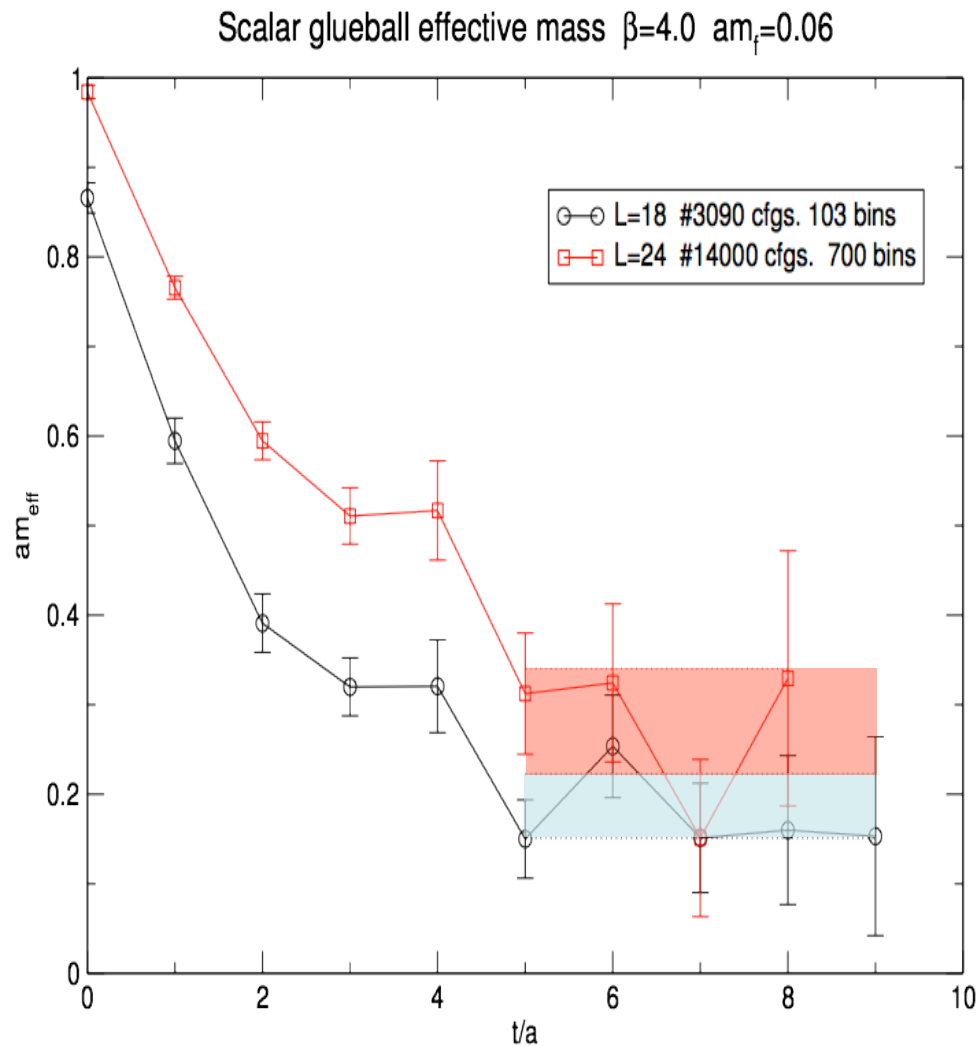
$$L=18 \rightarrow am=0.623(46)$$

$$L=24 \rightarrow am=0.534(58)$$

$$L=30 \rightarrow am=0.598(42)$$

Finite-size effects appear to be under control even when the bare fermion mass is lowered

# Results: scalar glueball



$$m_{0++} \leq m_{\pi}$$

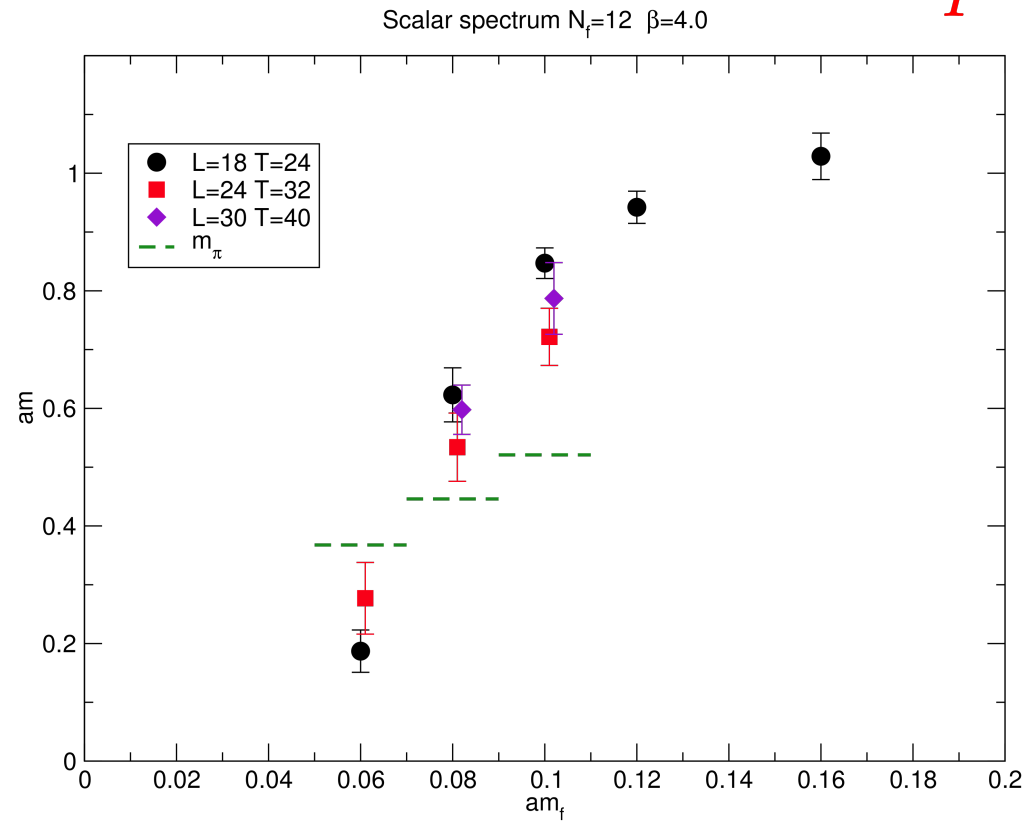
L=18  $\rightarrow$  am=0.187(36)

L=24  $\rightarrow$  am=0.277(61)

L=30  $\rightarrow$  now calculating

The state dominating at large time separation is lighter than the pion

*preliminary*

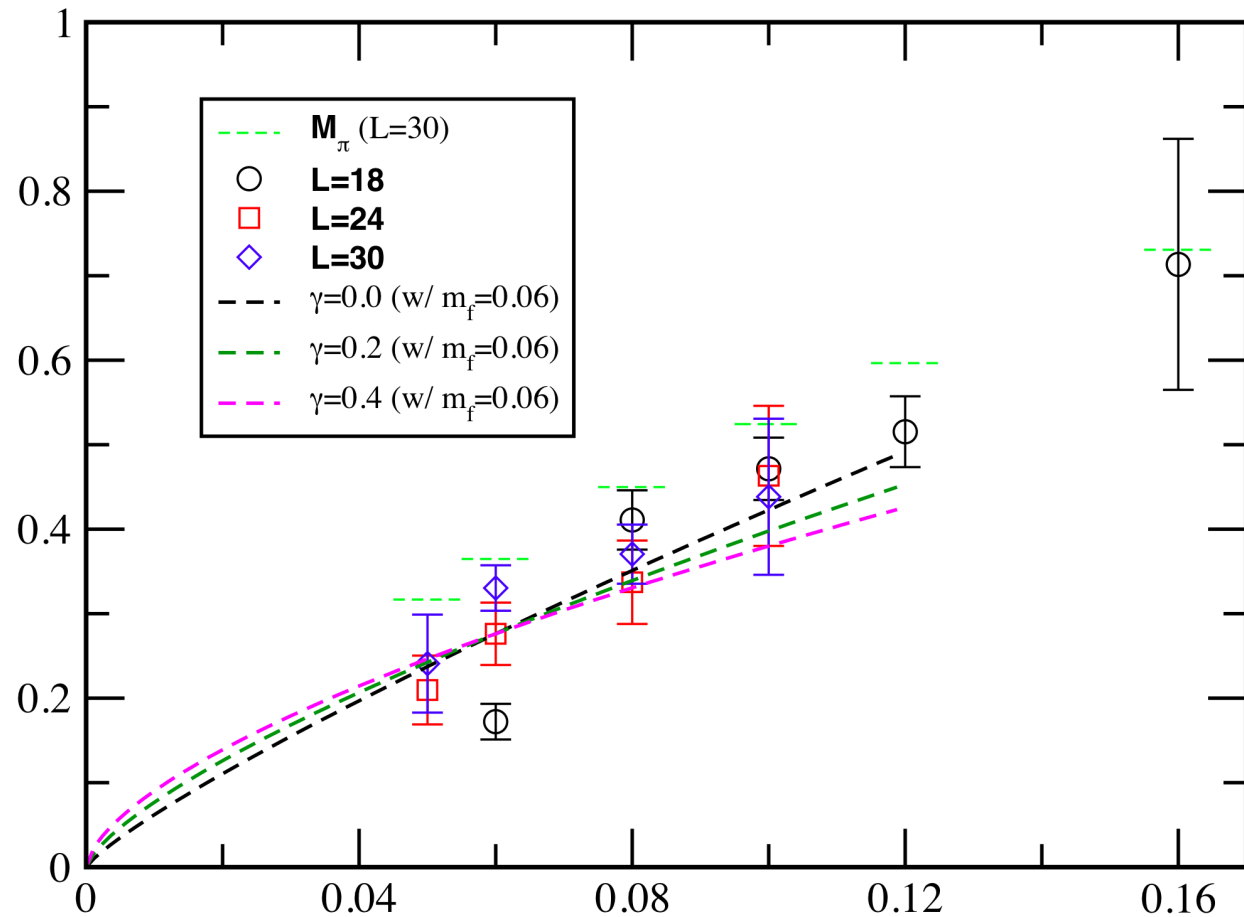


- Fermion mass dependence in glueball mass is observed.
- Glueball operator in large mass region is noisy.
- Glueball mass at  $m_f=0.06$  is smaller than pion mass.

Discussion(I)

Hyperscaling

# Fermion mass dependence (fermion scalar)



$$m_\sigma = C m_f^{1/(1+\gamma)}$$

with input  $m_\sigma$  at  $m = 0.06, L = 24$

- Difficult to precisely determine the value of gamma
- Consistent with hyperscaling observed in pion.

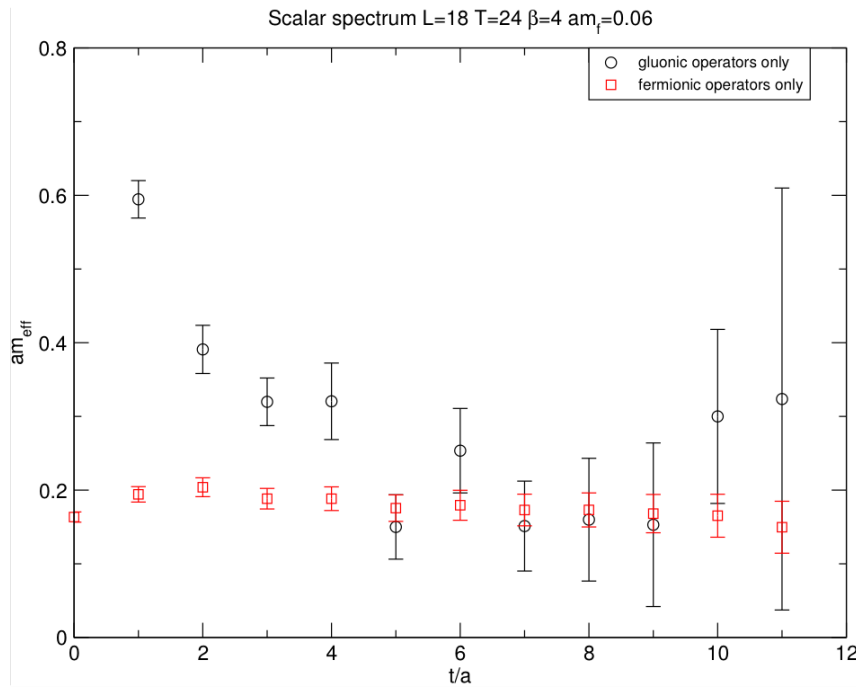
# Discussion(II)

Comparison of both observables

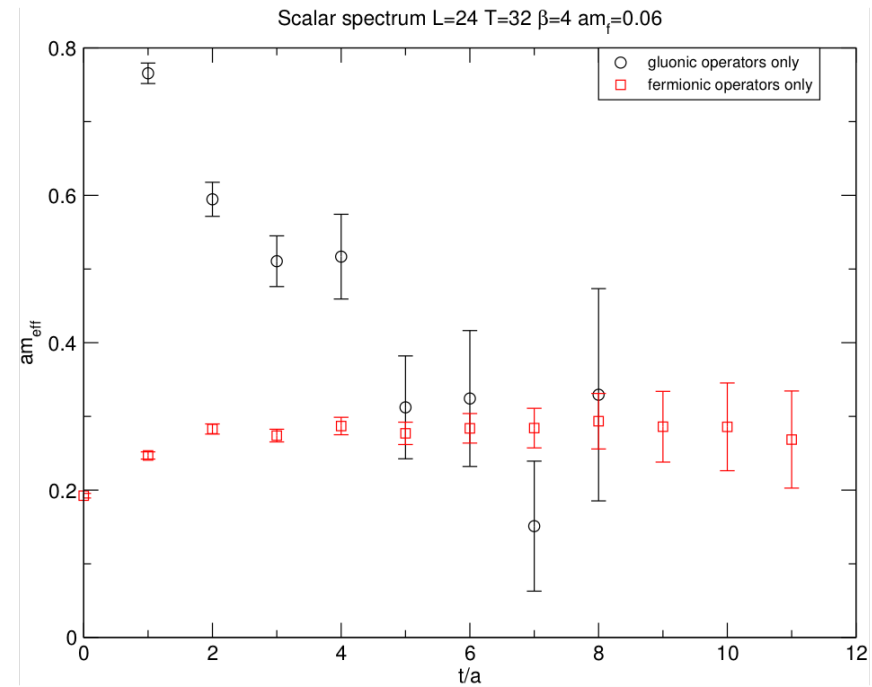


# Results: comparison with gluonic and fermionic observables, $m=0.06$

## L=18



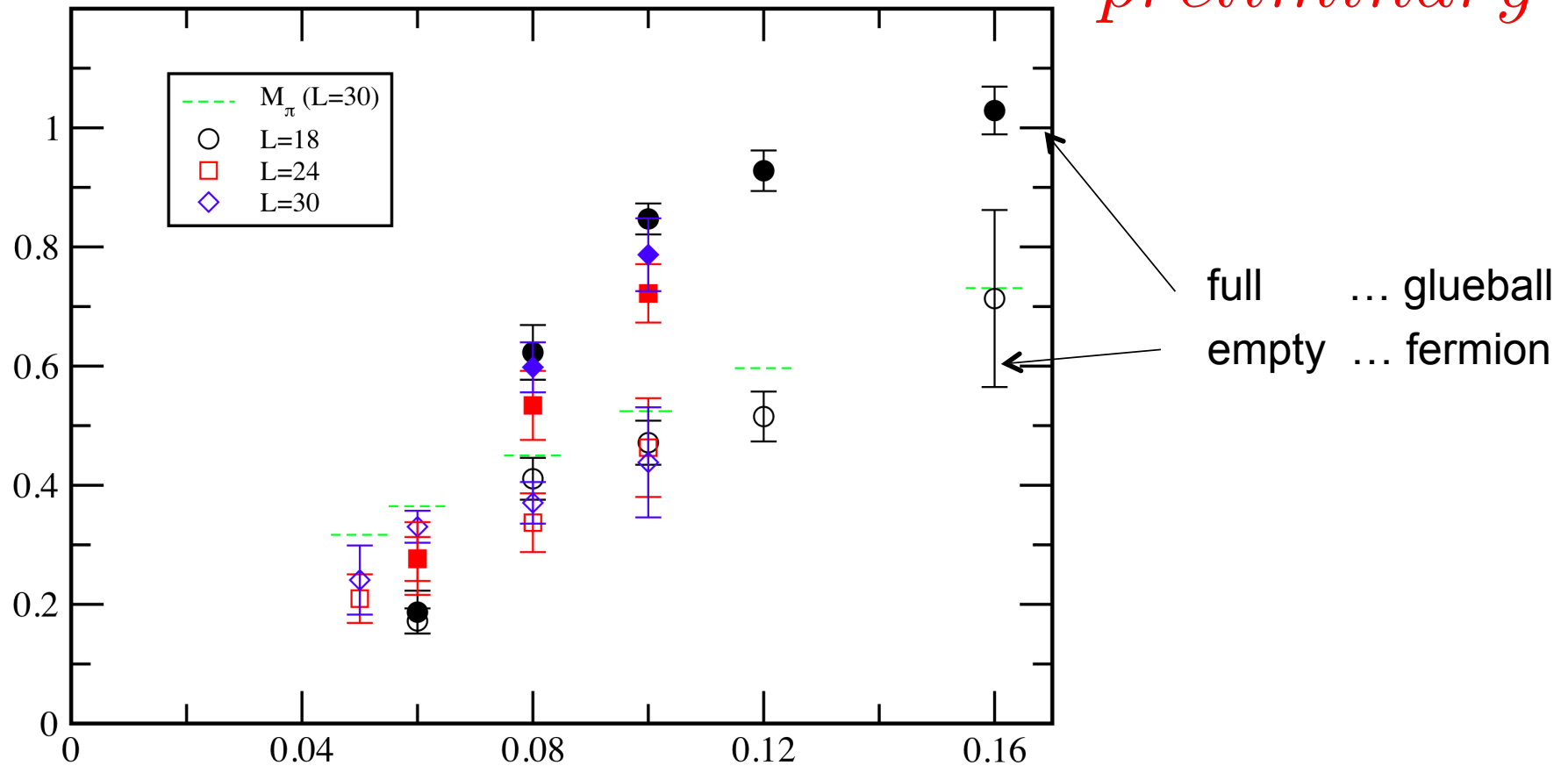
## L=24



Both results from gluonic and fermionic observables are consistent with each other. (and lighter than other mesons)

# Comparison of both the results

*preliminary*



- In smaller fermion mass region, both the results are consistent with each other.
- In larger fermion mass region, there is a difference in glueball and fermionic observables. (Effective mass plateau is not good at larger mass region.)

# Why difference in larger mass region?

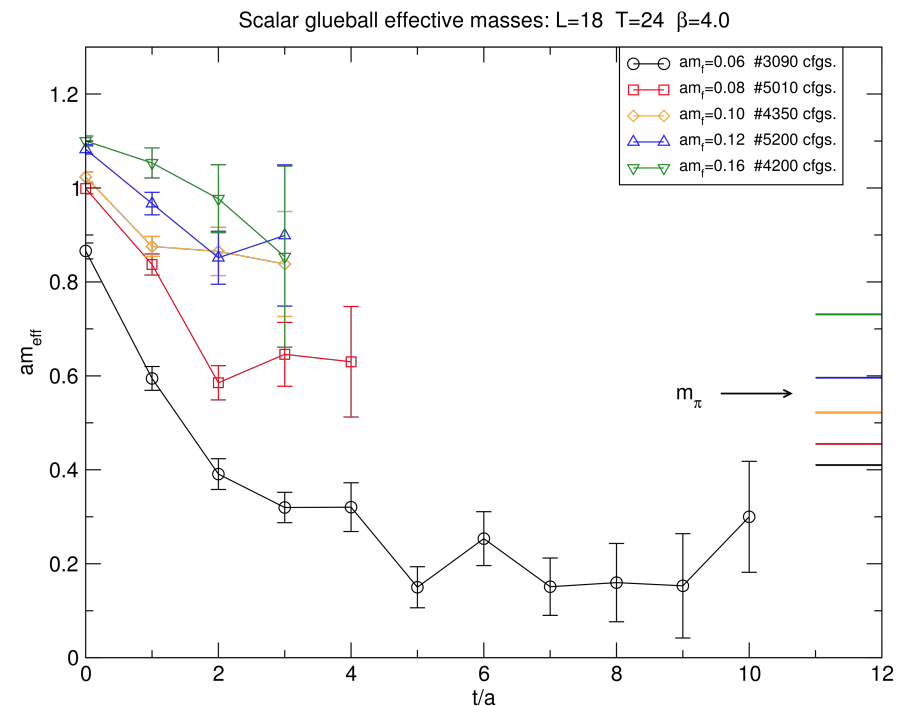
Both gluonic and fermionic operators have same quantum number:  $0^+$   
(Both can couple with ground state)  $\rightarrow$  same mass should be obtained.

possible reasons:

- difficult to get plateau at larger fermion mass region.
- Fermionic operator is better than gluonic operator to overlap with the ground mass eigenstate.

$\rightarrow$  Need large time separation to get the ground state in gluonic op.

$\rightarrow$  **More statistics!**



# Summary

- Large flavor SU(3) gauge theory is being investigated.
- In this talk, We focus on the  $N_f=12$  case.

- We measure the flavor singlet scalar mass

We obtain consistent results from gluonic and fermionic operators at small mass ( $m=0.06$ ).

The ground state scalar is lighter than pion.

- It is interesting to survey mixing of fermion and gluon operators
- $N_f=12$  favor conformal gauge theory.

How about other # of fermions??

-> e.g. 8 flavor case, talk by Kurachi san (Next!).

END  
Thank you